

数理解析研究所講究録 1232

Proceedings of the Workshop
“Algebraic Geometry and Integrable Systems
related to String Theory”

京都大学数理解析研究所

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Preface

From June 12 to June 16, 2000, we had an international workshop on “Algebraic Geometry and Integrable Systems related to String Theory” in RIMS, Kyoto University. This volume is the proceedings of the workshop, which was the concluding meeting of RIMS project 1999/2000, “Geometry related to string theory”. The project was organized by Kenji Fukaya (Kyoto Univ.), Masa-Hiko Saito (Kobe Univ.), Yuji Shimizu (Kyoto Univ.), Kaoru Ono (Hokkaido Univ.), Mikio Furuta, (RIMS), Yoichi Miyaoka (RIMS), Kyoji Saito (RIMS). For further information of the project, see the homepage at

<http://www.kurims.kyoto-u.ac.jp/kenkyubu/proj99/index.html>

In the last decade, many ideas from physics have been inspiring new ideas in several mathematical fields. In particular, the impacts of (super) string theory on mathematics are enormous. Gromov–Witten theory of counting curves in manifolds and Mirror symmetry of Calabi–Yau 3-folds may be first two good examples. Furthermore, some physicists are insisting that the mirror symmetry is only an appearance of deeper dualities, the so-called string dualities. These dualities connect many mathematical objects to each other which have been considered as those living in the different worlds of mathematics. The main motivation of the project was to understand the geometrical aspect of such theories.

As one can see from the title of the talks in the program of the workshop (cf. program of the workshop), the following topics were discussed in the workshop:

- Quantum cohomology, Gromov–Witten theory and BPS states
- Mirror symmetry, T-dualities and McKay correspondence
- Symplectic topology, Symplectic geometry and Floer cohomology
- Frobenius manifolds and Painlevé equations
- Moduli of vector bundles and G-bundles

We would like to thank all invited speakers, participants of the workshop and also contributors to the proceedings.

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Last, but not least, we would like to thank all staffs in RIMS for their contribution to make our project successful.

Organizer, Masa-Hiko SAITO (Kobe University)

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**Program of the Workshop “Algebraic Geometry
and Integrable Systems related to String Theory”**

弦理論に関わる代数幾何と可積分系

June, 12 – June, 16, 2000

	Mon	Tues	Wed	Thurs	Fri
9:30–10:30	no talk	Ohta	Eliashberg	Morrison	Morrison
10:50–11:50	no talk	Faber	Faber	Behrend	Getzler
13:30–14:30	Fukaya	Ito	Morava	Bryan	no talk
14:50–15:50	Nakajima	Sugiyama		Leung	no talk
16:10–17:10	Saito	Yoshioka		Takahashi	no talk

June 12, (Monday)

- 13:30–14:30, Kenji Fukaya (Kyoto)
Floer homology for family
- 14:50–15:50, Hiraku Nakajima (Kyoto):
Moduli of sheaves on blown up surfaces
- 16:10–17:10, Masa-Hiko Saito (Kobe)
Deformations of Okamoto–Painlevé pairs and Painlevé equations. (Joint works with Takebe and Terajima)

June 13, (Tuesday)

- 9:30–10:30, Hiroshi Ohta (Nagoya):
Obstruction to and deformation of Lagrangian intersection Floer homology. (Joint works with Fukaya, Kontsevich, Oh, Ohta, Ono)
- 10:50–11:50 Carel Faber (Oklahoma State):
Hodge integrals and Gromov-Witten theory I
- 13:30–14:30, Yukari Ito (Tokyo Metropolitan):
McKay correspondence and T-duality
- 14:50–15:50, Ken-ichi Sugiyama (Chiba):
Zeta functions of smooth manifolds and elliptic cohomology
- 16:10–17:10, Kota Yoshioka (Kobe):
Brill-Noether problem for sheaves on K3 surfaces

June 14, (Wednesday)

- 9:30–10:30, Y. Eliashberg, (RIMS/Stanford):
Few Examples from Symplectic Field Theory
- 10:50–11:50, Carel Faber (Oklahoma State)
Hodge integrals and Gromov-Witten theory II
- 13:30–14:30, Jack Morava (The Johns Hopkins)
Topological gravity in dimensions two and four

June 15, (Thursday)

- 9:30–10:30, David Morrison (Duke):
Theta-functions and six-manifolds (after Hopkins and Singer)
- 10:50–11:50, Kai Behrend (RIMS/British Columbia)
Differential Graded Schemes
- 13:30–14:30, Jim Bryan (Tulane):
Multiple coverings, BPS states, and integrality in Gromov-Witten theory
- 14:50–15:50, Naichung Conan Leung (Minnesota):
Moduli of G -bundles over T^4 and hyperkahler manifolds
- 16:10–17:10, Atushi Takahashi (RIMS)
Gromov-Witten Invariants and moduli of sheaves

June 16, (Friday)

- 9:30–10:30, David Morrison (Duke):
Mirror symmetry and topology
- 10:50–11:50 Ezra Getzler (RIMS/Northwestern):
higher genus Frobenius manifolds

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