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AWARENESS, BELIEF AND COMMUNICATION REACHING CONSENSUS

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ABSTRACT. We present a communication process according to a protocol which is associated with an awareness and belief model. In the model we impose none of the requirements for player's knowledge as those in the standard model with partitional information structure. We show that consensus on the posteriors for an event among all players can still be guaranteed in the communication if the protocol contains no cycle.

1. INTRODUCTION

Geanakoplos and Polemarchakis [5] investigated a communication process in which two players announce their posteriors to each other. In the process players learn and revise their posteriors and they reached consensus.

Krasucki [6] investigated the communication process according to a protocol in which value of function are communicated privately through messages among at least three players. He showed that in the process that in the process, consensus on the values of a decision function can be guaranteed if the protocol contains no cycle. They both extend the agreement theorem of Aumann [1]. The information structures of players for their model are given by partitions.

Bacharach [2] showed that the model with the partitional information structure is equivalent to his knowledge operator model with the three axioms for operators: K1 axiom of knowledge, K2 axiom of transparency and K3 axiom of wisdom. Matsuhisa and Kamiyama [7] introduced the lattice structure of knowledge for which non requirements is not imposed such as the three axioms. In lattice structure Aumann's notion of common-knowledge is the same as their notion of common-belief. Matsuhisa and Usami [9] presented an awareness structure in which the players are not required to have logically omniscient. Matsuhisa and Usami obtained an extension of the 'Agreeing to disagree' that Aumann showed in the awareness model.

The purpose of this paper is to introduce the revision process of the values of a common decision function through the communication associated with the awareness structure. The main result of ours is an extension of Krasucki [6] as follows:

Key words and phrases. Awareness, belief, communication process, consensus, protocol, agreeing to disagree, no trade theorem.

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Theorem 1. *In the communication process associated with awareness structure, consensus can be guaranteed if the communication protocol is an acyclic graph.*

This paper organizes as follows: Section 2 defines an awareness belief model, associated information structure with awareness structure and communication process. Further we present the notion of consensus of the values of a decision function. Section 3 illustrates Theorem 1 by using a simple example. In section 4 we show that consensus of the value of a decision function that satisfies the sure thing principle and is preserved under difference can still be guaranteed in this communication process if the protocol contains no cycle.

2. The Model

Let Ω be a non-empty *finite* set called a *state space*, N a set of finitely many *players* at least two, and let $\mathcal{F}$ denote the field 2^{Ω} that is the family of all subsets of Ω. Each member of $\mathcal{F}$ is called an *event* and each element of Ω called a *state.*

2.1. **Belief and Common-Belief.** *A belief structure* is a tuple $\langle \Omega, (B_i)_{i\in N}\rangle$ in which Ω is a state-space and (B_i) is a class of i's *belief* operators on 2^{Ω}. The *mutual* belief operator B_EF on 2^{Ω} is defined by

$$B_EF = \bigcap_{i\in N} B_iF \ .$$

A *common-belief* operator is an operator B_C on Ω satisfying the *fixed point property*:

FP $\qquad B_CF \subseteq B_E(F \cap B_CF) \quad$ for every F of $\mathcal{F}$.

The interpretation of B_iF is the event that 'i believes F,' whereas B_EF is interpreted as the event that 'everybody believes F.' We say that an event E is *common-belief at* ω if ω belongs to B_CE.

2.2. **Awareness and Belief.** We present a model of awareness according to Matsuhisa and Usami [9]. This model follows from E. Dekel, B. L. Lipman and A. Rustichini [3]. A different approach of awareness models is discussed in R. Fagin, J.Y. Halpern, Y. Moses and M.Y. Vardi [4] .

An *awareness structure* is a tuple $\langle \Omega, (A_i)_{i\in N}, (B_i)_{i\in N}\rangle$ in which $\langle \Omega, (B_i)_{i\in N}\rangle$ is a belief structure and $(A_i)_{i\in N}$ is a class of i's *awareness* operators on $\mathcal{F}$ such that Axiom **PL** (axiom of plausibility) is valid:

PL $\qquad B_iF \cup B_i(\Omega \setminus B_iF) \subseteq A_iF \quad$ for every F in $\mathcal{F}$.

The *mutual* awareness operator A_E on $\mathcal{F}$ is defined by

$$A_EF = \bigcap_{i\in N} A_iF \ .$$

An awareness structure is called *finite* if the state-space is a finite set.

The interpretation of A_iF is the event that 'i is aware of F,' whereas A_EF is interpreted as the event 'everybody is aware of F.' The axiom **PL** says that i is aware of F if he believes it or if he believes that he dose not believe it.[1]

[1]The axiom **PL** dues to E. Dekel, B. L. Lipman and A. Rustichini [3]

We say that an event F is *self-aware* of i if $F \subseteq A_i F$ and it is said to be *publicly aware* if $F \subseteq A_E F$. An event T is said to be *i's evident belief* if $T \subseteq B_i T$, and it is said to be *public belief* at state ω if $\omega \in T \subseteq B_E T$. An event is public belief (or respectively, it is publicly aware) if whenever it occurs all players believe it (or they are all aware of it.) We can think of public belief as embodying the essence of what is involved in all players making their direct observations.

2.3. **Associated Information Structure.** M. Bacharach [2] introduces the *strong epistemic model* that is just the Kripke semantics corresponding to the modal logic **S5**.[2] Further he defines the information partition of the state-space induced from the knowledge operator.[3] Following his line we generalize the notion of information partition to the associated information structure as below.

The *associated* information structure $(P_i)_{i\in N}$ with an awareness structure $\langle \Omega, (A_i), (B_i)\rangle$ is a class of the mappings P_i of Ω into $\mathcal{F}$ defined by

$$P_i(\omega) = \bigcap_{T \in 2^\Omega} \{T \mid \omega \in T \subseteq B_i T\}.$$

(If there is no event T for which $\omega \in T \subseteq B_i T$ then we take $P_i(\omega)$ to be non-defined.) We call $P_i(\omega)$ the *i's evidence set* at ω.

An evidence set is interpreted as the basis for all i's evident beliefs since each i's evident belief T is decomposed into a union of all evidence sets contained in T.

Remark 1. The strong epistemic model in M. Bacharach [2] can be interpreted as an awareness structure $\langle \Omega, (A_i), (B_i)\rangle$ such that B_i satisfies **N**, **K**, **T**, **4** and **5** and so A_i is the *trivial* awareness operator; i.e. $A_i(E) = \Omega$ for every $E \in \mathcal{F}$. This says that an awareness structure is an extension of the strong epistemic model.

2.4. **Posterior Revised.** We improve on the notion of posterior as follows: Let $\langle \Omega, (A_i), (B_i), \mu\rangle$ be an awareness structure with a common-prior μ. For every real number q_i, we denote

$$[q_i] = \{\omega \in \Omega \mid \mu(X \cap A_i(X) \, | P_i(\omega)) = q_i\}.$$

We say q_i to be the i's *posterior* of X at ω if ω belongs to $[q_i]$. We denote by q the profile $(q_i)_{i\in N}$. An event $[q]$ is the intersection of the sets $[q_i]$ for all i of N; that is,

$$[q] = \bigcap_{i \in N} [q_i].$$

We say that the players *commonly believe their posteriors* q_i of X at ω if $[q]$ is common-belief at ω; that is, $\omega \in B_C([q])$.

[2]The strong epistemic model is a tuple $\langle \Omega, (K_i)_{i\in N}\rangle$, in which Ω is a state-space and K_i is an i's knowledge operator satisfying the following axioms: For every E, F of 2^Ω,

N $K_i\Omega = \Omega$; **K** $K_i(E \cap F) = K_i E \cap K_i F$; **T** $K_i F \subseteq F$;
4 $K_i F \subseteq K_i K_i F$; **5** $\Omega \setminus K_i F \subseteq K_i(\Omega \setminus K_i F)$.

[3]The i's information partition Π_i induced from the knowledge operator K_i is defined by $\Pi_i(\omega) = \bigcap_T \{T \in 2^\Omega \mid \omega \in K_i T\}$.

An interpretation of $\mu(X \cap A_i(X) \,|P_i(\omega))$ is the conditional probability of the i's *awareness section* of X under his evidence set at ω. For the above example, letting $A_i(E) = B_i(E) \cup B_i(\Omega \setminus B_i(E))$ we obtain that $A_2(\{\alpha\}) = \{\beta\}$. Therefore it follows that the player 2's improved posterior of $\{\alpha\}$ at state α is $\mu(\{\alpha\} \cap A_i(\{\alpha\}) \,|P_i(\alpha)) = 0$, as desired.

2.5. **Decision Function** [4]**.** Let Z be a set of decisions for all players in N. An i's *decision function* is a mapping f_i of $\mathcal{F}$ into Z. It is said to satisfy the *sure thing principle* if it is preserved under disjoint union; that is, for every pair of disjoint events S and T such that if $f_i(S) = f(T) = d$ then $f_i(S \cup T) = d$.

A decision function f_i is said to be *preserved under difference* if for all events S and T such that $S \subsetneqq T, f_i(S) = f_i(T) = d$ then we have $f_i(T \setminus S) = d$.

If f_i is intended to be a posterior probability, we assume given a probability measure μ which is common for all players and some event X. Then f_i is the mapping of the domain of μ into the closed interval [0, 1] such that

$$f_i(E) = \mu(X \cap A_i(X)|E),$$

where $\mu(E) \neq 0$. We plainly observe that this f satisfies the sure thing principle and is preserved under difference.

2.6. **Communication with Awareness Structure.** We assume that players communicate by sending *messages*. A *protocol* is a mapping Pr of the set of non-negative integers $\mathbb{Z}_+$ into the product set $N \times N$ that assigns to each t a pair of players $(s(t), r(t))$. Here t stands for *time* and $s(t)$ and $r(t)$ are, respectively, the *sender* and the *recipient* of the communication which takes place at time t. We consider a protocol as the directed graph whose vertices are the set of all players N and such that there is an edge (or an arc) from i to j if and only if there are infinitely many t such that $s(t) = i$ and $r(t) = j$.

A protocol is said to be *fair* if the graph is strongly-connected; in words, every player in this protocol communicates directly or indirectly with every other player infinitely often. It is said to *contain a cycle* if the graph contains a cyclic path; that is, there are players $i_1, i_2, \dots, i_k$ with $k \geqq 3$ such that for all $m < k$, i_m communicates directly with i_{m+1}, and such that i_k communicates directly with i_1

Definition 1. A *communication process* π of revisions of the values of decision functions $(f_i)_{i\in N}$ is a triple $\langle \mathrm{Pr}, (Q_i^t)_{(i,t)\in N\times T}, (f_i)_{i\in N}\rangle$, in which $\mathrm{Pr}(t) = (s(t), r(t))$ is a fair protocol such that for every t, $r(t) = s(t+1)$, communications proceed in *rounds*[5], and Q_i^t is the mapping of Ω into $\mathcal{F}$ for i at time t that is defined inductively as follows:

- We assume given a mapping $Q_i^0 := P_i$.
- Suppose Q_i^t is defined.
  - $d_i^t(\omega)$ denotes the value $f_i(Q_i^t(\omega))$:
  - W_i^t is the mapping of Ω into $\mathcal{F}$ that assigns to each state ω the event $W_i^t(\omega)$ consisting of all the states ξ such that $d_i^t(\xi) = d_i^t(\omega)$:
  - B_i^t is the belief operator induced by Q_i^t defined by $B_i^t E = \{\omega \in \Omega | Q_i^t(\omega) \subseteqq E\}$:

[4]C.f. Bacharach [2]

[5]That is, there exists a natural number m such that for all t, $s(t) = s(t+m)$

- The recipient $j = r(t)$ at t send the message $B_j^t(W_j^t(\omega))$ at $t+1$ when $\omega \in B_j^t(W_j^t(\omega))$, and he does not send the message otherwise.
- Then $Q_i^{t+1}(\omega)$ is defined as follows:
  - If i is not a recipient of a message at time $t+1$, then $Q_i^{t+1}(\omega) = Q_i^t(\omega)$.
  - If i is a recipient of a message at time $t+1$, then $Q_i^{t+1}(\omega) = Q_i^t(\omega) \cap B_{s(t)}^t(W_{s(t)}^t(\omega))$.

Specifically the sender j sends to i the message that he believes that his decision is $d_j^t(\omega)$. The communication is said to be *cut off in a state* ω if $\omega \notin B_j^t(W_j^t(\omega))$ for some t and the recipient $j = r(t)$ at t.

2.7. **Consensus.** The family $\{Q_i^t(\omega) \mid t = 0, 1, 2, \dots\}$ is a descending chain in $\mathcal{F}$ and so the limit Q_i^∞ exists in each state where the communication is never cut off: In fact, there exists sufficiently large $T \in \mathbb{Z}_+$ such that for all $t \geq T$, $Q_i^t = Q_i^T$ because Ω is finite. Since the protocol is fair, there exist infinitely many t such that $r(t) = i$, it follows that $Q_i^\infty(\omega) = \lim_{t\to\infty} Q_i^t(\omega)$.

Remark 2. For each i the sequence of the domains of $Q_i^t(t = 0, 1, \cdots)$ makes a descending chain, and thus it may be occurred that for $Q_i^\infty(\omega) = \emptyset$ in some ω.

It can be easily observed that

Lemma 1. *For every player i, the sequence* $\{d_i^t(\omega) \mid t = 0, 1, 2, \dots\}$ *converges to the limiting value* $d_i^\infty(\omega)$ *of* f *at each state* ω *where the communication is never cut off and* $Q_i^\infty(\omega)$ *is defined;* i.e., *there exists*

$$d_i^\infty(\omega) = \lim_{t\to\infty} d_i^t(\omega).$$

We call $d_i^\infty(\omega) = f(Q_i^\infty(\omega))$ the *limiting value* of f at ω for i. We say that *consensus* on the limiting values of decision functions $(f_i)_{i\in N}$ can be guaranteed in a communication process if $d_i^\infty(\omega)$ and $d_j^\infty(\omega)$ are equal for each player i, j and in all the states ω that the communication is never cut off and that Q_i^∞ is defined for all $i \in N$.

Remark 3. In the communication process, the limiting value of f can be reached in finitely many rounds; i.e., there exists a non-negative integer T independent on ω such that for every player i and for all $t \geqq T$, $d_i^\infty(\omega) = d_i^t(\omega)$ by Lemma 1.

3. Example

We show a simple example below in order to explain Theorem 1. We consider three players A, B and C, and give their associated information structures at time $t = 0$ defined by Figure 1.

In this case, we assume the event $X = \{1, 3, 5, 7\}$ and the protocol $A \rightleftarrows B \rightleftarrows C$ that contains no cycle. Let us give the common prior $\mu(\omega) = \frac{1}{8}$ for each state $\omega \in \Omega$, and their decision functions are given by the common function f defined by $f(E) = \mu(X|E)$. It is noted that the common decision function satisfies the sure thing principle and is preserved under difference.

Then the calculation result with player A at $t = 0$ will be given in Table 1.

Sender A sends the message $B_A^0(\omega)$ to the recipient B. B mixes the message and his associated information structure, and he makes his revised information structure then.

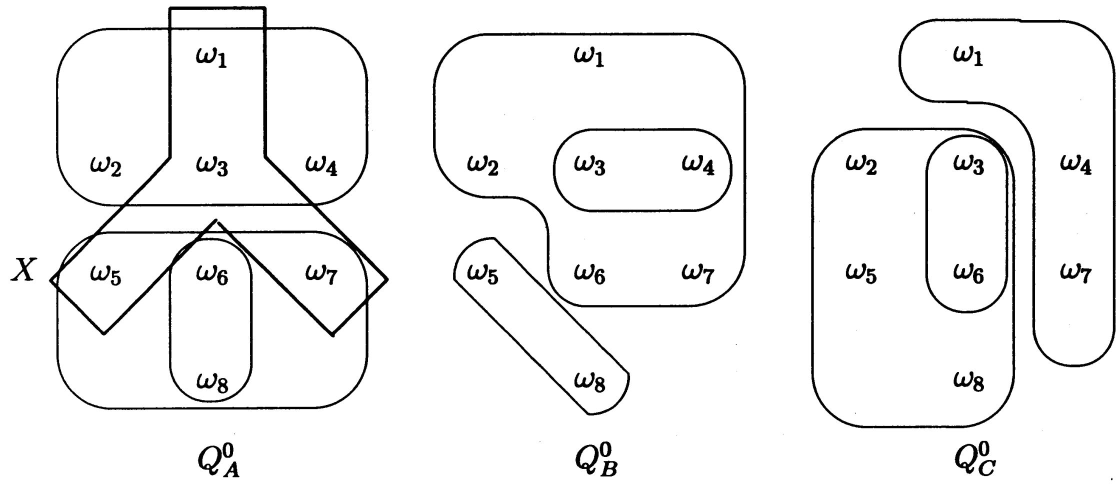

FIGURE 1. Associated information structure at the early time $t = 0$

$t=1$	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8
d^0_A	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0
W^0_A	123457	123457	123457	123457	123457	68	123457	68
$B^0_A(W^0_A)$	1234	1234	1234	1234	1234	68	1234	68

TABLE 1. The calculation result with A

$$
\begin{aligned}
Q^1_B(\omega_1) &= Q^1_B(\omega_2) = \{\omega_1, \omega_2, \omega_3, \omega_4\} \\
Q^1_B(\omega_3) &= Q^1_B(\omega_4) = \{\omega_3, \omega_4\}, \quad Q^1_B(\omega_6) = \{\omega_6\}, \quad Q^1_B(\omega_8) = \{\omega_8\}
\end{aligned}
$$

Note that the state ω_5, ω_7 of Q^1_B are not defined here. Next the results in Table 2 is also obtained by the same way.

$t=2$	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8
d^1_B	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\times$	0	$\times$	0
W^1_B	1234	1234	1234	1234	$\times$	68	$\times$	68
$B^1_B(W^1_B)$	1234	1234	1234	1234	$\times$	68	$\times$	68

TABLE 2. The calculation result with B

Then player C's revised information structure at $t = 2$ is as follows:

$$
\begin{aligned}
Q^2_C(\omega_1) &= Q^2_C(\omega_4) = \{\omega_1, \omega_4\} \quad Q^2_C(\omega_2) = \{\omega_2, \omega_3\}, \\
Q^2_C(\omega_3) &= \{\omega_3\}, \quad Q^2_C(\omega_6) = \{\omega_6\}, \quad Q^2_C(\omega_8) = \{\omega_6, \omega_8\}
\end{aligned}
$$

Similarly, the communication goes on as follows.

$t=3$	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8
d_C^2	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	×	0	×	0
W_C^2	124	124	3	124	×	68	×	68
$B_C^2(W_C^2)$	14	14	3	14	×	68	×	68

TABLE 3. The calculation result with C

Then Player B's revised information structure at $t=3$ is the next result.

$$\begin{aligned} Q_B^3(\omega_1) &= Q_B^3(\omega_4) = \{\omega_1, \omega_4\} \\ Q_B^3(\omega_3) &= \{\omega_3\}, \quad Q_B^3(\omega_6) = \{\omega_6\}, \quad Q_B^3(\omega_8) = \{\omega_8\} \end{aligned}$$

$t=4$	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8
d_B^3	$\frac{1}{2}$	×	1	$\frac{1}{2}$	×	0	×	0
W_B^3	14	×	3	14	×	68	×	68
$B_B^3(W_B^3)$	14	×	3	14	×	68	×	68

TABLE 4. The calculation result with B

Player A's revised information structure at $t=4$ is given by as follows:

$$\begin{aligned} Q_A^4(\omega_1) &= Q_A^4(\omega_4) = \{\omega_1, \omega_4\} \\ Q_A^4(\omega_3) &= \{\omega_3\}, \quad Q_A^2(\omega_6) = Q_A^2(\omega_8) = \{\omega_6, \omega_8\} \end{aligned}$$

$t=5$	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8
d_A^4	$\frac{1}{2}$	×	1	$\frac{1}{2}$	×	0	×	0
W_A^4	14	×	3	14	×	68	×	68
$B_A^4(W_A^4)$	14	×	3	14	×	68	×	68

TABLE 5. The calculation result with A

Player B's revised information structure at $t=5$ is given by as follows:

$$\begin{aligned} Q_B^5(\omega_1) &= Q_B^5(\omega_4) = \{\omega_1, \omega_4\} \\ Q_B^5(\omega_3) &= \{\omega_3\}, \quad Q_B^5(\omega_6) = \{\omega_6\}, \quad Q_B^5(\omega_8) = \{\omega_8\} \end{aligned}$$

$t=6$	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8
d_B^5	$\frac{1}{2}$	×	1	$\frac{1}{2}$	×	0	×	0
W_B^5	14	×	3	14	×	68	×	68
$B_B^5(W_B^5)$	14	×	3	14	×	68	×	68

TABLE 6. The calculation result with B

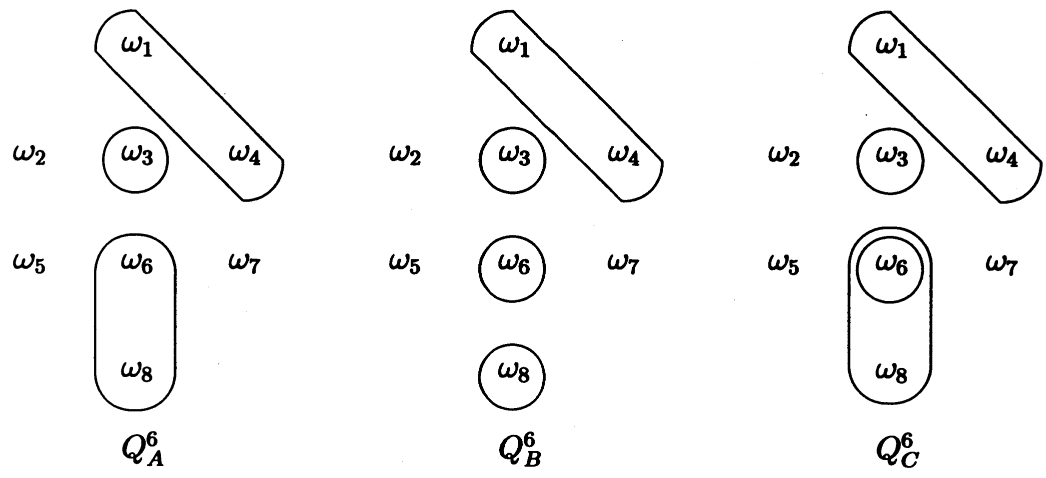

FIGURE 2. Each player's revised information structure at $t = 6$

For $t \geqq 5$ each Q_i^t is stationary given by Figure 2. All values of decision functions are equal. Therefore consensus can be guaranteed in this example. (Note that the states ω_2, ω_5 and ω_7 are not defined.) □

4. THE RESULT

4.1. We prove the generalized version of Theorem 1 in Matsuhisa et al [8] as follows:

Theorem 2. *In a communication process associated with awareness structure, suppose that the all decision functions are common for all players, and that the common decision function satisfies the sure thing principle and is preserved under difference. Consensus on the limiting values of the decision function can be guaranteed if the protocol contains no cycle.*

In the case that the decision function is given by posterior (Section 2.5), it immediately follows from Theorem 2 that

Corollary 1. *Suppose that all players have a common-prior. Consensus on the limiting values of the posteriors for a publicly aware event among all players can still be guaranteed in the communication if the protocol contains no cycle.*

4.2. ***RT*-Information Structure.** Before proceeding to the proof, we need the notion of *RT*-information structure and the fundamental lemma:

A mapping $Q : \Omega \to \mathcal{F}$ is said to be an *RT-information structure*[6] on Ω if the following two conditions are true: For each i and for every state ω such that $Q(\omega)$ is defined,

(Ref) $\quad \omega \in Q(\omega)$;

(Trn) $\quad \xi \in Q(\omega)$ implies $Q(\xi) \subseteqq Q(\omega)$.

The following lemma is a key to proving the theorem:

[6]The *RT*-information structure stands for *reflexive* and *transitive* information structure

Fundamental Lemma. *Let ω be a state in Ω, and Q an RT-information structure on Ω. Suppose that f be a decision function that satisfies the sure thing principle and is preserved under difference.*

If there exists an event H of $\mathcal{F}$ such that the two conditions are true: For each ξ of H,

(a) *$Q(\xi)$ is defined and it always contained in H, and*

(b) *it is always satisfied that $f(Q(\xi)) = f(Q(\omega))$,*

then we obtain that

$$f(H) = f(Q(\omega)).$$

A similar lemma may be found in Matsuhisa and Usami [9].

4.3. **Proof of Theorem 2.** It is sufficient to prove the following theorem because Theorem 2 immediately follows from it.

Theorem 3. *In a communication process associated with an awareness structure, suppose that the the decision functions are common for all players, and that the common decision function satisfies the sure thing principle and is preserved under difference, if player i communicates his/her message directly to player j then their limiting values of the decision function must be equal.*

Proof. Let $\pi = \langle \Pr, (Q_i^t), f \rangle$ be a communication process associated with awareness structures where f is the common decision function. The protocol has the property that, if $s(t) = i$ and $r(t) = j$ then $r(t+m) = i$ and $s(t+m) = j$ for some m.

It should be noted that P_i is an RT-information structure, and so is Q_i^∞ for every i. Therefore in view of (Ref) and (Trn), it can be plainly observed that

T $\quad B_i^\infty(F) \subseteqq F$;

4 $\quad B_i^\infty(F) \subseteqq B_i^\infty(B_i^\infty(F))$.

For each state ω at which Q_i^∞ is defined, set $H = B_i^\infty(W_i^\infty(\omega)) \cap B_j^\infty(W_j^\infty(\omega))$ and $Q = Q_l^\infty$ for $l = i, j$. To complete the proof, it suffices to verify the two properties:

(a) If ξ is a member of H then $Q(\xi) \subseteqq H$, and
(b) H is contained in the set consisting of all the states ξ such that $d_l^\infty(\xi) = d_l^\infty(\omega)$.

Indeed, if it is the case then, viewing of Fundamental Lemma we obtain that $f(H) = f(Q_l^\infty(\omega))$ for each $l \in \{i, j\}$, and thus $d_i^\infty(\omega) = d_j^\infty(\omega)$ as required.

Proof of (a): It follows from **4** that $Q_l^\infty(\xi) \subseteqq B_l^\infty(W_l^\infty(\omega))$, and it follows from the definition of Q_l^{t+1} that $\xi \in Q_l^\infty(\xi) \subseteq B_{l'}^\infty(W_{l'}^\infty(\xi))$ for $l' \in \{i, j\} \setminus \{l\}$, and thus $W_{l'}^\infty(\xi) = W_{l'}^\infty(\omega)$ by **T**. We observe that for every $\xi \in H$, $Q(\xi) \subseteq B_i^\infty(W_i^\infty(\omega)) \cap B_j^\infty(W_j^\infty(\omega))$.

Proof of (b): In view of **T** it is easily observed that $H \subseteqq W_l^\infty(\omega) = \{\xi \in \Omega \mid d_l^\infty(\xi) = d_l^\infty(\omega)\ \}$, in completing the proof. $\square$

4.4. As a consequence of Theorem 2 we can obtain a generalized version of the Agreement theorem of Geanakoplos and Polemarchakis [5] and that of Matsuhisa and Kamiyama [7].

Corollary 2. *In a communication process associated with awareness structure, suppose that the all decision functions are common for all players, and that the common decision function satisfies the sure thing principle and is preserved under difference, if the players in the process are only two then their limiting values of the decision function must be equal.*

Proof. Immediately follows from Theorem 2. □

4.5. **Conclusion.** In this paper we have shown that the nature that consensus can be guaranteed in a communication process is not the restrictions on the players' knowledge operators but their awareness and belief.

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