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エネルギー散逸を伴う遷音速流に対する二次元固有値問題

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Self-similar solutions play a crucial role in many branches of physics, in particular, for such fields as hydrodynamic phenomena in astrophysics. For example, the Larson-Penston (LP) solution[1,2] facilitates qualitative analysis of complex hydrodynamic flows of gravitational collapse of an isothermal gaseous sphere,[3,4] which is proposed to explain the qualitative dynamics in the early stage of star formation. However, the effect of radiative heat conduction is expected to play an important role in such a temporal domain that substantial dissociation and ionization of molecules and atoms proceed with contraction of the system and that the isothermal assumption is not appropriate any more. Still, among a large variety of self-similar solutions of the hydrodynamic equations, those which include heat conduction are relatively few.[5-7] Still more, to the best of our knowledge, there has been no other publications on the self-similar solution, which simultaneously treats both the self-gravity and the non-linear heat conductivity. A striking difference between the outputs of the LP model, for example, and the present model is found in the physical picture of the core formation. The former and the latter respectively describe a decreasing and increasing core mass with time. Figure 1 highlights the output of the present work showing the temporal evolution of the density and velocity profiles at sequential times. As can be seen in Fig. 1, the core shrinks with time, where $t = 0$ corresponds to the collapse time. The central density increases in proportion to t^{-2}, that is the universal scaling regardless of the degree of the heat conductivity. At the same time, the core mass also increases due to mass accretion. One more important feature of the present model, which is essentially different from the conventional ones obtained under the isothermal or the adiabatic assumptions, is that all the scales of the physical quantities are uniquely determined as a function of time only.

The one-dimensional spherical gas-dynamical equations with both self-gravity and heat conductivity are

$$\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho u)=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}-\frac{\partial \phi}{\partial r}, \tag{2}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \phi}{\partial r}\right)=4\pi G\rho, \tag{3}$$

$$\rho\left(\frac{\partial \varepsilon}{\partial t}+u\frac{\partial \varepsilon}{\partial r}\right)+\frac{p}{r^2}\frac{\partial}{\partial r}(r^2 u)=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\kappa\frac{\partial T}{\partial r}\right) \tag{4}$$

where p is the pressure, r the mass density, e the specific internal energy, u the flow velocity, ϕ the gravity potential, and G the gravity constant. We assume the ideal gas equation of state (EOS) in the form,

$$T=p/\rho=(\gamma-1)\varepsilon, \tag{5}$$

where γ is the specific heats ratio. Equation (4), described by the one-temperature model, includes the non-linear heat conduction term on the right hand side, where we assume a power-law dependence for the conduction coefficient,

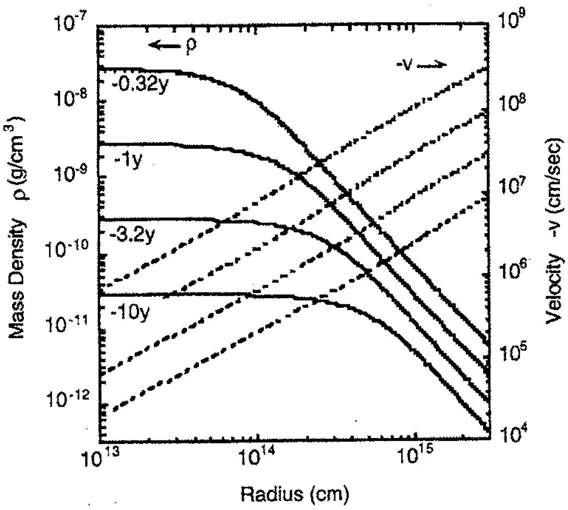

Fig. 1 Temporal evolution of the density and velocity profiles at different sequential times.

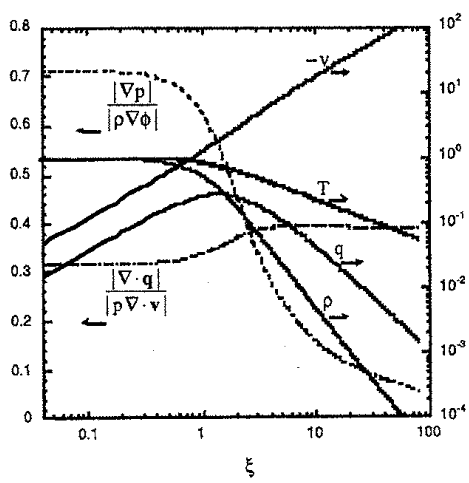

Fig. 2 The eigenstructure of the case, m = 2, n = 13/2, and γ = 5/3.

$$\kappa = \kappa_0 T^n/\rho^m , \tag{6}$$

with κ_0, m, and n being constants. For normal physical values, n > 0 and m > 0 are assumed. In the numerical calculations given below, we keep the generality in terms of the parameters, m, n, and γ, but also show specific forms using the values of the reference set, m = 2 and n = 13/2, which describing the opacity due to inverse bremsstrahlung in a fully ionized hydrogen plasma together with $\gamma = 5/3$.

We introduce the following similarity ansatz,

$$\xi = r/R(t), \quad R(t) = A\,|\,t\,|^{1/a}, \tag{7}$$

$$u = \frac{A}{a}|\,t\,|^{b/a}\, v(\xi), \quad \rho = B\,|\,t\,|^{-2}\, g(\xi), \quad T = \left(\frac{A}{a}\right)^2 |\,t\,|^{c/a}\, \tau(\xi), \tag{8}$$

$$\frac{\partial\phi}{\partial r} = \frac{ABG}{\xi^2}|\,t\,|^{(e-1)/a}\,\Omega(\xi), \quad \Omega(\xi) = 4\pi\int_0^{\xi}\xi^2 g(\xi)d\xi , \tag{9}$$

where $1-a = b = c/2 = 1+d/2 = e/2 = (1+2m)/(3+2m-2n)$, and R(t) is the temporal characteristic scale length of the system; A and B are positive constants defining the scales of the radius and the mass density, respectively. Then, Eqs. (1), (2), and (4) are reduced respectively to the following ordinary differential equations,

$$-(\pm\xi - v)g' + (\pm d + v' + 2v/\xi)g = 0, \tag{10}$$

$$\pm bv - (\pm\xi - v)v' + (g\tau)'/g + K_1\Omega/\xi^2 = 0, \tag{11}$$

$$\frac{\pm c\tau - (\pm\xi - v)\tau'}{\gamma - 1} + (v' + 2v/\xi)\tau = K_2\frac{(\xi^2 g^{-m}\tau^n\tau')'}{g\xi^2}, \tag{12}$$

where the prime denotes the derivative with respect to x , and concerning the double signs, $\pm$, the upper and lower sign correspond to t > 0 and t < 0, respectively. In the following analysis, we focus on the time domain, t < 0, and therefore $|\,t\,| = -t$. Since Eq. (3) is automatically satisfied, its reduced form does not appear in the above set of equations. As can be seen in Eqs. (11) and (12), the present system is characterized by the two dimensionless parameters, K_1 and K_2, defined by

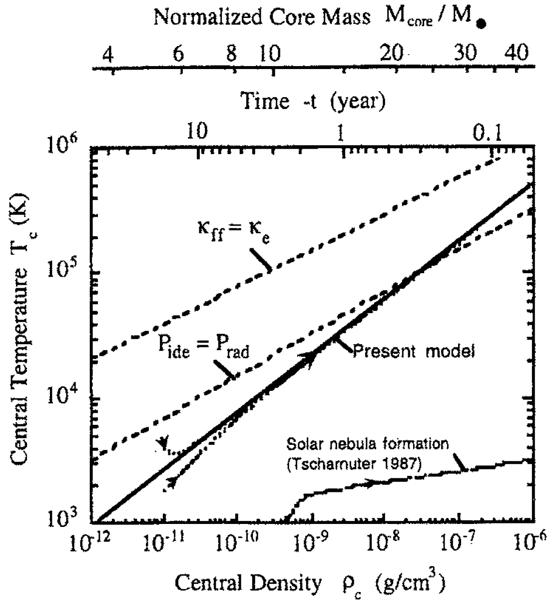

Fig. 3 Temporal evolution of the core parameters, r_c vs T_c, for the reference case, m = 2 and n = 13/2 (solid line). Both ρ_c and the core mass are directly related to time in the present model. The dashed line labeled by $P_{ide} = P_{rad}$ denotes temperature vs. density, on which the ideal EOS pressure, P_{ide}, balances with the radiation pressure, P_{rad}. The dashed line labeled by $k_{ff} = k_e$ denotes the temperature vs density, on which the absorption coefficient due to inverse bremsstrahlung, k_{ff}, balances with the absorption coefficient due to Thomson scattering, k_e. The dotted curve showing solar nebula formation process is just for comparison with the present model, for which the time and mass axes do not apply.

$$K_1 = a^2 GB, \quad K_2 = \frac{\kappa_0}{aB^{m+1}}\left(\frac{A}{a}\right)^{2n-2}. \tag{13}$$

To determine a unique set of parameters, K_1 and K_2, we need two more physical conditions. The first one is quite an orthodox prescription that the right integration curve smoothly passes through the singular point which is located somewhere at a finite distance from the center. Note that the LP solution does not need energy equation (4) under the isothermal assumption, and the system is described in terms of only a single unknown constant, K_1, which is determined by this first condition. The second one is somewhat less obvious compared with the first one, but still seems enough natural, namely, that both the density and the temperature converge to zero simultaneously with increasing the radius. Thus, the system under consideration is reduced to a two-dimensional eigenvalue problem.

Figure 2 shows the eigenstructure for the temperature, $\tau \propto T$, the density, $g \propto \rho$, the velocity, $v \propto u$, and the heat flux, $\mathbf{q} = -\kappa \nabla T$, of the system under the eigenvalues thus obtained, where the curves are assigned with labels corresponding to the original physical quantities just for simplicity. Also two other dimensionless quantities of interest are shown in Fig. 2. The first one is the ratio of the plasma pressure to the gravity, $\Psi \equiv |\nabla p|/|\rho \nabla \phi|$, which is given in the form,

$$\Psi \equiv \frac{|\nabla p|}{|\rho \nabla \phi|} = \begin{cases} 1-(n-1)^2/6\pi(n-m-3/2)^2 K_1 & (\xi << 1) \\ K_1 \xi^{-2(n-1)(2+3m-n)/n(1+2m)} & (\xi >> 1) \end{cases} \tag{14}$$

The second one is the Péclet number mentioned in the introduction, i.e., the ratio of the heating to the mechanical compression (pdV) work, which is given in the form,

$$\mathrm{Pe} \equiv \left|\frac{p\nabla \cdot \mathbf{u}}{\nabla \cdot \mathbf{q}}\right| = \begin{cases} 4(n-1)/(4n-6m-7) & (\xi << 1) \\ \{1-(m+2/3)/(\gamma-1)n\}^{-1} & (\xi >> 1) \end{cases} \tag{15}$$

For the reference case, m = 2, n = 13/2, and $\gamma = 5/3$, Eqs. (14) and (15) are reduced to

$$\begin{cases} Pe \to 3.1, & \Psi \to 0.71 \qquad (\xi \ll 1) \\ Pe \to 2.6, & \Psi \to 0.62\xi^{-0.51} \qquad (\xi \gg 1) \end{cases} \tag{16}$$

For the LP solution, $\Psi \sim 0.6$ ($\xi \ll 1$), which is close to that derived above. In the outer region for $\xi \gg 1$, however, Ψ for the LP solution remains constant ~ 0.2, which contrasts with $\Psi \to 0$ in Eq. (16). The constancy of Pe tells that the entropy is persistently emitted outward in the entire space. For such a non-adiabatic implosions, the self-similar solution predicts that the flow pattern should approach an asymptotical regime in which it ceases to depend on the initial entropy, in other words, the system will "forget" certain aspects of the initial state. In that sense, the radiative heat conductivity is expected to substantially affect the self-organization process of the core formation.

Figure 3 shows the temporal evolution of the core parameters, ρ_c versus T_c, as the solid line, which are both expressed as a function of time. Here it should be noted that the core mass is also uniquely measured as a function of time, which appears as the uppermost axis in Fig. 3. At temperatures of few thousands K and densities of about 10^{-11} g/cm^3, hydrogen atoms are expected to be enough ionized. Thus, we can roughly fix the applicable parameter domain of the present solution from Fig. 3: T = a few 10^3 - a few 10^4 K, $\rho_c = 10^{-11} - 10^{-9}$ g/cm^3, $|t|$ = a few - a few tens of years, and the core mass of about ten times the solar mass. The core evolves at rather high temperatures compared, for example, with the solar nebula formation process.[8] Note that, as is clear from the model description, the present analysis is not limited to such a special combination of physical effects as the ideal EOS and the inverse bremsstrahlung opacity, but more extensively applies to other physical situations, under which one will obtain a different solution and scalings correspondingly.

In summary, taking both the gravity and the radiative heat conductivity into account simultaneously, a new class of self-similar solution for spherical implosions of gaseous masses has been found. The solution has been investigated in detail in terms of the constants, m and n, which characterize the radiative heat conductivity. Under the appropriate similarity ansatz and variable transformations, the hydrodynamic system is reduced to the novel two-dimensional eigenvalue problem. The physical implication is that a unique quantitative relations between the gravity and the heat conductivity indwells in the self-similar dynamics. Also, it has turned out that the present system has no free parameter to control the system as in the cases under adiabatic or isothermal assumptions. Furthermore, in contrast to the LP solution, the core mass increases with time due to mass accretion, which results from the persistent entropy emission via radiation.

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