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Cantor bouquet of holomorphic stable manifolds for a periodic indeterminate point

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Abstract

In the rational dynamics of the complex projective plane, we construct a full-2-shift family of holomorphic stable manifolds of a periodic indeterminate point with two periodic orbits.

1 Introduction

The aim of this paper is a geometric study of the local dynamics of a rational mapping $\varphi : \mathbf{PC}^2 \to \mathbf{PC}^2$ with a periodic indeterminate point $Q_0 \in \mathbf{PC}^2$. A point Q_0 is called an indeterminate point of φ if the intersection of the closures $\overline{\varphi(U)}$, where $U \subset \mathbf{PC}^2$ runs a neighborhood of Q_0, is not a point.

A periodic indeterminate point naturally arises in the dynamics of Newton's method as a multiple root of a system of equations. Newton's method for the system of polynomial equations $F(x,y) = (F_1(x,y), F_2(x,y)) = (0,0)$ is defined by the rational map NF : $(x,y) \mapsto (x,y) - (DF_{(x,y)})^{-1}F(x,y)$. If F has a multiple root (x_0, y_0), i.e., $F(x_0,y_0) = 0$ and $\det DF_{(x_0,y_0)} = 0$, then the image $NF(x_0,y_0)$ is not a single-point but an algebraic curve that passes through (x_0,y_0). A multiple root is thus an 'indeterminate attractor.' An open problem of geometric dynamics is to give a complete description of the local convergence of Newton's method about a multiple root.

In this paper we study a general periodic indeterminate point Q_0 in the plane with the simplest indeterminancy. If a point on the image-curve returns to Q_0 and satisfies a stability condition, then the periodic orbit has a local holomorphic stable manifold. If, moreover, two points q_1, q_2 on the image-curve return to Q_0, the local stable set is not two curves but a family of holomorphic curves indexed by the full-2-shift $\Sigma(2) = \{0,1\}^{\mathbf{N}}$, which we call a Cantor bouquet of holomorphic stable manifolds. The shift operator acts naturally on the family. In the case that the orbits of q_1 and q_2 come only close to Q_0, we still have the Cantor family of holomorphic curves as the maximal local invariant set.

The local transformation that generates the Cantor bouquet seems like a couple of pieces of 'dango', a traditional sweet of the author's country, pierced by a bamboo stick. It can also be compared to 'barbecue'. It is defined on two (once-punctured) polydisks and it maps each of them homeomorphically onto a long-and-narrow region that intersects through them.

The Cantor bouquet is obtained by defining two contraction mappings in the function space. Our argument of the construction of each holomorphic curve is parallel to the standard unstable manifold theorem for a hyperbolic fixed point in [1]. A keypoint is that the blow-down transformation is naturally super-contracting in one direction.

Numerical experiments are given at the end of the paper.

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2 Result

Here we state the main result.

In the complex projective plane $\mathbf{PC}^2$, we choose a point Q_0. Let $\pi : E \to \mathbf{PC}^2$ be a blow-up over Q_0, where $\pi^{-1}(Q_0) \simeq \mathbf{PC}^1$. Let U be a neighborhood of Q_0 and let $V = \pi^{-1}(U)$. Consider local coordinates $(x,y)_{\mathbf{PC}^2}$ and $(u,v)_E$ such that $Q_0 = (0,0)_{\mathbf{PC}^2}$ and $\pi : (u,v)_E \mapsto (u,uv)_{\mathbf{PC}^2}$.

Let $i = 1,2$ throughout this paper. Choose two points $q_i = (0,\alpha_i)_E \in \pi^{-1}(Q_0)$ and their neighborhoods $V_i \ni q_i$. Let $\mathbf{B}_0 = \bar{\mathbf{D}}(0,\rho) \times \bar{\mathbf{D}}(0,r_0) \subset U$, $\mathbf{B}_i = \bar{\mathbf{D}}(0,\rho) \times \bar{\mathbf{D}}(\alpha_i, r) \subset V_i$ be closed polydisks with $\rho, r_0, r > 0$. For $M > 0$, let $\mathbf{L}_i = \mathrm{Lip}_M(\bar{\mathbf{D}}(0,\rho), \bar{\mathbf{D}}(\alpha_i, r))$ be the set of all Lipschitz functions $\tau : \bar{\mathbf{D}}(0,\rho) \to \bar{\mathbf{D}}(\alpha_i, r)$ with Lipschitz constants $\mathrm{Lip}(\tau) \leq M$, which is a bounded and complete metric space under the uniform topology.

Let $\Sigma(2) = \{1,2\}^{\mathbf{N}}$ be a Cantor set and denote its element by $w = w_0 w_1 w_2 \cdots \in \Sigma(2)$, $w_k \in \{1,2\}$, $0 \leq k < \infty$. Let $s : \Sigma(2) \to \Sigma(2)$ be the shift operator $s(w_0 w_1 w_2 \cdots) = w_1 w_2 w_3 \cdots$. Let $\mathrm{C}(\Sigma(2), \mathbf{L}_1 \cup \mathbf{L}_2)$ be the space of continuous maps $\sigma : \Sigma(2) \to \mathbf{L}_1 \cup \mathbf{L}_2$.

Definition 1 *A 'Cantor family of holomorphic curves' is a continuous injective map $\sigma : \Sigma(2) \to \mathbf{L}_1 \cup \mathbf{L}_2$ such that the restriction to the open disk $\sigma(w)|\mathbf{D}(0,\rho) : \mathbf{D}(0,\rho) \to \mathbf{D}(\alpha_{w_0}, r)$ is holomorphic for each $w \in \Sigma(2)$. It is called a 'Cantor bouquet' if* $\mathrm{graph}(\sigma(w)) \cap \mathrm{graph}(\sigma(w')) = \{q_{w_0}\}$ *for any $w, w' \in \Sigma(2)$ with $w_0 = w'_0$. Its 'graph' is the union of the curves $G(\sigma) = \bigcup_{w \in \Sigma(2)} \mathrm{graph}(\sigma(w)) = \left\{\sigma(w)(z) \mid (w,z) \in \Sigma(2) \times \bar{\mathbf{D}}(0,\rho)\right\} \subset \mathbf{B}_1 \cup \mathbf{B}_2$.*

Definition 2 *Let $f : V_1 \cup V_2 \to V$ be a mapping that may have an indeterminate point in each V_i. But suppose that the inverse f^{-1} has two well-defined differentiable branches $V \to V_i$. Let σ be a Cantor family of holomorphic curves defined above. We say that (i) σ is 'invariant' under f if* $\mathrm{graph}(\sigma(w)) = \mathbf{B}_{w_0} \cap f^{-1}(\mathrm{graph}(\sigma(s(w))))$ *for each $w \in \Sigma(2)$, and (ii) $G(\sigma)$ is the 'maximal local invariant set' of f in $\mathbf{B}_1 \cup \mathbf{B}_2$ if $G(\sigma) = \bigcap_{n=0}^{\infty} f^{-n}(\mathbf{B}_1 \cup \mathbf{B}_2)$.*

Suppose that σ is a Cantor bouquet. We say that (iii) $G(\sigma)$ is the 'local stable set' of $\{q_1, q_2\}$, denoted by $W^s_{\mathrm{loc}}(\{q_1,q_2\})$, if $G(\sigma)$ is the maximal local invariant set of f in $\mathbf{B}_1 \cup \mathbf{B}_2$ and $f^n(z) \to \{q_1, q_2\}$ as $n \to \infty$ for every $z \in G(\sigma) \setminus \{q_1, q_2\}$.

Let $n_i > 0$ be integers. Let R be the space of rational maps $\varphi : \mathbf{PC}^2 \to \mathbf{PC}^2$ of a fixed degree ≥ 2 such that (i) Q_0 is an indeterminate point of φ, (ii) $\varphi\pi : E \to \mathbf{PC}^2$ is a well-defined differentiable map on V, (iii) there is no degenerate set passing through Q_0

(a degenerate set is a curve that has a Zariski open subset whose well-defined image is a point), and (iv) $\varphi^{n_i}\pi|V_i : V_i \to U$ is a diffeomorphism onto its image. For each $\varphi \in R$, we define the local dynamics $f_\varphi : V_1 \cup V_2 \to V$ by $f_\varphi|V_i = \pi^{-1}\varphi^{n_i}\pi : V_i \to V$.

Theorem 3 (Main Theorem) *Let $\varphi_0 \in R$. Suppose that $\varphi_0^{n_i}\pi(q_i) = Q_0$ and that the inverse $g_{0i} = (\varphi_0^{n_i}\pi)^{-1}$ has a Taylor expansion $g_{0i} : (x,y)_{\mathbf{PC}^2} \mapsto (a_i x + b_i y + \cdots, \alpha_i + c_i x + d_i y + \cdots)_E$ with $1 < |a_i + b_i\alpha_j|$. There exist $\rho, r_0, r, M > 0$ defining $\mathbf{L}_i$, and there exists a Cantor bouquet σ^{φ_0} of holomorphic curves invariant under f_{φ_0} such that $G(\sigma^{\varphi_0}) = W^s_{\mathrm{loc}}(\{q_1, q_2\})$. There exists a neighborhood $X \subset R$ of φ_0 and a continuous mapping $X \ni \varphi \mapsto \sigma^\varphi \in \mathrm{C}(\Sigma(2), \mathbf{L}_1 \cup \mathbf{L}_2)$ such that σ^φ is a Cantor family of holomorphic curves invariant under f_φ and $G(\sigma^\varphi)$ is the maximal local invariant set of f_φ for each φ.*

3 Proof

Here we prove the Main Theorem.

Let $i, j = 1, 2$ in the following. Let $p_1 : V \ni (u,v) \mapsto u \in \mathbf{C}$, $p_2 : V \ni (u,v) \mapsto v \in \mathbf{C}$, and $S_i : U \ni (x,y) \mapsto (a_i x + b_i y, \alpha_i + c_i x + d_i y) \in V$. Let $M_0 > 0$. Choose $M > 0$ and $\rho, r_0, r, \epsilon > 0$ such that $\rho(|\alpha_i| + r) \le r_0$, $M(\rho + \epsilon) + \epsilon \le r$, $\epsilon \le \rho(|a_i + b_i\alpha_j| - 1 - \delta)$, $\delta + \left|\frac{c_i + d_i\alpha_j}{a_i + b_i\alpha_j}\right|(1+\delta) \le M$, and $\rho C_0 < 1$, where $\delta = 2rC_1 + \epsilon C_2$, $C_0 = (M+1)(C_1 + \epsilon)$, $C_1 = \max(|b_1|, |b_2|, |d_1|, |d_2|)$, and $C_2 = \max(1, |\alpha_1| + 2r, |\alpha_2| + 2r)$. Choose a neighborhood $X \subset R$ of φ_0 such that $|p_1 g_i(0,0)| \le \epsilon$, $|p_2 g_i(0,0) - \alpha_i| \le \epsilon$ and $\mathrm{Lip}(g_i - S_i) \le \epsilon$ for $g_i = (\varphi^{n_i}\pi|V_i)^{-1}$ of $\varphi \in X$.

Fix a $\varphi \in X$. Let $\tau_j \in \mathbf{L}_j$. We are going to define two contractions Γ_{g_i} in the function space $\mathbf{L}_1 \cup \mathbf{L}_2$ by

$$\Gamma_{g_i}(\tau_j) = p_2 g_i \pi(\mathrm{id}, \tau_j)[p_1 g_i \pi(\mathrm{id}, \tau_j)]^{-1} \in \mathbf{L}_i.$$

Let $\tau_{j0} \in \mathbf{L}_j$ be a constant function $\tau_{j0}(u) = \alpha_j$. Since the mapping $u \mapsto u(\tau_j(u) - \alpha_j)$ has Lipschitz constant $\le 2r$, we have $\mathrm{Lip}(\pi(\mathrm{id}, \tau_j)) \le C_2$, and $\mathrm{Lip}(p_k S_i \pi(\mathrm{id}, \tau_j) - p_k S_i \pi(\mathrm{id}, \tau_{j0})) \le 2rC_1$, $k = 1, 2$.

Lemma 4 *The mapping $\Gamma_{g_i}(\tau_j) : \bar{\mathbf{D}}(0, \rho) \to \mathbf{C}$ is well-defined.*

(proof) For $k = 1, 2$,

$$\begin{aligned} & \mathrm{Lip}(p_k g_i \pi(\mathrm{id}, \tau_j) - p_k S_i \pi(\mathrm{id}, \tau_{j0})) \\ \le\ & \mathrm{Lip}(p_k)\mathrm{Lip}(g_i - S_i)\mathrm{Lip}(\pi(\mathrm{id}, \tau_j)) + \mathrm{Lip}(p_k S_i \pi(\mathrm{id}, \tau_j) - p_k S_i \pi(\mathrm{id}, \tau_{j0})) \\ \le\ & \epsilon C_2 + 2rC_1 = \delta. \end{aligned}$$

We can apply the Lipschitz inverse function theorem in [1], Appendix I. Thus $p_1 g_i \pi(\mathrm{id}, \tau_j)$ is a homeomorphism onto its image with Lipschitz constant

$$\mathrm{Lip}([p_1 g_i \pi(\mathrm{id}, \tau_j)]^{-1}) \le \frac{1}{|a_i + b_i\alpha_j| - \delta} < 1, \tag{1}$$

and the image $p_1 g_i \pi(\mathrm{id}, \tau_j)(\bar{\mathbf{D}}(0, \rho))$ contains the closed disk with center $p_1 g_i \pi(\mathrm{id}, \tau_j)(0)$ and radius $\rho(|a_i + b_i\alpha_j| - \delta)$, which contains $\bar{\mathbf{D}}(0, \rho)$. ∎

Lemma 5 *The graph transform $\Gamma_{g_i} : \mathbf{L}_j \to \mathbf{L}_i$ is well-defined.*

(proof) First,

$$\begin{aligned}
& |\Gamma_{g_i}(\tau_j)(u) - \alpha_i| \\
\leq\ & |\Gamma_{g_i}(\tau_j)(u) - \Gamma_{g_i}(\tau_j)(p_1 g_i \pi(\mathrm{id}, \tau_j)(0))| + |\Gamma_{g_i}(\tau_j)(p_1 g_i \pi(\mathrm{id}, \tau_j)(0)) - \alpha_i| \\
\leq\ & \mathrm{Lip}(\Gamma_{g_i}(\tau_j))\,|u - p_1 g_i \pi(\mathrm{id}, \tau_j)(0)| + |p_2 g_i \pi(\mathrm{id}, \tau_j)(0) - \alpha_i| \\
\leq\ & M(\rho + \epsilon) + \epsilon \leq r.
\end{aligned}$$

Since $\mathrm{Lip}(f^{-1} - g^{-1}) \leq \mathrm{Lip}(f^{-1})\mathrm{Lip}(g-f)\mathrm{Lip}(g^{-1})$ in general, we have $\mathrm{Lip}([p_1 g_i \pi(\mathrm{id}, \tau_j)]^{-1} - [p_1 S_i \pi(\mathrm{id}, \tau_{j0})]^{-1}) \leq \delta |a_i + b_i \alpha_j|^{-1}$. So

$$\begin{aligned}
& \mathrm{Lip}(\Gamma_{g_i}(\tau_j) - \Gamma_{S_i}(\tau_{j0})) \\
=\ & \mathrm{Lip}(p_2 g_i \pi(\mathrm{id}, \tau_j)[p_1 g_i \pi(\mathrm{id}, \tau_j)]^{-1} - p_2 S_i \pi(\mathrm{id}, \tau_{j0})[p_1 S_i \pi(\mathrm{id}, \tau_{j0})]^{-1}) \\
\leq\ & \mathrm{Lip}(p_2 g_i \pi(\mathrm{id}, \tau_j) - p_2 S_i \pi(\mathrm{id}, \tau_{j0}))\mathrm{Lip}([p_1 g_i \pi(\mathrm{id}, \tau_j)]^{-1}) \\
& + \mathrm{Lip}(p_2 S_i \pi(\mathrm{id}, \tau_{j0}))\mathrm{Lip}([p_1 g_i \pi(\mathrm{id}, \tau_j)]^{-1} - [p_1 S_i \pi(\mathrm{id}, \tau_{j0})]^{-1}) \\
\leq\ & \delta + \delta\,|c_i + d_i \alpha_j|\,|a_i + b_i \alpha_j|^{-1},
\end{aligned}$$

and thus $\mathrm{Lip}(\Gamma_{g_i}(\tau_j)) \leq \mathrm{Lip}(\Gamma_{g_i}(\tau_j) - \Gamma_{S_i}(\tau_{j0})) + \mathrm{Lip}(\Gamma_{S_i}(\tau_{j0})) \leq M$. ∎

Lemma 6 *The graph transform Γ_{g_i} is a contraction:*

$$\left\|\Gamma_{g_i}(\tau_j) - \Gamma_{g_i}(\tau_j')\right\| \leq \rho C_0 \left\|\tau_j - \tau_j'\right\|.$$

(proof) Consider a point $(u, v) \in \mathbf{B}_j$ with $p_1 g_i \pi(u, v) \in \bar{\mathbf{D}}(0, \rho)$. First,

$$\begin{aligned}
& |p_k g_i \pi(u, v) - p_k g_i \pi(u, \tau_j u)| \\
\leq\ & \mathrm{Lip}(p_k)\mathrm{Lip}(g_i - S_i)\,|\pi(u, v) - \pi(u, \tau_j u)| + |p_k S_i \pi(u, v) - p_k S_i \pi(u, \tau_j u)| \\
\leq\ & \epsilon\rho\,|v - \tau_j(u)| + C_1 \rho\,|v - \tau_j(u)|
\end{aligned}$$

for $k = 1, 2$. Since $p_2 g_i \pi(u, \tau_j u) = \Gamma_{g_i}(\tau_j)(p_1 g_i \pi(u, \tau_j u))$, we have

$$\begin{aligned}
& |p_2 g_i \pi(u, v) - \Gamma_{g_i}(\tau_j)(p_1 g_i \pi(u, v))| \\
\leq\ & |p_2 g_i \pi(u, v) - p_2 g_i \pi(u, \tau_j u)| + \mathrm{Lip}(\Gamma_{g_i}(\tau_j))\,|p_1 g_i \pi(u, \tau_j u) - p_1 g_i \pi(u, v)| \\
\leq\ & \rho(\epsilon + C_1)\,|v - \tau_j(u)| + M\rho(\epsilon + C_1)\,|v - \tau_j(u)| \\
=\ & \rho C_0\,|v - \tau_j(u)|. \qquad (2)
\end{aligned}$$

By taking $v = \tau_j'(u)$, we obtain the lemma. ∎

Let $\Gamma_{g_{w_0} \cdots g_{w_{n-1}}} = \Gamma_{g_{w_0}} \cdots \Gamma_{g_{w_{n-1}}}$ be the composite of the graph transforms. We define the mapping $\sigma^\varphi : \Sigma(2) \to \mathbf{L}_1 \cup \mathbf{L}_2$ by

$$\{\sigma^\varphi(w)\} = \bigcap_{n=1}^{\infty} \Gamma_{g_{w_0} \cdots g_{w_{n-1}}}(\mathbf{L}_{w_n}).$$

Lemma 7 *The mapping σ^φ is a Cantor family of holomorphic curves invariant under f_φ. The graph $G(\sigma^\varphi)$ is the maximal local invariant set of f_φ in $\mathbf{B}_1 \cup \mathbf{B}_2$. The mapping $X \ni \varphi \mapsto \sigma^\varphi \in \mathrm{C}(\Sigma(2), \mathbf{L}_1 \cup \mathbf{L}_2)$ is also continuous.*

For $\varphi = \varphi_0$, σ^{φ_0} is a Cantor bouquet and $G(\sigma^{\varphi_0}) = W^s_{\mathrm{loc}}(\{q_1, q_2\})$.

(proof) First, we have $\Gamma_{g_{w_0}}(\sigma^\varphi(s(w))) \in \Gamma_{g_{w_0}}\Gamma_{g_{w_1}\cdots g_{w_{n-1}}}(\mathbf{L}_{w_n})$. By the uniqueness of $\sigma^\varphi(w)$, we obtain $\Gamma_{g_{w_0}}(\sigma^\varphi(s(w))) = \sigma^\varphi(w)$. This implies the invariance of σ^φ.

For w, $w' \in \Sigma(2)$ with $w_k = w'_k$, $0 \le k \le n$, we have $\sigma^\varphi(w), \sigma^\varphi(w') \in \Gamma_{g_{w_0}\cdots g_{w_{n-1}}}(\mathbf{L}_{w_n})$, and hence $\|\sigma^\varphi(w) - \sigma^\varphi(w')\| \le 2r(\rho C_0)^n$. Thus σ^φ is continuous.

If $w \ne w' \in \Sigma(2)$, there exist n such that $w_n \ne w'_n$. But we have $\sigma^\varphi(w) \in \Gamma_{g_{w_0}\cdots g_{w_{n-1}}}(\mathbf{L}_{w_n})$, and the graph transform Γ_{g_i} is injective because $\pi|(V \backslash \pi^{-1}(Q_0))$ and g_i are diffeomorphisms. Thus $\sigma^\varphi(w) \ne \sigma^\varphi(w')$ and σ^φ is injective.

If $(u,v) \in \mathbf{B}_{w_0} \cap (g_{w_0}\pi \cdots g_{w_{n-1}}\pi)(\mathbf{B}_{w_n})$, there exists a sequence $(u_k, v_k) \in \mathbf{B}_{w_k}$, $0 \le k \le n$, such that $(u_0, v_0) = (u,v)$ and $g_{w_{k-1}}\pi(u_k, v_k) = (u_{k-1}, v_{k-1})$. By (2), we have $|v - \sigma^\varphi(w)(u)| \le 2r(\rho C_0)^n$. Taking $n \to \infty$, we obtain $(u,v) \in \mathrm{graph}(\sigma^\varphi(w))$.

Let $\mathbf{H}_i = \{\tau_i \in \mathbf{L}_i \mid \tau_i|\mathbf{D}(0,\rho) \text{ is holomorphic}\}$. Then $\mathbf{H}_1 \cup \mathbf{H}_2$ is a closed subset of $\mathbf{L}_i$ invariant under the graph transform: $\Gamma_{g_i}(\mathbf{H}_j) \subset \mathbf{H}_i$. Thus $\sigma^\varphi(w) \in \mathbf{H}_1 \cup \mathbf{H}_2$.

For $\varphi = \varphi_0$, we have $g_{0i}\pi(\mathbf{B}_j) \cap g_{0i}(\mathbf{B}_i) = \{q_i\}$. Thus σ^{φ_0} is a Cantor bouquet of holomorphic curves. By (1) and $g_{0i}\pi(q_j) = q_i$, we obtain $G(\sigma^{\varphi_0}) = W^s_{\mathrm{loc}}(\{q_1, q_2\})$.

The mapping $\varphi \mapsto \Gamma_{g_i}$ is continuous on X in the uniform topology. For two pairs of contraction mappings $(\Gamma_{g_1}, \Gamma_{g_2})$ and $(\Gamma_{g'_1}, \Gamma_{g'_2})$ with $\left\|\Gamma_{g_i} - \Gamma_{g'_i}\right\| < \epsilon$ on the bounded metric space $\mathbf{L}_1 \cup \mathbf{L}_2$ with contraction constants $< \rho C_0$, we obtain $\left\|\sigma^\varphi(w) - \sigma^{\varphi'}(w)\right\| \le \epsilon/(1 - \rho C_0)$ for each $w \in \Sigma(2)$. Thus $\varphi \mapsto \sigma^\varphi$ is continuous. ∎

4 Experiments

Here is an example of a periodic indeterminate point with two periodic orbits, having the Cantor bouquet of stable manifolds.

Let $\varphi_0 : (x,y) \mapsto (y^2 + x^3/2 - 6x^2y + 18xy^2 - 33y^3/2)/x^2, (xy + xy^2 - 3y^3)/x^2)$. The origin $Q_0 = (0,0)$ is an indeterminate point. Its image is the conic $x - y^2 = 0$ that passes through Q_0 itself. We also have $\varphi_0(4,2) = Q_0$. So there are two periodic orbits $Q_0 \mapsto [4,2,1] \mapsto Q_0$ (of period two) and $Q_0 \mapsto Q_0$ (of period one). This example also satisfies the conditions of our main theorem.

Figure 1 is the region $V'_1 = \{|x| \le 0.1, |y| \le 0.1\}$ in the real (x,y)-plane, in 1024×1024 bitmap image. Figure 2 is $V'_2 = \{|x - 4| \le 1, |y - 2| \le 1\}$. In Figure 1, the gray region represents the subset $V'_1 \cap \varphi_0^{-1}(V'_1 \cup V'_2)$; the black region is the union of the subsets $V'_1 \cap \varphi_0^{-1}(V'_1) \cap \varphi_0^{-2}(V'_1 \cup V'_2)$ with $V'_1 \cap \varphi_0^{-1}(V'_2) \cap \varphi_0^{-2}(V'_1) \cap \varphi_0^{-3}(V'_1 \cup V'_2)$.

In Figure 2, the gray region is the subset $V'_2 \cap \varphi_0^{-1}(V'_1)$; the black region is the subset $V'_2 \cap \varphi_0^{-1}(V'_1) \cap \varphi_0^{-2}(V'_1 \cup V'_2)$.

References

[1] Michael Shub, Global Stability of Dynamical Systems, Springer (1987).

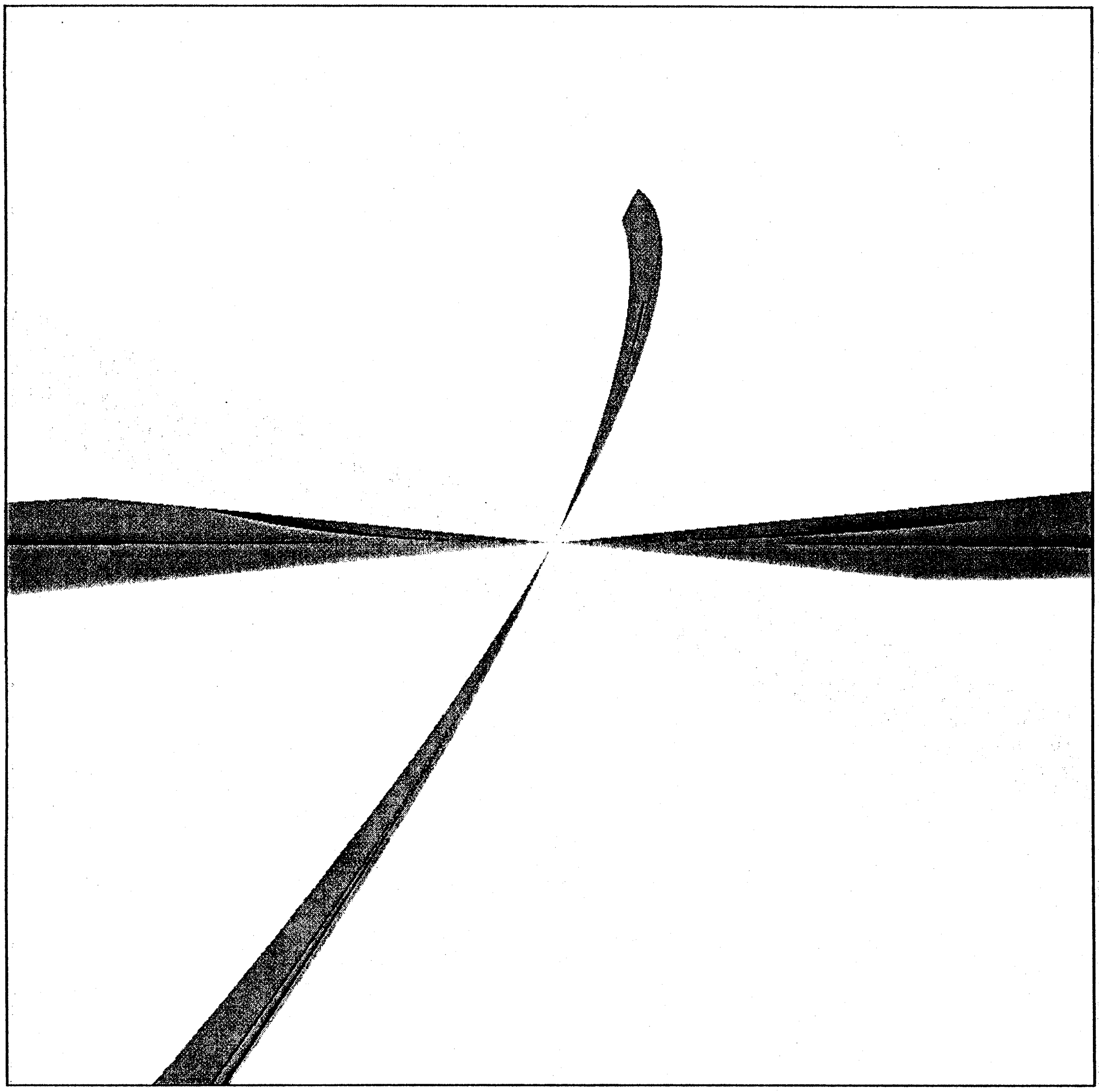

Fig. 1

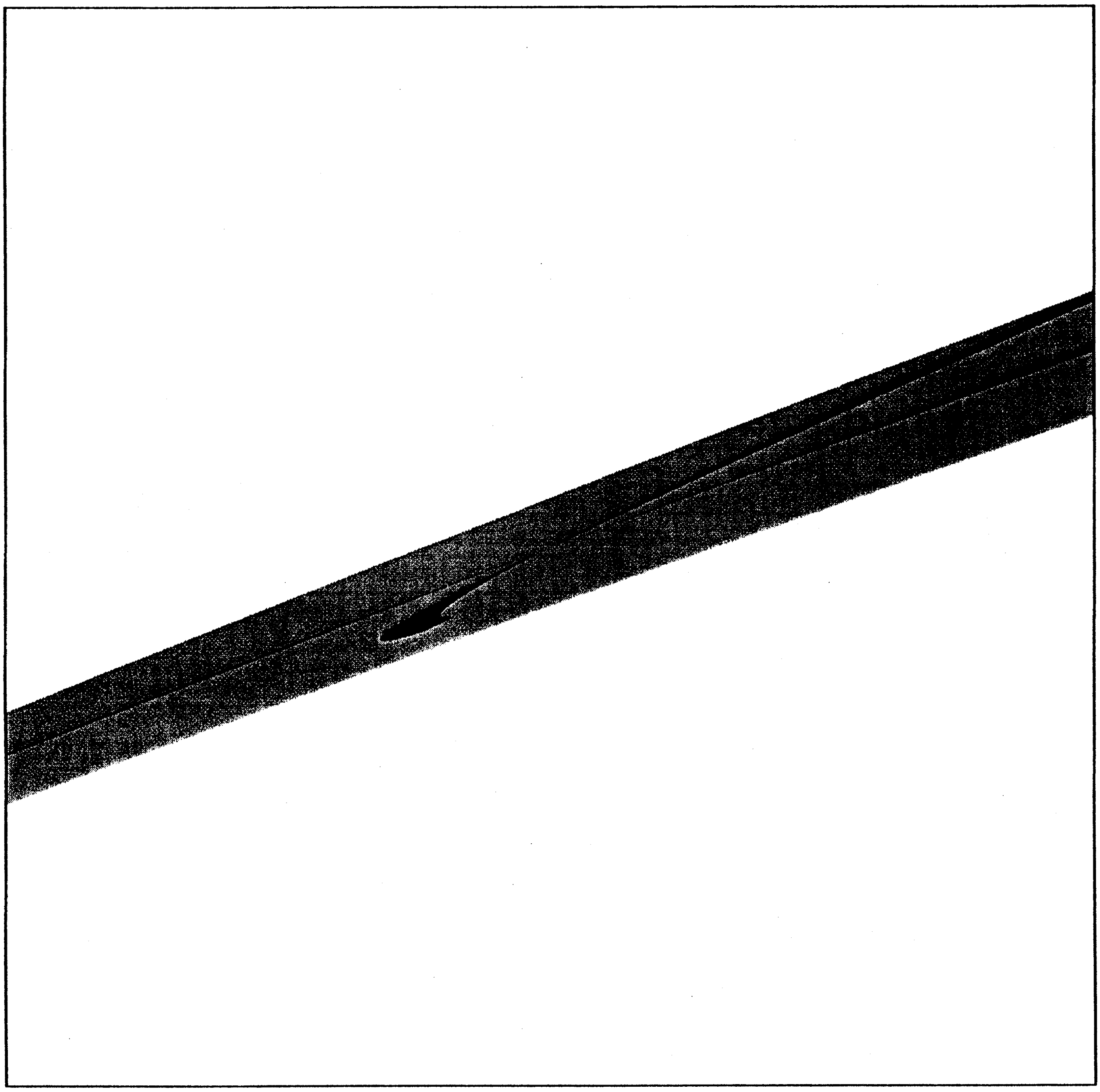

Fig. 2