数理解析研究所講究録 1217 巻 2001 年 158-164

Strong Normalization of Second Order Symmetric Lambda-mu Calculus

Yoriyuki Yamagata 山形頼之

Department of Mathematical Science, University of Tokyo
yoriyuki@ms.u-tokyo.ac.jp
東京大学数理科学研究科

Abstract. Parigot suggested symmetric structural reduction rules for application to μ-abstraction in [7] to ensure unique representation of data type. We prove strong normalization of second order $\lambda\mu$-calculus with these rules.

1 Introduction

Originally, $\lambda\mu$-calculus was defined to clarify correspondence between classical logic and control operators in functional programming languages. In this respect, $\lambda\mu$-calculus seems quite successful [3] [4] [5] [10]. In addition, Parigot was also motivated in [6] by possibility of witness extraction from classical proofs of Σ_1^0-sentences. Unfortunately, reduction rules of $\lambda\mu$-calculus seems not sufficient for this purpose. For example, let $A(x)$ be an atomic formula of arithmetic and $A'(x)$ be its code in second order predicate logic. We represent $\exists x.A(x)$ as $\forall X.\forall x(A(x) \to X) \to X$ in second order language, where X is a variable over propositions. We expect that a closed normal deduction of $\exists x.A'(x)$ somehow contains a unique first order term t such that $A(t)$ holds. However, consider the following situation. Suppose that $A(t)$ holds but $A(u)$ does not hold. Let M be a deduction of $A'(t)$ represented as $\lambda\mu$-terms. $\Lambda X.\lambda\alpha.\mu\beta.[\beta]\alpha u(\mu\gamma.[\beta]\alpha t M)$ is a closed and normal deduction of $\exists x A'(x)$ but apparently contains two terms t, u. Moreover, u is not a witness of $\exists x A(x)$. This suggests that we need additional reduction to extract the witness. In fact, Parigot proposed addition of new reduction rules $M(\mu\alpha.N) \Rightarrow \mu\beta.N[M^*/\alpha]$ to solve similar problem on normal forms of the natural number type. $N[M^*/\alpha]$ is defined by inductively replacing all occurrence of $[\alpha]L$ in N to $[\alpha]M(L[M^*/\alpha])$. We will prove that adding these rules suffices to ensure that a closed normal term of type $\exists x A(x)$ for an atomic $A(x)$ contains one and only one first order term t and $A(t)$ holds. Non-confluency of this calculus could be used to analyze non-determinism in classical logic.

Obviously, to use such calculus for witness extraction, we need normalization property of it. In addition, if we expect that reduction rules represent extraction algorithm of witness, strong normalization is desirable. However, symmetric nature of reduction of application to μ-abstraction seems to prevent obvious adaption of the proof of strong normalization of original $\lambda\mu$-calculus [8]. Luke Ong and Charles Stewart addressed strong normalization of call-by-value restriction

Lemma 5. *Assume that $(t_i)_{i\in I}, (\mathcal{A}_i)_{i\in I}$ is defined as Definition 6. If $M \in \bigwedge^1_{i\in I}\mathcal{A}_i$, $Mt_i \in \mathcal{A}_i$. Similarly, for $(T_j)_{j\in J}$ and $(\mathcal{A}_j)_{j\in J}$ defined as Definition 6, if $M \in \bigwedge^2_{j\in J}\mathcal{A}_j$, $MT_j \in \mathcal{A}_j$.*

Proof. The proof goes on the same line of that of Lemma 4. We concentrate the second order case. Let $D^{\omega_1}(S) = \bigwedge_{i\in I}\mathcal{A}_i$. Assume that κ is the least ordinal such that $t \in D^{\kappa}(S)$. We will prove that for all L such that $MT_j \Rightarrow_1 L$, $L \in \mathcal{A}_j$ holds, by induction on κ and $w(M)$.

The case where $L \equiv M'T_j$ and $M \Rightarrow_1 M'$. From induction hypothesis of $w(M')$, the thesis follows.

The case where $M \equiv \lambda X.M_1$ and $L \equiv M_1[T_j/X]$. Since $M \in \Pi^2_{j\in J}\mathcal{A}_j$, we have the thesis.

The case where $M \equiv \mu\alpha.M_1$ and $L \equiv \mu\beta.M_1[\mu\gamma.[\beta](\gamma T_i)/\alpha]$. Let $J \in \bullet\mathcal{A}_i$ and $K \in D^{\kappa_1}(S)$. By induction hypothesis on κ_1, we have $KT_i \in \mathcal{A}_i$. From arbitrariness of K and κ_1, it follows

$$\mu\gamma.[J](\gamma T_i) \in \bullet \bigcup_{\kappa_1<\kappa} D^{\kappa_1}(S).$$

Since M has a μ-form, $M \in \bullet\bullet\bigcup_{\kappa_1<\kappa} D^{\kappa_1}(S)$. We can infer $M_1[\mu\gamma.[J](\gamma T_i)/\alpha] \in \bot$. Hence we have $L \in \bullet\bullet\mathcal{A}_i$.

The rest of the proof runs similarly to the usual method of reducibility candidates. Let $\mathcal{T}$ be the set of all first order terms. $\mathcal{F}^n$ denotes the set of all functions from $\mathcal{T}^n$ to $\mathbf{R}$. Suppose that ξ is a map sending first order variables to first order terms, a predicate variable X^n to n-ary function from the set of first order terms to $\mathbf{R}$. We extend xi to be a map on the whole types using $\xi(\bot) = \bot$ and the following clauses.

$$\xi(\bullet A) = \bullet\xi(A) \tag{7}$$

$$\xi(A \to B) = \xi(A) \to \xi(B) \tag{8}$$

$$\xi(\forall x A) = \bigwedge^1_{t\in\mathcal{T}} \xi[t/x](A) \tag{9}$$

$$\xi(\forall X^n A) = \bigwedge^2_{f\in\mathcal{F}^n} \xi[f/X^n](A) \tag{10}$$

where $\xi[a/b]$ is defined as a map $\xi[a/b](b) = a$ and for $c \neq b$, $\xi[a/b](c) = \xi(c)$.

Proposition 2. *Let M be a term of type A. Assume that free first order variables of M are $x_1, \cdots, x_m$, free predicate variables of M are $X_1, \cdots, X_n$ and free variables of M are $\alpha_1^{A_1}, \cdots, \alpha_l^{A_l}$. Suppose that ξ is a map sending first order variables to first order terms, a predicate variable X^k to k-ary function from the set of first order terms to $\mathbf{R}$. For each $1 \leq i \leq n$ and $t_1, \cdots, t_k \in \mathcal{T}$ (k is the arity of $\xi(X_i)$) $\xi(X_i)t_1 \cdots t_n \in \mathbf{R}_{B_i[t_1/x_1,\cdots,t_n/x_k]}$. Let $N_j \in \xi(A_j)$ for*

7. *$\lambda x.M$ is a term of type $\forall xA$ for a term M of type A and a first order variable x. Variables of M do not contain x as a free variable.*
8. *Mt is a term of type $A[t/x]$ for a term M of type A and a first order term t.*
9. *$\lambda X^n.M$ is a term of type $\forall X^nA$ for a predicate variable X^n and a term M of type A. Variables of M do not contain X^n as a free variable.*
10. *$M\{T\}$ is a term of a type $A[T/X^n]$ for a term M of type $\forall X^nA$ and an abstraction term $T \equiv \lambda x_1 \cdots x_n.B$.*

Definition 3. Reduction rules *are the followings. Let β, γ, δ and y, Y be fresh variables.*

$$
\begin{array}{lll}
(\lambda_1)\ (\lambda\alpha.M)N & \Rightarrow M[N/\alpha] & \\
(\lambda_2)\ (\lambda x.M)t & \Rightarrow M[t/x] & \\
(\lambda_3)\ (\lambda X^n.M)T & \Rightarrow M[T/X^n] & \\
(\mu)\ [M]\mu\alpha.N & \Rightarrow N[M/\alpha] & [\mu\alpha.M]N \Rightarrow M[N/\alpha] \\
(\zeta_1)\ (\mu\alpha.M)N & \Rightarrow \mu\beta.M[\mu\gamma.[\beta](\gamma N)/\alpha] & M(\mu\alpha.N) \Rightarrow \mu\beta.N[\mu\gamma.[\beta](M\gamma)/\alpha] \\
(\zeta_2)\ (\mu\alpha.M)t & \Rightarrow \mu\beta.M[\mu\gamma.[\beta](\gamma t)/\alpha] & \\
(\zeta_3)\ (\mu\alpha.M)T & \Rightarrow \mu\beta.M[\mu\gamma.[\beta](\gamma T)/\alpha] &
\end{array}
$$

As usual, compatible closure of the rules above is called *one-step reduction relation* (denoted $\Rightarrow_1$) and reflexive and transitive closure of one-step reduction is called *reduction relation* (denoted $\Rightarrow$). $w(t)$ is the maximal length of sequences $t \Rightarrow_1 t_1 \cdots \Rightarrow_1 t_n$ if the maximum exists. Otherwise $w(t)$ is undefined. t is *strongly normalizable* if and only if $w(t)$ is defined.

Using μ and ζ-rules, Parigot's structural reduction [6] and symmetric one [7] mentioned in Section 1 can be derived.

$$(\mu\alpha.\ \ldots[\alpha]u\ldots)v \Rightarrow_\zeta \mu\beta.\ \ldots[\mu x.[\beta](xv)]u\ldots \Rightarrow_\mu \mu\beta.\ \ldots[\beta](uv)\ldots$$

$$u(\mu\alpha.\ \ldots[\alpha]v\ldots) \Rightarrow_\zeta \mu\beta.\ \ldots[\mu x.[\beta](ux)]v\ldots \Rightarrow_\mu \mu\beta.\ \ldots[\beta](uv)\ldots$$

If we understand $\bullet$ as the usual negation symbol, our ζ-rules resemble to Andou's reduction for $\perp_c$ [1].

By induction on a term, it is easy to prove the following lemma.

Lemma 1. *If M is a normal $\lambda\mu$-term,*

$$M \equiv \lambda\alpha.\cdots\lambda X.\cdots\lambda x.\alpha M_1 \cdots M_m \text{ or}$$
$$M \equiv \lambda\alpha.\cdots\lambda X.\cdots\lambda x.\mu\beta.[\gamma]M_1$$

where $\lambda\alpha.\cdots\lambda X.\cdots\lambda x.$ is an arbitrary sequence of λ-abstraction.

We assume that predicates and function symbols for primitive recursive arithmetic are included in our language. Then we can code second order Peano arithmetic in second order predicate logic. In particular, a Σ^0_1-sentence $\exists x.A$ is coded as $\exists x.N(x) \wedge A(x)$, where $N(x)$ is defined as $\forall X^1.X^1 0 \rightarrow \forall y(X^1 y \rightarrow X^1 Sy) \rightarrow X^1 y$ and $A(x)$ is atomic. Since we can deduce $\exists x.A(x)$ from $\exists x.N(x) \wedge A(x)$, we extract witness from a formula $\exists x.A(x)$.

Proposition 1. *Let $A(x)$ be an atomic formula and M be a normal closed term of type $\exists x.A(x)$. M contains one and only one first order term t and $A(t)$ holds.*

Proof. By Lemma 1, M has the form $E[\alpha M_1 \cdots M_m]$ where $E[\cdot]$ consists of abstraction and $[\cdot]\cdot$. By consideration on type, we have that α has a type $\forall x.(A \to X)$, M_1 is a first order term and M_2 is a term of type $A(t)$. Since $A(t)$ is atomic and does not contain X, we can see that M_2 consists of axioms alone. We have the thesis.

3 Strong normalization

Definition 4. *First we prepare several notations.*

1. *A term beginning with μ is called a μ-form.*
2. *For a set S of terms of type C, $Cl(S)$ is defined as the smallest set which satisfies clauses*
   (a) *$S \subset Cl(S)$ and contains all variables of type C.*
   (b) *$MN \in Cl(S)$ if $L \in Cl(S)$ for all L such that $MN \Rightarrow_1 L$.*
   (c) *$Mt \in Cl(S)$ if $L \in Cl(S)$ for all L such that $Mt \Rightarrow_1 v$ for a first order term t.*
   (d) *$MT \in Cl(S)$ if $L \in Cl(S)$ for all L such that $MT \Rightarrow_1 v$ for an abstraction term T.*
3. *The set of strong normalizable terms of type $\bot$ is also denoted $\bot$.*
4. *For a set S of terms of type $C \neq \bot$,*

$$\bullet S := \{\mu\alpha.M | \forall N \in S, M[N/x] \in \bot\}$$

   where α is a variable of type C and M has a type $\bot$.
5. *the operator $D(\mathcal{X})$ is defined as $Cl(\mathcal{X} \cup \bullet\bullet \mathcal{X})$. Note that $\bullet\bullet$ and hence D are monotone operators. For ordinal κ,*

$$D^\kappa(\mathcal{X}) := D(\bigcup_{\tau<\kappa} D^\tau(\mathcal{X})).$$

Definition 5 (Reducibility candidates). *Let ω_1 be the first uncountable ordinal and A be a proposition. Let S be a set of strong normalizable terms of type A. Suppose S does not contain a μ-form and S is closed under reduction relation. Then, a set $D^{\omega_1}(S)$ is called a reducibility candidate of the proposition A. Note that from monotonicity of D, a reducibility candidate is a fixed point of D. The set of candidates of the proposition A is denoted by $\mathbf{R}_A$. $\mathbf{R}$ is the union of all $\mathbf{R}_A$.*

Lemma 2. *Let $\mathcal{R}$ be a candidate $D^{\omega_1}(S)$. $\mathcal{R} = Cl(S \cup \bullet\bullet \mathcal{R})$.*

Proof. Since $\mathcal{R}$ is a fixed point of D, we have $\mathcal{R} = Cl(\mathcal{R} \cup \bullet\bullet \mathcal{R}) \supset Cl(S \cup \bullet\bullet \mathcal{R})$, while $D^\kappa(S) \subset Cl(S \cup \bullet\bullet \mathcal{R})$.

Lemma 3. *For $M \in \bullet\mathcal{R}$ and $N \in \mathcal{R}$, $[M]N \in \bot$.*

Proof. It suffices to prove that all L such that $[M]N \Rightarrow_1 L$ are strong normalizable. We consider each possibilities of the reduction of $[M]N$.

The case where L has the form $[M']N'$ and $M \Rightarrow M'$ and $N \Rightarrow N'$. The thesis follows from induction hypothesis on $w(M) + w(N)$.

The case where $M \equiv \mu\alpha.M_1$ and $L \equiv M_1[N/\alpha]$. By the hypothesis $M \in \bullet\mathcal{R}$.

The case where $N \equiv \mu\alpha.N_1$ and $L \equiv N_1[M/\alpha]$. By Lemma 2, N should be an element of $\bullet\bullet\mathcal{R}$. We have the thesis.

Definition 6. *Let $\mathcal{A} \in \mathbf{R}_A$ and $\mathcal{B} \in \mathbf{R}_B$. Assume that $(t_i)_{i\in I}$ is a non-empty family of first order terms and $(T_j)_{j\in J}$ is a non-empty family of abstraction terms. Further, $\mathcal{A}_i$ is a candidate of the proposition $A[t_i/x]$ for each $i \in I$ and $\mathcal{A}_j$ is a candidate of the proposition $A[T_j/X]$ for each $j \in J$. Candidates $\mathcal{A} \to \mathcal{B}$ $\bigwedge^1_{i\in I}\mathcal{A}_i$, $\bigwedge^2_{j\in J}\mathcal{A}_j$ are defined by the following steps.*

$$L(\mathcal{A},\mathcal{B}) := \{\lambda\alpha^A.M | \forall N \in \mathcal{A}, M[N/\alpha^A] \in \mathcal{B}\} \tag{1}$$

$$\Pi^1_{i\in I}\mathcal{A}_i := \{\lambda x.M | \forall i \in I, M[t_i/x] \in \mathcal{A}_i\} \tag{2}$$

$$\Pi^2_{j\in J}\mathcal{A}_j := \{\lambda X.M | \forall j \in J, M[T_j/X] \in \mathcal{A}_j\} \tag{3}$$

$$\mathcal{A} \to \mathcal{B} := D^{\omega_1}(L(\mathcal{A},\mathcal{B})) \tag{4}$$

$$\bigwedge^1_{i\in I}\mathcal{A}_i := D^{\omega_1}(\Pi^1_{i\in I}\mathcal{A}_i) \tag{5}$$

$$\bigwedge^2_{j\in J}\mathcal{A}_i := D^{\omega_1}(\Pi^2_{j\in J}\mathcal{A}_i) \tag{6}$$

Lemma 4. *Let $\mathcal{A} \in \mathbf{R}_A$ and $\mathcal{B} \in \mathbf{R}_B$. If $M \in \mathcal{A} \to \mathcal{B}$ and $N \in \mathcal{A}$, $MN \in \mathcal{B}$.*

Proof. Let $\mathcal{A} = D^{\omega_1}(S)$. Assume that κ is the least ordinal such that $M \in D^\kappa(L(\mathcal{A},\mathcal{B}))$ and τ is the least ordinal such that $N \in D^\tau(S)$. By induction on the natural sum $\kappa \oplus \tau$ and $w(M) + w(N)$, we will prove that if $MN \Rightarrow_1 L$, $L \in \mathcal{B}$, which is the exact condition of $MN \in \mathcal{B}$.

The case $L \equiv M'N'$ and either $M \Rightarrow_1 M'$ and $N \equiv N'$ or $M \equiv M'$ and $N \Rightarrow_1 N'$. The thesis follows from induction hypothesis on $w(M) + w(N)$.

The case $M \equiv \lambda\alpha.M_1$ and $L \equiv M_1[N/\alpha]$. Since $M \in L(\mathcal{A},\mathcal{B})$, we have the thesis.

The case where M has a form $\mu\alpha.M_1$ and L is obtained from reduction of the outermost redex. Then, L has a form $\mu\beta.M_1[\mu\gamma.[\beta](\gamma N)/\alpha]$. Let $J \in \bullet\mathcal{B}$, $K \in D^{\kappa_1}(L(\mathcal{A},\mathcal{B}))$ for $\kappa_1 < \kappa$. By induction hypothesis on κ_1, we have $KN \in \mathcal{B}$. It follows $[J](KN) \in \bot$. From arbitrariness of K and κ_1, $\mu\gamma.[J](\gamma N) \in \bullet\bigcup_{\kappa_1<\kappa} D^{\kappa_1}(L(\mathcal{A},\mathcal{B}))$ follows. Since M is a μ-form, $M \in \bullet\bullet\bigcup_{\kappa_1<\kappa} D^{\kappa_1}(L(\mathcal{A},\mathcal{B}))$. We can infer $M_1[\mu\gamma.[J](\gamma N)/\alpha] \in \bot$. Since $J \in \bullet\mathcal{B}$, we have $L \in \bullet\bullet\mathcal{B}$. Now, from $\bullet\bullet\mathcal{B} \subset \mathcal{B}$, the thesis follows.

The case where N has a form $\mu\alpha.N_1$ and L is obtained from reduction of the outermost redex. L has a form $\mu\beta.N_1[\mu\gamma.[\beta](M\gamma)/\alpha]$. Let $J \in \mathcal{B}$ and $K \in D^{\tau_1}(S)$ for $\tau_1 < \tau$. From induction hypothesis on τ_1, we have $MK \in \mathcal{B}$. Similarly as above, it follows $\mu\gamma.[J](M\gamma) \in \bullet\bigcup_{\tau_1<\tau} D^{\tau_1}(S)$. Since N has a μ-form, $N \in \bullet\bullet\bigcup_{\tau_1<\tau} D^{\tau_1}(S)$. We have $N_1[\mu\gamma.[\beta](M\gamma)/\alpha] \in \bot$ and hence, $L \in \mathcal{B}$.

of this calculus [5]. Their calculus $\lambda\mu_v$ is confluent, hence useful as a programming language. However, imposing reduction strategy seems to be an alien idea in a logical calculus, and non-determinism is lost.

Barbanera and Berardi proved strong normalization of a non-deterministic calculus for propositional classical logic using fixed point construction for reducibility candidates [2]. We will prove strong normalization of second order $\lambda\mu$-calculus with the rules above based on this method.

2 Symmetric $\lambda\mu$-calculus

Our formalization is exactly a second order extension of symmetric $\lambda\mu$-calculus in [9]. Usually, a term of $\lambda\mu$-calculus is understood as a proof with multiple conclusions. On the contrary, we consider a $\lambda\mu$-term as a proof with a single conclusion but two kinds of hypothesis, ordinary hypothesis and denials of propositions, which correspond conclusions other than a principal formula in usual $\lambda\mu$-calculus. Moreover, we do not distinguish λ-variables and μ-variables. $x, y, x_1, \cdots$ and $t, u, t_1, \cdots$ stand for first order variables and terms. X^n, Y^n, X_i^n denotes n-ary predicate variables and constants.

Definition 1. *A* proposition *is that of second order predicate logic built up by predicate variables X_i^n and logical connectives $\to$, $\forall$. Formally,*

$$A ::= X_i^n t_1 \cdots t_n \mid A \to A \mid \forall x_i A \mid \forall X_i^n A.$$

A formula *is a proposition A or a denial $\bullet A$ of proposition or contradiction $\bot$. Note that $\bot$ is not counted as a proposition. Other connectives are defined by using second order construct. For example, $\exists x.A(x)$ is defined as $\forall X^0.\forall x(A(x) \to X^0) \to X^0$.*

We assume axioms of our calculus is limited to those for atomic propositions or formulae with a form $A_1 \to A_2 \to \cdots \to A_n$ for atomic proposition A_i. We denote axioms and variable by Greek letters $\alpha, \beta, \cdots$.

Definition 2. *For each formula A,* $\lambda\mu$-terms of type A *are defined inductively as follows.*

1. *A variable or an axiom α_i^C is a term of type C. We assume that there is no variable of type $\bot$.*
2. *$[M]N$ is a term of type $\bot$ for a term M of type $\bullet A$ and a term N of type A.*
3. *$\mu\alpha.M$ is a term of type A for a variable α of type $\bullet A$ and a term M of type $\bot$.*
4. *$\mu\alpha.M$ is a term of type $\bullet A$ for a variable α of type A and a term M of type $\bot$.*
5. *$\lambda\alpha.M$ is a term of type $A \to B$ for variable α of type A and a term M of type B.*
6. *MN is a term of type B for a term M of type $A \to B$ and a term N of type*

$1 \leq j \leq l$. *We define* $\tilde{M}$ *by simultaneous substitution* $\xi(x_1), \cdots, \xi(x_m)$ *into* $x_1, \cdots, x_m$, $B_1, \cdots, B_n$ *into* $X_1, \cdots, X_n$, $N_1, \cdots, N_l$ *into* $\alpha_1, \cdots, \alpha_l$ *on* M. *Then we have* $\tilde{M} \in \xi(A)$.

Proof. By induction on the construction of M.

As a special case, $t \in \xi(A)$ holds. From Lemma 2, we have the following theorem.

Theorem 1. *All terms are strongly normalizable.*

Acknowledgement. I am grateful to Ken-etsu Fujita, Ryu Hasegawa and Charles Stewart for their helpful comments and discussion.

References

1. Yuuki Andou. A normalization-procedure for the first order classical natural deduction with full symbols. *Tsukuba Journal of Mathematics*, 19(1):153–162, 1995.
2. F. Barbanera and S. Berardi. A strong normalization result for classical logic. *Ann. Pure Appl. Logic*, 76:99–116, 1995.
3. Ph. de Groote. A cps-translation of the $\lambda\mu$-calculus. In *Trees in algebra and programming, CAAP '94*, number 787 in Lect. Notes Comput. Sci, pages 85–99. Springer-Verlag, 1994.
4. Ph. de Groote. On the relation between $\lambda\mu$-calculus and the syntactic theory of sequential control. In *Logic programming and automated reasoning*, volume 822 of *Lect. Notes Comput. Sci*, pages 31–43. Springer-Verlag, 1994.
5. C.-H. L. Ong and C. A. Stewart. A curry-howard foundation for functional computation with control. In *Proceedings of the 24th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*. ACM press, January 1997.
6. M. Parigot. $\lambda\mu$-calculus: an algorithmic interpretation of classical natural deduction. In A. Voronkov, editor, *Logic Programming and Automated Reasoning*, volume 624 of *Lecture Notes in Artificial Intelligence*, pages 190–201. Springer-Verlag, 1992.
7. M. Parigot. Classical proofs as programs. In *Computational logic and proof theory*, volume 713 of *Lect. Notes Comput. Sci*, pages 263–276. Springer-Verlag, 1993.
8. M. Parigot. Strong normalization for second order classical natural deduction. *J. Symb. Log.*, 62(4):1461–1479, 1997.
9. M. Parigot. On the computational interpretation of negation. In P. Clote and H. Schwichtenberg, editors, *Computer Science Logic*, volume 1862 of *Lect. Notes Comput. Sci*, pages 472–484. Springer-Verlag, 2000.
10. Th. Streicher and B. Reus. Classical logic, continuation semantics and abstract machines. *Journal of Functional Programming*, 8(6):543–572, 1998.