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INTERIOR ESTIMATES IN CAMPANATO SPACES RELATED TO QUADRATIC FUNCTIONALS

Maria Alessandra Ragusa & Atsushi Tachikawa

Abstract

In this paper we obtain interior estimates in Campanato spaces for the derivatives of the minimizers of quadratic functionals.

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1 Introduction and Preliminary Tools

The papers concerned with the regularity problem almost always have as a common starting point the Euler's equation related to a generic functional I. In the paper by Giaquinta and Giusti [11], the authors investigate the hölder continuity of the minima working directly with the functional I instead of Euler's equation.

In the present paper, following the method in [11], we have studied regularity properties of the minima of variational integrals of the type:

$$\mathcal{A}(u,\Omega)=\int f(x,u,Du)dx,$$

$\Omega\subset\mathbb{R}^n, n\geq 3$ is a bounded opens set $u:\Omega\to\mathbb{R}^N$, $u(x)=(u^1(x),\dots,u^N(x))$, $Du=D_\alpha u^i$, $D_\alpha=\frac{\partial}{\partial x^\alpha}$, $i=1,\dots,N$, $\alpha=1,\dots,n$,

$$f:\Omega\times\mathbb{R}^N\times\mathbb{R}^{nN}\to\mathbb{R}$$

defined by

$$f(x,u,Du) \;=\; A_{ij}^{\alpha\beta}(x,u)D_\alpha u^i D_\beta u^j \;+\; g(x,u,Du).$$

Let us now give the following definitions useful in the sequel.

Definition 1.1. *(see [16], [20]). Let* $1 \le p < \infty, 0 \le \lambda < n$.
By $L^{p,\lambda}(\Omega)$ *we denote the linear space of functions* $f \in L^p(\Omega)$ *such that*

$$\|f\|_{L^{p,\lambda}(\Omega)} = \left\{ \sup_{\substack{0<\rho<\operatorname{diam}\Omega \\ x\in\Omega}} \rho^{-\lambda} \int_{\Omega\cap B(x,\rho)} |f(y)|^p dy \right\} < +\infty$$

where $B(x,\rho)$ *ranges in the class of the balls of* $\mathbb{R}^n$ *of radius* ρ *around* x.

We have that $\|f\|_{L^{p,\lambda}(\Omega)}$ is a norm respect to which $L^{p,\lambda}(\Omega)$ is a Banach space, and also that

$$\|f\|_{L^p(\Omega)} \le (\operatorname{diam}\Omega)^{\frac{\lambda}{p}} \|f\|_{L^{p,\lambda}(\Omega)}.$$

Before the definition of the Campanato spaces let us define $f_{B(x,\rho)}$ as the integral average

$$f_{B(x,\rho)} = \frac{1}{|B(x,\rho)|} \int_{B(x,\rho)} f(y)dy$$

of the function $f(x)$ over the balls $B(x,\rho)$ of $\mathbb{R}^n$. When no confusion may arise, we will write f_{B_ρ} or f_ρ instead of $f_{B(x,\rho)}$.

Definition 1.2. *(see e. g. [3], [9]). Let* $1 \le p < \infty$ *and* $\lambda \ge 0$.
By **Campanato spaces** $\mathcal{L}^{p,\lambda}(\Omega)$ *we denote the linear space of functions* $u \in L^p(\Omega)$ *such that*

$$[f]_{p,\lambda} \;=\; \left\{ \sup_{x\in\Omega,\ 0<\rho<\operatorname{diam}\Omega} \rho^{-\lambda} \int_{\Omega\cap B(x,\rho)} |f(y) - f_\rho|^p dy \right\}^{\frac{1}{p}} < +\infty.$$

$\mathcal{L}^{p,\lambda}(\Omega)$ are Banach spaces with the following norm

$$\|f\|_{\mathcal{L}^{p,\lambda}(\Omega)} = \|f\|_{L^p(\Omega)} + [f]_{p,\lambda}$$

which simply demonstrate that $u \in \mathcal{L}^{p,\lambda}(\Omega)$ if and only if

$$\sup_{x\in\Omega, 0<\rho<\operatorname{diam}\Omega} \rho^{-\lambda} \inf_{c\in\mathbb{R}} \int_{\Omega\cap B(x,\rho)} |f - c|^p dy < \infty.$$

Using Hölder inequality we have that

$$\mathcal{L}^{p_1,\lambda_1}(\Omega) \subset \mathcal{L}^{p,\lambda}(\Omega)$$

where

$$p \leq p_1, \qquad \frac{n-\lambda}{p} \geq \frac{n-\lambda_1}{p_1}.$$

Let us observe that

$$\int_{\Omega\cap B(x,\rho)} |f - f_\rho|^p dy \leq C \cdot \int_{\Omega\cap B(x,\rho)} \left(|f|^p + |\Omega \cap B(x,\rho)| \cdot |f_\rho|^p \right) dy$$

and also

$$|f_\rho|^p \leq \frac{1}{|\Omega \cap B(x,\rho)|} \cdot \int_\Omega |f|^p dy.$$

Below, we consider $0 \leq \lambda < n$.

We use

$$[f]_{p,\lambda} \leq C\|f\|_{L^{p,\lambda}(\Omega)}.$$

to obtain the following relation between Morrey and Campanato spaces

$$L^{p,\lambda}(\Omega) \subset \mathcal{L}^{p,\lambda}(\Omega). \tag{1.1}$$

Let us now recall the definitions of the BMO and VMO classes.

Definition 1.3. *(see [15]). We say that a function f belongs to the John-Nirenberg space BMO, or that f has "bounded mean oscillation", if*

$$\|f\|_* \equiv \sup_{B_\rho \subset \mathbb{R}^n} \frac{1}{|B_\rho|} \int_{B_\rho} |f(y) - f_\rho| dy < \infty$$

where f_ρ is the integral average of the function f over the balls B_ρ.

Let us define, for a function $f \in BMO$,

$$\eta(r) = \sup_{x\in\mathbb{R}^n,\ \rho\leq r} \frac{1}{|B_\rho|} \int_{B_\rho} |f(x) - f_\rho| dx.$$

Definition 1.4. *(see [22]). A function $f \in BMO$ belongs to the class VMO, or f has "vanishing mean oscillation" if*

$$\lim_{r \to 0^+} \eta(r) = 0.$$

We are now ready to formulate the hypothesis on the terms $A_{ij}^{\alpha\beta}(x,u)$ and $g(\cdot, u, Du)$.

We suppose that $A_{ij}^{\alpha\beta}(x,u)$ are bounded functions in $\Omega \times \mathbb{R}^N$, such that:

(A1) $A_{ij}^{\alpha\beta} = A_{ji}^{\beta\alpha}$.

(A2) For every $u \in \mathbb{R}^N$, $A_{ij}^{\alpha\beta}(\cdot, u) \in VMO(\Omega)$.

(A3) For every $x \in \Omega$ and $u, v \in \mathbb{R}^N$,

$$\left|A_{ij}^{\alpha\beta}(x,u) - A_{ij}^{\alpha\beta}(x,v)\right| \le \omega(|u-v|^2)$$

for some monotone increasing concave function ω with $\omega(0) = 0$.

(A4) There exists a positive constant ν such that

$$\nu|\xi|^2 \le A_{ij}^{\alpha\beta}(x,u)\xi_\alpha^i \xi_\beta^j$$

for a.e. $x \in \Omega$, all $u \in \mathbb{R}^N$ and $\xi \in \mathbb{R}^{nN}$.

We suppose that the function g is a Charathéodory function, that is:

(g1) $g(\cdot, u, Du)$ is measurable in x $\forall u \in \mathbb{R}^N, \forall z \in \mathbb{R}^{nN}$;

(g2) $g(x, \cdot, \cdot)$ is continuous in (u, z) $a.\ e.\ x \in \Omega$;

moreover we consider g satisfying the condition:

(g3)

$$|g(x,u,z)| \le g_1(x) + H\,|z|^\gamma,$$

$g_1 \ge 0$, a. e. in Ω, $g_1 \in L^p(\Omega)$, $2 < p \le \infty$, $H \ge 0$, $0 \le \gamma < 2$.

We point out that since C^0 is a proper subset of VMO, the continuity of $A_{ij}^{\alpha\beta}(x,u)$ with respect to x is not assumed.

Let us make some remarks on VMO class. It was at first defined by Sarason in 1975 and later it was considered by many others. For instance we recall the papers by Chiarenza, Frasca and Longo [4] where the authors answer a question raised thirty years before by C. Miranda in [18]. In his paper he considers a linear elliptic equation where the coefficients a_{ij} of the higher order derivatives are in the class $W^{1,n}(\Omega)$ and asks whether the gradient of the solution is bounded, if $p > n$. Chiarenza, Frasca and Longo suppose $a_{ij} \in VMO$ and prove that Du is hölder continuous for all $p \in]1, +\infty[$. We point out that $W^{1,n} \subset VMO$ because, using Poincare's inequality

$$\fint_B |f(x) - f_B| \leq c(n) \left(\int_B |\nabla u| dx \right)^{\frac{1}{n}}$$

and the term on the right-hand side tending to zero as $|B| \to 0$. Later the interior estimates obtained by Chiarenza, Frasca and Longo were extended to boundary estimates in [5]. From these papers on, many authors have used this space VMO to obtain regularity results for partial differential equations and systems with discontinuous coefficients. We recall for example Bramanti and Cerutti [2] for parabolic equations and many others.

With this useful assumption we investigate the regularity of the minimizers for the quadratic functional. Its existence is guaranteed, being the functional $\mathcal{A}$ sequentially lower semicontinuous with respect to the $H^{1,2}$–weak topology (see [10]).

2 Main Results

Theorem 2.5. *Let $u \in W^{1,2}(\Omega, \mathbb{R}^N)$ be a minimum of the functional $\mathcal{A}(u, \Omega)$ defined above. Suppose that assumptions (A-1), (A-2), (A-3), (A-4), $1 < q \leq 2$ and (g-1), (g-2) and (g-3) are satisfied. Then for $\lambda = n(1 - \frac{q}{p})$ we have*

$$Du \in \mathcal{L}^{q,\lambda}_{\text{loc}}(\Omega_0, \mathbb{R}^{nN}) \tag{2.2}$$

where

$$\Omega_0 = \{ x \in \Omega : \liminf_{R \to 0} \frac{1}{R^{n-2}} \int_{B(x,R)} |Du(y)|^2 dy = 0 \}.$$

The set Ω_0 is obligatory, in fact when we pass from the regularity theory for scalar minimizers of solutions of elliptic equations to the regularity theory for vector-valued minimizers of solutions of elliptic systems, the situation changes completely: regularity is an exceptional occurence everywhere, excluding the two dimensional case. In 1968 De Giorgi in [7] showed that his regularity result for solutions of second order elliptic equations with measurable bounded coefficients cannot be extended to solutions of elliptic systems. He presented the quadratic functional

$$\mathcal{S} = \int_\Omega A_{ij}^{\alpha\beta}(x) D_\alpha u^i D_\beta u^j dx$$

with $A_{ij}^{\alpha\beta} \in L^\infty(\Omega)$, such that

$$\exists \nu > 0 : A_{ij}^{\alpha\beta} \chi_\alpha^i \chi_\beta^j \geq \nu |\chi|^2 \text{a.e.} x \in \Omega, \forall \chi \in \mathbb{R}^{nN}$$

De Giorgi proves that $\mathcal{S}$ has a minimizer that is a function having a point of discontinuity in the origin. Later, Souček in [23] showed that minimizers of functionals of the type $\mathcal{S}$ can be discontinuous, not only in a point, but also on a dense subset of Ω. Modifying De Giorgi's example, Giusti and Miranda in [13] showed that solutions of elliptic quasilinear systems of the type

$$\int_\Omega A_{ij}^{\alpha\beta}(u) D_\alpha u^i D_\beta \varphi^j dx = 0, \forall \varphi \in C_0^\infty(\Omega, \mathbb{R}^N)$$

with analytic elliptic coefficients $A_{ij}^{\alpha\beta}$ have singularities in dimension $n \geq 3$. We observe that we can get global regularity for some special cases, see for example [24].

Similar examples were presented in the meantime independently by Maz'ya in [17]. Even Giaquinta in 1993 in [10], Morrey in [19] and others were interested in these problems of solutions of elliptic systems, solutions in general non regular. Then we can prove regularity except on a set, hopefully not too large.

For linear systems, regularity results assuming $A_{ij}^{\alpha\beta}$ constant or in $C^0(\Omega)$, have been obtained by Campanato (see [3]). Without assuming continuity of coefficients, we mention the study by Acquistapace [1] where Campanato's results are refined considering that $A_{ij}^{\alpha\beta}$ belongs to a class that neither contains nor is being contained by $C^0(\Omega)$, hence in general discontinuous. Moreover, we recall the study made by Huang in [14] where he shows the regularity of weak solutions of linear elliptic systems with coefficients $A_{ij}^{\alpha\beta}(x) \in VMO$. So, it seems to be natural to expect partial regularity results under the condition that the coefficients of the principal terms $A_{ij}^{\alpha\beta} \in VMO$, even for nonlinear cases. Daneček and Viszus in [6] consider the regularity of minimizer for the functional

$$\int_\Omega \{A_{ij}^{\alpha\beta}(x) D_\alpha u^i D_\beta u^j + g(x, u, Du)\} dx$$

where the term $g(x,u,Du)$ is such that

$$|g(x,u,z)| \leq f(x) + |z|^{\gamma}$$

where $f \in L^p(\Omega)$, $2 < p \leq \infty$, $f \geq 0$ a. e. on Ω, L is a non-negative constant and $0 \leq \gamma < 2$.

They obtained hölder regularity of minimizer assuming that $A_{ij}^{\alpha\beta}(x) \in VMO$.

We also recall the paper [8] where the authors obtain regularity results for minimizers of the quasilinear functionals

$$\int_{\Omega} A_{ij}^{\alpha\beta}(x,u) D_\alpha u^i D_\beta u^j dx,$$

where the coefficients $A_{ij}^{\alpha\beta}(x,u)$ have VMO dependence on the variable x and continuous dependence on u.

In the paper [21] we improve the last mentioned result in Morrey spaces because we have considered inside the integral the term $g(x,u,Du)$ and the result by Danečĕk and Viszus because we consider $A_{ij}^{\alpha\beta}$ dependent not only on x but also on u. In the present note we extend our above cited regularity results because we consider the more general class Campanato spaces.

Before an outline of the proof we state a preliminary Lemma by Campanato.

Lemma 2.6. *Let $B(x_0,R)$be a fixed ball and $u \in W^{1,2}(B(x_0,R);\mathbb{R}^N)$ be a weak solution of the system*

$$D_\alpha(A_{ij}^{\alpha\beta} D_\beta u^j) = 0, \quad i = 1,\dots,N$$

where $A_{ij}^{\alpha\beta}$ are constant and satisfy the ellipticity condition. Then $\forall t \in (0,1]$

$$\int_{B(x_0,tR)} |Du|^2 dx \leq c \cdot t^n \int_{B(x_0,R)} |Du|^2 dx.$$

PROOF OF THE THEOREM

For simplicity we 'll consider the case $g = 0$. Let $R > 0$ and $x_0 \in \Omega$ such that $B(x_0,R) \subset\subset \Omega$.

Let v be the minimum of the "freezing" functional $\mathcal{A}$, that is

$$\mathcal{A}_0(v, B(x_0,\frac{R}{2})) = \int_{B(x_0,\frac{R}{2})} A_{ij}^{\alpha\beta}(x_0, u_{\frac{R}{2}}) D_\alpha v^i D_\beta v^j dx$$

with $v \equiv u$ on $\partial B(x_0 \frac{R}{2})$. The idea of freezing is the same used by Chiarenza, Frasca and Longo in [4].

For $0 \le \lambda < n$ and $q \le 2$ we have that

$$\|Du\|_{\mathcal{L}^{q,\lambda}(\Omega)} = \|Du\|_{L^q(\Omega)} + [Du]_{q,\lambda} \le$$

$$\le \mathcal{K}\,\|Du\|_{L^{q,\lambda}(\Omega)} \le \mathcal{K}\,\|Du\|_{L^{2,\lambda}(\Omega)}$$

where the constant $\mathcal{K}$ is independent of u. We observe that $A_{ij}^{\alpha\beta}(x_0, u_{\frac{R}{2}})$ are constant coefficients, then from the above Lemma, $\forall t \in (0,1]$,

$$\int_{B(x_0,t\frac{R}{2})} |Du|^2 dx \le c\cdot t^n \int_{B(x_0,\frac{R}{2})} |Du|^2 dx.$$

Let $w = u - v$, then $w \in W_0^{1,2}(B(x_0, \frac{R}{2}))$

$$\int_{B(x_0,\frac{tR}{2})} |Du|^2 dx \le$$

$$\le \; c\cdot \left\{ t^n \int_{B(x_0,\frac{R}{2})} |Du|^2 dx + \int_{B(x_0,\frac{R}{2})} |Dw|^2 dx \right\}.$$

Let us estimate:

$$\int_{B(x_0,\frac{R}{2})} |Dw|^2\, dx.$$

From the hypothesis and a Lemma in [12] we have

$$\nu \int_{B(x_0,\frac{R}{2})} |Dw|^2 dx \le \left\{ \mathcal{A}^0(u, B(x_0,\frac{R}{2})) - \mathcal{A}^0(v, B(x_0,\frac{R}{2})) \right\}$$

adding and subtracting:

$$A_{ij}^{\alpha\beta}(x, u_{\frac{R}{2}}) D_\alpha u^i D_\beta u^j, \quad A_{ij}^{\alpha\beta}(x, u_{\frac{R}{2}}) D_\alpha v^i D_\beta v^j$$

$$A_{ij}^{\alpha\beta}(x, u) D_\alpha u^i D_\beta u^j\,, \quad A_{ij}^{\alpha\beta}(x, v)\, D_\alpha v^i D_\beta v^j$$

we obtain some different kinds of integrals that we now examine.

Using L^p estimate, we can estimate the terms with $|Du|^2$ as follows:

$$\int_{B(x_0,\frac{R}{2})} |A_{ij}^{\alpha\beta}(x_0,u_{\frac{R}{2}}) - A_{ij}^{\alpha\beta}(x,u_{\frac{R}{2}})| \cdot |Du|^2\,dx \le$$

$$\le c\left\{\eta(A(\cdot,u_{\frac{R}{2}});R)\right\}^{1-\frac{2}{p}} \int_{B(x_0,R)} |\,Du\,|^2\,dx.$$

We also observe that:

$$\int_{B(x_0,\frac{R}{2})} |A_{ij}^{\alpha\beta}(x,u_{\frac{R}{2}}) - A_{ij}^{\alpha\beta}(x,u)|\,D_\alpha u^i D_\beta u^j\;dx \le$$

$$\le c\left(\int_{B(x_0,R)}|Du|^2dx\right)\left(\fint_{B(x_0,\frac{R}{2})}\omega(|u_{\frac{R}{2}}-u|^2)dx\right)^{1-\frac{2}{p}} \le$$

$$\le c\left(\int_{B(x_0,R)}|Du|^2dx\right)\left(\omega\left(R^{2-n}\int_{B(x_0,\frac{R}{2})}|D\,u|^2\,dx\right)\right)^{1-\frac{2}{p}}.$$

Moreover, we can estimate the terms having $|Dv|^2$ similarly using L^p estimates for Dv.

Then if $\rho = tR$

$$\int_{B(x_0,\rho)}|Du|^2dx \le$$

$$\le c\left\{\left(\frac{\rho}{R}\right)^n + \left(\omega\left(R^{2-n}\int_{B(x_0,\frac{R}{2})}|D\,u|^2\,dx\right)\right)^{1-\frac{2}{p}}\right.$$

$$\left. +\left(\eta(A(\cdot,u_{\frac{R}{2}}),R)\right)^{1-\frac{2}{p}}\right\}\cdot\left(\int_{B(x_0,\frac{R}{2})}|D\,u|^2dx\right).$$

Using a Lemma contained in [9] and selecting ρ sufficiently small, specifically $\rho < \frac{R}{2}$, we have

$$\int_{B(x_0,\rho)}|Du|^2dx \le c\cdot\rho^\lambda.$$

References

[1] P.ACQUISTAPACE, On $BMO-$regularity for linear elliptic systems, *Annali Mat. Pura Appl.* **161** (4) (1992), 231–269.

[2] M. BRAMANTI, M. C. CERUTTI, $W_p^{1,2}$Solvability for the Cauchy-Dirichlet Problem for Parabolic Equations with VMO coefficients, *Comm. in Partial Differential Equations*18 **(9 and 10)** (1993), 1735–1763.

[3] S. CAMPANATO, Sistemi ellittici in forma di divergenza. Regolarita' all'interno, *Quaderni Scuola Norm. Pisa* (1980).

[4] F. CHIARENZA, M. FRASCA, P. LONGO, Interior $W^{2,p}$ estimates for nondivergence elliptic equations with discontinuous coefficients, *Ricerche di Matematica* **40** (1991), 149–168.

[5] F. CHIARENZA, M. FRASCA, P. LONGO, $W^{2,p}$-solvability of the Dirichlet problem for nondivergence elliptic equations with VMO coefficients, *Trans. Amer. Math. Soc.* **336** (1993), 841-853.

[6] J. DANĚČEK, E. VISZUS, $L^{2,\lambda}-$Regularity for minima of variational integrals, *Boll. U.M.I.* **8** 6-B (2003), 39–48.

[7] E. DE GIORGI, Un esempio di estremali discontinue per un problema variazionale di tipo ellittico, *Boll. U.M.I.* **4** (1968), 135–137.

[8] P. DI GIRONIMO, L. ESPOSITO, L SGAMBATI, A remark on $L^{2,\lambda}-$regularity for minimizers of quasilinear functionals, *Manuscripta Math.* **113** (2004), 143–151.

[9] M. GIAQUINTA, Multiple integrals in the calculus of variations and nonlinear elliptic systems, *Ann. of Math. Studies* **105** (1983), Princeton University Press.

[10] M. GIAQUINTA, Introduction to regularity theory for nonlinear elliptic systems, *Lectures in Mathematics, ETH Zürich, Birkhäuser Verlag* (1993).

[11] M. GIAQUINTA, E. GIUSTI, On the regularity of the minima of variational integrals, *Acta Math.* **148** (1982), 31–46.

[12] M. Giaquinta, E. Giusti, Differentiability of minima non-differentiable functionals, *Invent. Math.* **72** (1983), 285–298.

[13] E. Giusti, M. Miranda, Un esempio di soluzioni discontinue per un problema di minimo relativo ad un integrale regolare del calcolo delle variazioni, *Boll. U.M.I.* **2** (1968), 1–8.

[14] Q. Huang, Estimates on the generalized Morrey Spaces $L_{\phi}^{p,\lambda}$ and BMO for linear elliptic systems, *Indiana Univ. Mah. Journal* **45** (2) (1996), 397–439.

[15] F. John, L. Nirenberg, On functions of bounded mean oscillation, *Comm. Pure Appl. Math.* **14** (1961), 415–476.

[16] A. Kufner, O. John, S. Fučik, Functions spaces, *Academia, Prague* (1977).

[17] V.G. Maz'ya, Examples of nonregular solutions of quasilinear elliptic equations with analytic coefficients, *Funktsional'nyi Analiz. i Ego Prilosheniya* **2** (1968), 53–57.

[18] C. Miranda, Sulle equazioni ellittiche del secondo ordine a coefficienti discontinui, *Annali Mat. Pura Appl.* **63** (1963), 353–386.

[19] C.B. Morrey Jr., Multiple integrals in the calculus of variations, *Springer Verlag, Berlin, Heidelberg, New York* (1966).

[20] M.A. Ragusa, Regularity of solutions of divergence form elliptic equations, *Proc. Amer. Math. Soc.* **128** (1999), 533–540.

[21] M. A. Ragusa and A. Tachikawa, Partial regularity of the minimizers of quadratic functionals with VMO coefficients, (preprint).

[22] D. Sarason, On functions of vanishing mean oscillation, *Trans. Amer. Math. Soc.* **207** (1975), 391–405.

[23] J. Souček, Singular solutions to linear elliptic systems, *Comment. Math. Univ. Carolinae* **23** (1984), 273–281.

[24] A. Tachikawa, On interior regularity and Liouville's theorem for harmonic mappings, *Manuscripta Math.* **42** (1983), 11–40.

M.A. Ragusa, Dipartimento di Matematica, Università di Catania, Viale A. Doria, 6, 95125 Catania, Italia
E-mail address: maragusa@dipmat.unict.it

A. Tachikawa, Department of Mathematics, Faculty of Science and Technology, Tokyo University of Science, Noda, Chiba, 278-8510, Japan
E-mail address: tachikawa_atsushi@ma.noda.tus.ac.jp