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A TRANSFER CONSTRUCTION IN THE EQUIVARIANT SURGERY EXACT SEQUENCE

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SECTION 1. INTRODUCTION: THE EQUIVARIANT SURGERY EXACT SEQUENCE

Let G be a finite group. The classification of G-manifolds can be approached through the equivariant surgery exact sequence. In the category of locally linear PL-G-manifolds with a certain stability condition ("the gap hypothesis"), a surgery exact sequence was set up by I. Madsen and M. Rothenberg in [MR 2], when the group G is of odd order. One of its central feature is equivariant transversality, which holds only in those circumstances.

Let X be a (locally linear PL) G-manifold with boundary. The main target we wish to investigate is expressed, in this context, as the "structure set" $\widetilde{\mathcal{S}}_G(X,\partial)$, which is the set of equivalence classes of G-simple homotopy equivalences $h: M \to X$ with ∂h a PL-homeomorphism, where two such objects are equivalent when they are connected (in a commutative diagram) with a PL-G-homeomorphism of the domain M.

When one wishes to analyze the surgery exact sequence, one needs to compute the set $\widetilde{\mathcal{N}}_G(X)$ of G-normal cobordism classes of G-normal maps. By virtue of G-transversality, this set is interpreted in terms of bundle theories, and therefore is classified by a G-space F/PL. (See [MR 2, §5].)

Madsen and Rothenberg set up the equivariant surgery exact sequence and identified $\widetilde{\mathcal{N}}_G(X)$ as a term in the sequence, in a suitable category of G-spaces when G is a group of odd order. Here we cite their main results:

The strong gap condition. [MR 2, Theorem 5.11] *If G is a group of odd order and X is a G-oriented PL-G-manifold which satisfies the gap conditions*

$$10 < 2\dim X^H < \dim X^K \qquad \textit{for} \quad K \subset H, X^H \neq X^K,$$

then $\widetilde{\mathcal{N}}_G(X/\partial X)$ *is in one-to-one correspondence with normal cobordism classes of restricted G-normal maps over* X*, as defined in* [MR 2, 5.9].

The equivariant surgery exact sequence. [MR 2, Theorem 5.12] *If* G, X *are as above and we assume that* X^H *is simply-connected for all* H*, then there is an exact sequence*

$$\to \widetilde{\mathcal{S}}_G(D^1 \times X, \partial) \to \widetilde{\mathcal{N}}_G(D^1 \times X, \partial) \to \mathcal{L}_{1+m} \to \widetilde{\mathcal{S}}_G(X, \partial) \to \widetilde{\mathcal{N}}_G(X/\partial X) \to \mathcal{L}_m(G)$$

where

$$\mathcal{L}_m(G) = \oplus_{(H)} L_{m(H)}(N_G H/H)$$

with $m(H) = \dim X^H$*, and the sum is over the conjugacy classes of subgroups of* G.

Madsen and Rotherberg ([MR 2]) identified the terms of the exact sequence in geometric and homotopy theoretic methods, and the author ([N 4, 5]) modified their methods to interpret the terms in a homotopy theoretic way.

Two of the terms in the equivariant surgery exact sequence, $\widetilde{\mathcal{N}}_G(X/\partial X)$ and $\mathcal{L}_m(G)$, are defined using homotopy-theoretic and algebraic methods, respectively. Therefore they naturally inherit a Mackey functor structure over the system of subgroups of G. However, the remaining term, the structure set $\widetilde{\mathcal{S}}_G(X, \partial)$, is concerned with homeomorphisms, and so it does not provide a straightforward way to construct a functorial (Mackey) structure with respect to the system of subgroups of G.

Ranicki ([R 1,2]) has identified the structure set term in the equivariant surgery exact sequence with an "algebraically defined structure set," in his terminology. He used categorical constructions to identify the surgery exact sequence itself using algebraically constructed objects, thus making it possible to apply various categorical techniques. Making use of his methods, it is possible to interpret the equivariant structure set $\widetilde{\mathcal{S}}_G(X, \partial)$ in a categorical manner. However, that approach puts one in a stabilization situation, and thus requires a very strong stability hypotheses.

In [N 1] we used geometric methods, rather than algebraic, to directly construct a Mackey structure in the terms of the equivariant surgery exact sequence, in the case where the manifold X is a very special one. We recall the construction in [N 1] in Sections 3 and 5, below. So, at least in that situation, the Mackey functor structure is realized in the equivariant surgery exact sequence, without going through the stable homotopy category, thus giving the result to the structure set of the manifold itself, that is considered here.

Section 2. Definition: The Mackey Functor Structure

The Mackey functor structure over the system of subgroups of the finite group G is defined as follows. For an $\mathbb{R}G$-module V, let $\mathrm{Iso}(V)$ be the set of isotropy subgroups of the G-module V.

Let $\mathcal{M}$ be an abelian group valued bifunctor over the category $\mathrm{Iso}(V)$, and for the morphisms in $\mathrm{Iso}(V)$, that is, inclusions of subgroups $H < K$, we use the notation $\mathrm{Res}_K^H : \mathcal{M}(K) \to \mathcal{M}(H)$ and $\mathrm{Ind}_H^K : \mathcal{M}(H) \to \mathcal{M}(K)$ for the corresponding morphisms. Also we suppose there is a conjugation morphism $c_g : \mathcal{M}(H) \to \mathcal{M}(H^g)$ for any H and and $g \in G$.

The system $\mathcal{M}, \mathrm{Res}_K^H, \mathrm{Ind}_H^K, c_g$ is called a Mackey functor if the following conditions are satisfied for all $H < K$ in $\mathrm{Iso}(V)$:

$$c_g = \mathrm{id}_{\mathcal{N}(H)} \quad \text{if} \quad g \in H; \qquad c_{g_1 \circ g_2} = c_{g_1} \circ c_{g_2}$$
$$\mathrm{Ind}_{H^G}^{K^g} \circ c_g = c_g \circ \mathrm{Ind}_H^K, \qquad \mathrm{Res}_{K^g}^{H^g} \circ c_g = c_g \circ \mathrm{Res}_K^H$$
$$\mathrm{Res}_G^H \circ \mathrm{Ind}_K^G = \sum_{H \backslash G / K} \mathrm{Ind}_{h \cap K^g}^H \circ c_g \circ \mathrm{Res}_K^{k \cap H^{g^{-1}}}$$

Let $A(G : V)$ be the Grothendieck group of finite G-sets X such that $\mathrm{Iso}(X) \subset \mathrm{Iso}(V)$. Then a Mackey functor $\mathcal{M}$ over $\mathrm{Iso}(V)$ becomes a natural $A(G : V)$-module, and thus traditional algebraic caluculations are applicable to compute such terms. See [MS] for example.

Section 3. The Transfer Construction for $X = D^k \times SU$

We now specialize to the following case: Let $X = D^k \times SU$ where D^k is the k-dimensional disk with the trivial G-action, U is an $\mathbb{R}G$-module with no G-trivial summand, that is, $U^G = 0$, $V = U \oplus \mathbb{R}^{k-1}$, and we assume that X satisfies the strong gap condition that was defined in the above.

We will construct a Mackey finctor structure for the structure set

$$\widetilde{S}_H(D^k \times SU, \partial) \qquad (H \in \mathrm{Iso}(V))$$

The restriction and the conjugation maps are defined naturally. That is, for $H < K$, with $H, K \in \mathrm{Iso}(V)$, we define the restriction map:

$$\mathrm{Res}_K^H : \widetilde{S}_K(D^k \times SU, \partial) \to \widetilde{S}_H(D^k \times SU, \partial)$$

by the natural restriction (forgetful map) of viewing a K-simple homotopy equivalence as an H-simple homotopy equivalence. Similarly, the conjugation map:

$$c_g : \widetilde{S}_H(D^k \times SU, \partial) \to \widetilde{S}_{H^g}(D^k \times SU, \partial)$$

is defined by sending a map $(f : M \to X)$ to $(f : M^g \to X)$, where the H^g-action on the manifold $M^g = M$ is given by the map $H^g \to H \to \operatorname{Aut} M$, in which the first map sends $x \in H^g$ to $g^{-1}hg \in H$.

Thus, it remains to define the induction map

$$\operatorname{Ind}_H^K : \widetilde{S}_H(D^k \times SU, \partial) \to \widetilde{S}_K(D^k \times SU, \partial)$$

for all subgroup inclusions $H < K$ in $\operatorname{Iso}(V) = \operatorname{Iso}(U \oplus \mathbb{R}^{k-1})$.

An element of the domain $\widetilde{S}_H(D^k \times SU, \partial)$ is represented by an H-simple homotopy equivalence

$$f : (M, \partial) \longrightarrow (D^k \times SU, \partial)$$

such that its restriction to the boundary ∂M is a PL homeomorphism. Thus, $\partial M \cong S^{k-1} \times SU$. Divide the $(k-1)$-dimensional sphere into northern and southern hemispheres: $S^{k-1} = D_+^{k-1} \cup D_-^{k-1}$. Thus the boundary manifold is divided into

$$\partial M = \partial_+ M \cup \partial_- M$$

where the map f can be assumed to be the identity on the southern hemispere part:

$$\partial_- M = D^{k-1} \times SU.$$

Using this identity map, we extend the H-homotopy equivalence f into:

$$\begin{aligned} \hat{f} : \hat{M} = M \cup_\partial (S^{k-1} \times DU) &\xrightarrow{f \cup \mathrm{id}} D^k \times SU \cup_\partial S^{k-1} \times DU \\ &\cong S(\mathbb{R}^k \times U) \end{aligned}$$

Next, we remove the interior of a small disk $D(\mathbb{R}^{k-1} \times U) = D_+^{k-1} \subset S^{k-1} \times DU$, out of $\hat{M}$, to get:

$$\begin{aligned} M_0 &= \hat{M} - \operatorname{int}\left(D(\mathbb{R}^{k-1} \times U)\right) \\ f_0 = \hat{f}|_{M_0} : (M_0, \partial)) &\longrightarrow \left(D(\mathbb{R}^{k-1} \times U), \partial\right) = (DV, \partial). \end{aligned}$$

Since the Whitehead torsion does not change:

$$\tau_H(f) = \tau_H(\hat{f}) = \tau(f_0)$$

because the D^k-direction has the trivial H-action, the result map f_0 is an H-simple homotopy equivalence. Furthermore, it is easily seen that $\partial f_0 = \mathrm{id}$ and that f_0 is a PL-homeomorphism in the neighborhood of $f_0^{-1}\left(D^{k-1} \times \{0\}\right)$.

Now, for each $H \in \mathrm{Iso}(V)$, choose a G-embedding

$$i_H : G/H \longrightarrow V$$

such that the isotropy subgroup of $i_H(eH)$ is H, and fix all the $\{i_H\}$ for the rest of the construction.

For any subgroup inclusion $H < K$ in $\mathrm{Iso}(V)$, choose a positive number ϵ small enough so that the G-embedding

$$\rho : V \longrightarrow V, \qquad v \mapsto \epsilon \frac{v}{1+|v|}$$

satisfies the condition that $i_H(gH) + \rho(DV)$ for all $g \in K/H$ are mutually disjoint. That is, $\rho(K \times_H DV)$ is embedded into DV. Since the map $f_0 : (M_0, \partial) \to (DV, \partial)$ has been defined so that it is the identity on $\partial M_0 = SV$, we can now paste them together to get a manifold N_0 and a map F_0:

$$\begin{aligned} N_0 &= (K \times_H M_0) \cup_\partial (DV - \mathrm{int}\,\rho\,(K \times_H DV)) \\ &\xrightarrow{F_0 = (K \times_H f_0) \cup \mathrm{id}} DV. \end{aligned}$$

Because the map F_0 is a PL homeomorphism in a neighborhood of $F_0^{-1}\left(D^{k-1} \times \{0\}\right)$, we can now remove the interior of its neighborhood to get:

$$\begin{aligned} N_1 &= N_0 - \mathrm{int}\,F_0^{-1}\left(D^{k-1} \times D_\epsilon V\right) \\ &\xrightarrow{f_1} D^k \times SU. \end{aligned}$$

This result map f_1 turns out to be a K-simple homotopy equivalence. That it is a K-homotopy equivalence is shown by the standard argument, becuase the construction is by pasting together H-homotopy equivalences via the group-level transfer construction $K \times_H DV$ inside the representation space DV. The Whitehead torsion doesn't change either, because the pasting and the removal were all done with respect to the trvial action directions. We now use this as the definition of $\mathrm{Ind}_H^K[f]$:

Definition 3.1. *For any class $[f] \in \widetilde{S}_H(D^k \times SU, \partial)$, define its induction image as follows:* $\mathrm{Ind}_H^K[f] = [f_1] \in \widetilde{S}_K(D^k \times SU, \partial)$

Theorem 3.2. *If $X = D^k \times SU$ satisfies the strong gap condition explained in the above, then the induction map*

$$\mathrm{Ind}_H^K : \widetilde{S}_H(D^k \times SU, \partial) \longrightarrow \widetilde{S}_K(D^k \times SU, \partial)$$

is well-defined, and, together with the restriction and conjugation maps, Res_K^H and c_g, that were defined in the beginning of this section, satisfies the conditions of Mackey functor (defined in Section 2).

The proof of this theorem will occupy the rest of this section.

We follow the argument in Section 3 of Madsen-Svensson's paper [MS], which checks the Mackey conditions in the homotopy-theoretic situation. In our geometric situation, where (simple) homotopy equivalences are constructed by pasting homeomorphisms together, we simply have additional need to check that the homotopy constructed in their paper would be able to made, in our situation, to become a shifting by homeomorphisms. In fact this can be done, thanks to the existence of collars ("fattening by identity maps") in our construction, and to the general position allowance provided by the codimention condition given by the strong gap condition.

So, we simply follow the Section 3 of [MS], adapted to our construction with $\widetilde{S}_{(-)}(D^k \times SU, \partial)$. The strong gap condition guarantees just enough trivial-action dimention that allows the existence of homotopies between maps of (3.5) of [MS], which they give by explicit parameter formula. We can use the same homotopy, glued together with the identity maps outside of the embedding neighborhoods, strictly following their construction.

As in Madsen-Svensson's argument, only the double-coset formula (the last equation in our definition of the Mackey conditions) and the commutation of Ind and c_g need real checking. For the commutation of Ind and c_g, we define our homotopy as:

$$\Psi|_{\theta(t)+V_\epsilon} : (\psi(t) i_H(t) + t i_H(gH) + \rho(v), t) \to f^g(v)$$

on the "core" $K \times_H M_0$, where $f^g(v)$ is the map twisted by the conjugation action c_g, $\psi(t)$ is a path modification in the trivial representation component so that the g-orbits avoids crossing together, and $\theta(t)$ is the result curves in $DV \times I$ that are disjoint each other. We paste this homotopy on the "core" with the identity maps on the outside of the core neighborhoods, and, thanks to the strong gap condition, the pasting can still be done without making the homeomorphisms crossing together in $DV \times I$.

Now the diagram

$$\begin{array}{ccc}
\widetilde{S}_H(D^k \times SU, \partial) & \xrightarrow{c_g} & \widetilde{S}_{H^g}(D^k \times SU, \partial) \\
{\scriptstyle \mathrm{Ind}_H^K}\downarrow & & \downarrow{\scriptstyle \mathrm{Ind}_H^K} \\
\widetilde{S}_K(D^k \times SU, \partial) & \xrightarrow{c_g} & \widetilde{S}_{K^g}(D^k \times SU, \partial)
\end{array}$$

commutes, with the same reason that the homotopy gives the commutative diagram in the homotopy sets in the situation of Section 3 of Madsen-Svensson [MS].

The (more complicated) diagram for the double-coset formula also holds with the similar construction of homotopies, again as in Madsen-Svensson's argument, and our Theorem 3.2 is proved.

The main point is the appropriate construction of the map, and once it is constructed properly, then the proof of the required Mackey functor condition is done by the standard argument.

SECTION 4. EXPANSION TO THE MAPPING CONE CASE

Let $X = C_\varphi$ be the mapping cone of an equivariant map $\varphi : S^\ell \times SW \longrightarrow S^k \times SU$. We claim the following:

Theorem 4.1. *If the mapping cone $X = C_\varphi$ satisfies the strong gap condition (in Section 1), then we can construct a transfer map*

$$\mathrm{Ind}_H^K : \widetilde{S}_H(C_\varphi, \partial) \longrightarrow \widetilde{S}_K(C_\varphi, \partial)$$

that is compatible with the other Mackey structures in the equivariant surgery exact sequence for $X = C_\varphi$.

The proof is done using a stratified surgery, that needs an isovariant data rather than just equivariant one. A map is called isovariant if $G_{f(x)} = G_x$ holds everywhere, that is, the map preserves the orbit type everywhere. In the case of manifolds with finite PL-G-triangulation, this results in a stratified surgery data. (See Section 13.2 of [We 1].)

The key tool to be used for the proof of the theorem is the following result of Browder ([Br], [Do]):

Theorem (Browder). *If M and N are G-manifolds with the strong gap condition, then for any G-homotopy equivalence $f : M \to N$ there is a G-isovariant homotopy equivalence $f' : M \to N$ that is G-homotopic to f.*

That is, if we start with a G-homotopy equivalence, we can equivariantly homotope it into an isovariant situation, which induces a stratified homotopy equivalence, making it possible to apply the stratified surgery theory in the sense of Browder and Quinn ([BQ]. See also [We 1].)

We start with an element of $\widetilde{S}_H(C_\varphi, \partial)$. That is a map from a G-manifold M to the mapping cone C_φ. Apply Browder's theorem to make it an isovariant homotopy equivalence. This provides a stratified surgery data, each of whose strata looks like:

$$H\text{-orbit} \quad \xrightarrow{f(H)} \quad \text{mapping cone of } H\text{-orbit}$$

Since each of the strata looks like a piece used in the previous Section 3, we get the transfer of the above data as:

$$K \times_H (H\text{-orbit}) \quad \xrightarrow{f(K)} \quad \text{mapping cone of } K \times_H (H\text{-orbit})$$

Now we paste those strata together. Since we have the strong gap condition, those pieces of maps can be assumed to be in the general position, and thus the stratified surgery can be applied. We use the following (See Section 7.1 of [We 1]):

Stratified π-π Theorem. *Suppose (Y, X) is a strongly stratified pair, $X = \partial Y$, and each pure stratum of Y touches exactly one stratum of X for which the inclusion is a 1-equivalence. If all strata of X are of dimension ≥ 5, then any normal invariant of $(W, V) \to (Y, X)$ can be surged into a simple homotopy equivalence.*

Since our strong gap condition is stronger than the condition needed here, our general position situation is enough to apply the Stratified π-π Theorem to our strafied data, we can surger the data to construct a K-homotopy equivalence. Pasting them together along the stratification structure, we get an equivariant homotopy equivalence map in the global mapping cone level.

That provides a transfer map between the structure set, thus we can complete thhe proof Theorem 4.1.

SECTION 5. THE TRANSFER COMPATIBILITY IN THE SURGERY EXACT SEQUENCE

Once we have a Mackey functor structure in each of the terms in the equivariant surgery exact sequence, we want to check if the maps in the exact sequence are compatible with those Mackey structures. In fact this is true, as in the following:

Theorem 5.1. *Let X be either $X = D^k \times SU$ (considered in Section 3) or $X = C_\varphi$ with $\varphi : S^\ell \times SW \longrightarrow S^k \times SU$ (considered in Section 4) and assume that the X satisfies the strong gap condition as in the above. Then, the equivariant surgery exact sequence for X consists of Mackey functor maps, where the structure set term is given the Mackey structure constructed in Sections 3 and 4 above, and the other terms are given the natural homotopy-theoretically and algebraically defined Mackey structures, that were explained in Section 1.*

Proof. The L-group term in the equivariant surgery exact sequence was interpreted by Madsen-Rothenberg ([MR 2]) as hierarchical strata-wise L-group classes, each of which is interpreted (by the original realization theorem of C. T. C. Wall ([W], Section 3)) as appropriate classes of equivariant normal maps. Therefore, we can re-interpret the construction of the induction maps in the L-group term with the geometric normal map level constructions, and once we do that, the exactly similar construction to our one in the above Section 3 (replacing equivariant homotopy equivalences with equivariant normal maps, homotopies with normal cobordisms, etc.) for the structure set term can be checked to be compatible with the induction maps in the L-group term. In the case of $X = D^k \times SU$, our construction of $K \times_H \rho(f_0)$ is compatible with the inductive splitting correspondence of Theorem 9.1 and Theorems 10.1 and 10.2 of Madsen-Rothenberg ([MR 2]).

Similarly, the normal invariant term in the equivariant surgery exact sequence is interpreted by homotopy classes of equivariant normal maps as done in Madsen-Rothenberg ([MR 2]), and, again, the comparison of constructions can be done, to provide the compatibility of induction maps between the structure set term and the normal invariant term.

Other Mackey structure maps, that is, the restriction maps and the conjugation maps, are obviously compatible with the maps in the surgery exact sequence, by definition, and thus we see that the exact sequence consists of maps of Mackey functors.

In the mapping cone case $X = C_\varphi$, the check for the compatibility is also routine. The construction was done with the application of Stratified π-π Theorem, and thus the naturality and the compatibility with the Mackey structures is part of the data provided with the stratified surgery. The point is that the strata-wise pasting is done using the dimension gap between trivial-action summands, and thus the homotopy providing the compatibility is allowed to make it compatible with all other strata. We will provide the details elsewhere.

Repeating the mapping cone construction, we can reach the situation with PL manifold with finite equivariant triangulation. We expect the same result to hold for more general G-manifolds X, with enough stability condition (we hope the same "strong gap condition" for the G-manifold X could be enough), but we haven't been able to provide a satisfactory construction for that general case, at this point. We hope to return to this generality in a future work.

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