数理解析研究所講究録
1150巻 2000年 p.165

GEOMETRIC STRUCTURES AND DIFFERENTIAL EQUATIONS ON FILTERED MANIFOLDS

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A *filtered manifold* (M, F) is a differential manifold M endowed with a filtration $F = \{F^p\}_{p\in\mathbb{Z}}$ of the tangent bundle TM of M satisfying the following conditions:

(1) Each F^P is a subbundle of TM and $F^P \subset F^{p+1}$.
(2) $F^0 = 0$, and $\bigcup F^p = TM$.
(3) $[\underline{F}^p, \underline{F}^q] \subset \underline{F}^{p+q}$, where $\underline{F}^{\cdot}$ denotes the sheaf of the sections of $F^{\cdot}$.

Let (M, F) be a filtered manifold and x be a point in M. Denoting by $F_x^{\cdot}$ the fiber of $F^{\cdot}$ over x and putting $gr_pF_x = F_x^p/F_x^{p-1}$, we form a graded vector space

$$grF_x = \bigoplus gr_pF_x.$$

This vector space has a natural Lie algebra structure induced from the bracket operation of vector fields and satisfies:

$$[gr_pF_x, gr_qF_x] \subset gr_{p+q}F_x.$$

Thus grF_x turns out to be a nilpotent graded Lie algebra and may be regarded as a tangent algebra to (M, F) at x.

We call *nilpotent geometry and nilpotent analysis* studies of geometric structures and differential equations based on these tangent nilpotent Lie algebras.

The nilpotent geometry has proved to be very fruitful: It gives us, on one hand, unified view points and on the other hand, refined method to study various geometric structures.

A systematic study of differential equations on a filtered manifold (M, F), on the basis of *weighted orders* of differential operators associated with grF, gives rise to a non-trivial generalization of Cartan-Kähler theorem, a general existence theorem of analytic solutions to system of non-linear analytic partial differential equations possibly with singularities.