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Rendezvous Search with Examination Cost on a Finite Graph

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Abstract. In a rendezvous search on a finite graph, two agents are placed randomly on nodes. At each step each moves to an adjacent node or stays where he is. It costs an amount for each step and when each agent moves from a node to an adjacent node and also when each examines a node. Their common purpose is to minimize the expected cost required to meet. We analyze a case where the graph is a star graph and the examination cost is the same for every node. Then we try to extend our study to the case where the graph is a star graph and the examination cost for every terminal node is the same.

Key words : rendezvous search, examination cost, optimal strategy.

1. Introduction.

In a rendezvous search two or more agents try to minimize the cost needed to find each other. In a discrete form of the rendezvous search problem, the agents move in discrete time from node to node on a finite and connected graph after they are placed randomly on nodes of the graph. The nodes can be marked if they have a name that both agents can recognize when they go there; they are markable if each agent knows whether it has visited a node or not or they are unmarkable if each agent forgets whether it has visited a node or not. When agents can choose different strategies, the model is called asymmetric, while it is called symmetric if agents must choose the same strategy. In a basic model of rendezvous search, the cost consists of only the time needed to find each other.

Anderson and Weber (1990) first proposed the graph formulation of a rendezvous search problem, and Alpern (1995) formalized the general continuous rendezvous. Since Alpern's work, many papers on rendezvous search have been published. Alpern and Beck (1999) assumed individual limits for the total distance that each player can travel. Alpern and Gal (2003) is a monograph dealing with search games and rendezvous search with an extensive bibliography.

In this paper we consider a discrete form and try to solve the asymmetric rendezvous search problem when the nodes are markable. It is well-known that in an area of the search game, the model becomes more practical and interesting by assuming the examination cost, but it becomes very difficult to solve. The most remarkable point in this paper is that we consider an examination cost and a traveling cost as well as the cost for the time needed to find each other. After seeing Theorems 1 and 2 in this paper, the reader would know that the problem becomes very difficult to solve when the examination cost is different from node to node.

2. Model and Notation.

We consider a finite graph (N, E) where $N = \{0, 1, \ldots, m\}$ is the set of nodes and $E \subseteq N \times N$ is the set of edges. Two agents (called Agents I and II) are placed on nodes of the graph (N, E) at random. They know their position up to the symmetry of the graph. We assume that the nodes can be markable, that is, an agent can remember nodes where he/she examines or stays in the previous steps. We measure time discretely, in *steps*. At each step each agent stays where he/she is or moves to an adjacent node. Furthermore, at each step he/she may or may not examine a node. So at each step each agent can choose one of four alternatives :

(i) ME: move to an adjacent node and examine the node ;
(ii)MN: move to an adjacent node and does not examine the node;
(iii)SE: stay where he/she is and examine the node; and
(iv)SN: stay where he/she is and does not examine the node.

They can find each other only when (i) they are at the same node, and (ii) at least one of them examines. They can not find when they transpose their positions between two adjacent nodes. It costs 1 when each moves from a node to an adjacent node (i.e., traveling cost) and a_i when each examines the node $i \in N$ (i.e., examination cost). It cost c for each step. Their common purpose is to minimize the expected cost required to meet as well as the expected number of steps. We assume that agents can choose different strategies, that is, the model is asymmetric. By this setting, at every step, at most one of the agents must examine a node. So without loss of generality, we have

Assumption 1. At every step Agent II does not examine a node.

In the remaining part of this section, Sections 3 and 4, we assume a star graph, that is,

$$E = \{(0, 1), \ldots, (0, m)\}.$$

The node 0 is called the central node and the other nodes are called terminal nodes. Each agent can distinguish whether he/she is at a terminal node or not. In Section 3 we analyze a special case where the examination cost is the same for every node. Then we try to extend our study to the case where the examination cost for every terminal node is the same.

The choices of players at the step t are denoted by $d_i(ter(cen), t), i = 1, 2$ for the choice of Agent i when he is at the terminal (central) node respectively. The nodes where players are at the end of the step t is denoted by $n_i(t), i = 1, 2$ for the node where Agent i is. $n_I(0)$ and $n_{II}(0)$ are nodes where Agents are put initially. We let $d_I(t) \equiv (d_I(ter, t), d_I(cen, t))$ and $d_{II}(t) \equiv (d_{II}(ter, t), d_{II}(cen, t))$. Strategies for Agents I and II are :

$d_I \equiv (d_I(1), d_I(2), \ldots)$ and $d_{II} \equiv (d_{II}(1), d_{II}(2), \ldots)$.

By Assumption 1, Agent II has only two choices at each step, that is, $d_{II}(ter, t) = SN$ or MN, and $d_{II}(cen, t) = SN$ or MN. We write as $f(d_I, d_{II}; n_I(0), n_{II}(0))$ the expected cost when Agents adopt the strategy pair (d_I, d_{II}) and they are put at $n_I(0), n_{II}(0)$ initially, and then as $f(d_I, d_{II})$ the expected cost when Agents adopt the strategy pair (d_I, d_{II}). Then

$$\begin{aligned} f(d_I, d_{II}) &= \sum_{n_I(0), n_{II}(0)} f(d_I, d_{II}; n_I(0), n_{II}(0)) P(n_I(0), n_{II}(0)) \\ &= \frac{m}{(m+1)^2} f(d_I, d_{II}; ter, cen) + \frac{m}{(m+1)^2} f(d_I, d_{II}; cen, ter) + \frac{m(m-1)}{(m+1)^2} f(d_I, d_{II}; ter, \overline{ter}) \quad (1) \\ &\quad + \frac{m}{(m+1)^2} f(d_I, d_{II}; ter, ter) + \frac{1}{(m+1)^2} f(d_I, d_{II}; cen, cen), \end{aligned}$$

where $\overline{ter}$ means a different terminal from ter. So $f(d_I, d_{II}; ter, \overline{ter})$ is the expected cost when both agents are at terminal nodes which are different. $f(d_I, d_{II}; ter, ter)$ is the expected cost when both agents are at the same terminal node.

3. Analysis of a Special Case

In this section, we analyze the case where the examination cost is the same for every node, that is,

$$a_i = a \text{ for every } i \in N.$$

For simplicity we put

Assumption 2. At the first step Agent I examines.

The problem is solved and given as the next theorem.

Theorem 1. If $c + a > \frac{2}{m+1}$ then an optimal strategy is

$$(S1) \qquad d_I(t) = (ME, SE), \text{ and } d_{II}(t) = (MN, SN), \forall t = 1, \ldots.$$

The expected cost for this strategy is $2 + c + a - \frac{2}{m+1}$.
If $c + a < \frac{2}{m+1}$ then an optimal strategy is

$$(S2) \qquad d_I(1) = (SE, SE), d_I(2) = (ME, SE) \text{ and } d_{II}(1) = (SN, SN), d_{II}(2) = (MN, SN).$$

The expected cost for this strategy is $2(\frac{m}{m+1})^2 + \frac{2m+1}{m+1}(c + a)$.

Roughly speaking, the theorem says that if $c + a$ is relatively large, then Agents must move to the central node first and then examine, while if $c + a$ is relatively small, Agents must examine first and then move to the central node. As m becomes large, the expected cost converges to $2 + c + a$, which is attained by the strategy $(S1)$ since the probability that both agents are at the central node converges to 0.

Proof: We calculate the expected cost for the strategy $(S1)$.

$$f(d_I, d_{II}; ter, cen) = 1 + c + a, \text{ since } d_I(ter, 1) = ME, \text{ and } d_{II}(cen, 1) = SN,$$

$$f(d_I, d_{II}; cen, ter) = 1 + c + a, \text{ since } d_I(cen, 1) = SE, \text{ and } d_{II}(ter, 1) = MN,$$

$$f(d_I, d_{II}; ter, \overline{ter}) = 2 + c + a, \text{ since } d_I(ter, 1) = ME, \text{ and } d_{II}(ter, 1) = MN,$$

$$f(d_I, d_{II}; ter, ter) = 2 + c + a, \text{ since } d_I(ter, 1) = ME, \text{ and } d_{II}(ter, 1) = MN,$$

$$f(d_I, d_{II}; cen, cen) = c + a, \text{ since } d_I(cen, 1) = SE, \text{ and } d_{II}(cen, 1) = SN.$$

Hence by the equality (1), we have $f(d_I, d_{II}) = 2 + c + a - \frac{2}{m+1}$.

We calculate the expected cost for the strategy $(S2)$.

$$f(d_I, d_{II}; ter, cen) = 1 + 2c + 2a, \text{ since } d_I(ter, 1) = SE, \text{ and } d_{II}(cen, 1) = SN, \\ d_I(ter, 2) = ME, \text{ and } d_{II}(cen, 2) = SN.$$

$$f(d_I, d_{II}; cen, ter) = 1 + 2c + 2a, \text{ since } d_I(cen, 1) = SE, \text{ and } d_{II}(ter, 1) = SN,$$
$$d_I(cen, 2) = SE, \text{ and } d_{II}(ter, 2) = MN.$$
$$f(d_I, d_{II}; ter, \overline{ter}) = 2 + 2c + 2a, \text{ since } d_I(ter, 1) = SE, \text{ and } d_{II}(ter, 1) = SN,$$
$$d_I(ter, 2) = ME, \text{ and } d_{II}(ter, 2) = MN.$$
$$f(d_I, d_{II}; ter, ter) = c + a, \text{ since } d_I(ter, 1) = SE, \text{ and } d_{II}(ter, 1) = SN,$$
$$f(d_I, d_{II}; cen, cen) = c + a, \text{ since } d_I(cen, 1) = SE, \text{ and } d_{II}(cen, 1) = SN.$$

Hence by the equality (1), we have $f(d_I, d_{II}) = 2(\frac{m}{m+1})^2 + \frac{2m+1}{m+1}(c + a)$.

So, by considering the strategies ($S1$) and ($S2$), the expected cost is at most $min\{2 + c + a - \frac{2}{m+1}, 2(\frac{m}{m+1})^2 + \frac{2m+1}{m+1}(c + a)\}$. Next before seeing that the expected cost is at least $min\{2 + c + a - \frac{2}{m+1}, 2(\frac{m}{m+1})^2 + \frac{2m+1}{m+1}(c + a)\}$, we need lemmas.

Lemma 1A.

$$f(d_I, d_{II}; ter, cen) \begin{cases} \geq 1 + 2c + 2a & \text{if } d_I(ter, 1) = SE \text{ and } d_{II}(cen, 1) = SN; \\ \geq 1 + a + c + \frac{m-1}{m}(2 + c + a) & \text{if } d_I(ter, 1) = SE \text{ and } d_{II}(cen, 1) = MN; \\ = 1 + c + a & \text{if } d_I(ter, 1) = ME \text{ and } d_{II}(cen, 1) = SN; \\ \geq 3 + 2c + 2a & \text{if } d_I(ter, 1) = ME \text{ and } d_{II}(cen, 1) = MN. \end{cases}$$

By the symmetry of the initial positions of the players, we have Lemma 1B. We get Lemmas 1C-1E in a similar way to Lemma 1A.

Lemma 1B.

$$f(d_I, d_{II}; cen, ter) \begin{cases} \geq 1 + 2c + 2a & \text{if } d_I(cen, 1) = SE \text{ and } d_{II}(ter, 1) = SN; \\ = 1 + c + a & \text{if } d_I(cen, 1) = SE \text{ and } d_{II}(ter, 1) = MN; \\ \geq 1 + c + a + \frac{m-1}{m}(2 + c + a) & \text{if } d_I(cen, 1) = ME \text{ and } d_{II}(ter, 1) = SN; \\ \geq 3 + 2c + 2a & \text{if } d_I(cen, 1) = ME \text{ and } d_{II}(ter, 1) = MN. \end{cases}$$

Lemma 1C.

$$f(d_I, d_{II}; ter, \overline{ter}) \begin{cases} \geq 2 + 2c + 2a & \text{if } d_I(ter, 1) = SE \text{ and } d_{II}(\overline{ter}, 1) = SN; \\ \geq 2 + 2c + 2a & \text{if } d_I(ter, 1) = SE \text{ and } d_{II}(\overline{ter}, 1) = MN; \\ \geq 2 + 2c + 2a & \text{if } d_I(ter, 1) = ME \text{ and } d_{II}(\overline{ter}, 1) = SN; \\ = 2 + c + a & \text{if } d_I(ter, 1) = ME \text{ and } d_{II}(\overline{ter}, 1) = MN. \end{cases}$$

Lemma 1D.

$$f(d_I, d_{II}; ter, ter) \begin{cases} = c + a & \text{if } d_I(ter, 1) = SE \text{ and } d_{II}(ter, 1) = SN; \\ \geq 2 + 2c + 2a & \text{if } d_I(ter, 1) = SE \text{ and } d_{II}(ter, 1) = MN; \\ \geq 2 + 2c + 2a & \text{if } d_I(ter, 1) = ME \text{ and } d_{II}(ter, 1) = SN; \\ = 2 + c + a & \text{if } d_I(ter, 1) = ME \text{ and } d_{II}(ter, 1) = MN. \end{cases}$$

Lemma 1E.

$$f(d_I, d_{II}; cen, cen) \begin{cases} = c + a & \text{if } d_I(cen, 1) = SE \text{ and } d_{II}(cen, 1) = SN; \\ \geq 2 + 2c + 2a & \text{if } d_I(cen, 1) = SE \text{ and } d_{II}(cen, 1) = MN; \\ \geq 2 + 2c + 2a & \text{if } d_I(cen, 1) = ME \text{ and } d_{II}(cen, 1) = SN; \\ \geq 2 + a + c + \frac{m-1}{m}[2 + c + a] & \text{if } d_I(cen, 1) = ME \text{ and } d_{II}(cen, 1) = MN. \end{cases}$$

Lemma 1F.

$$f(d_I, d_{II}) \geq min\{2 + c + a - \frac{2}{m+1}, 2(\frac{m}{m+1})^2 + \frac{2m+1}{m+1}(c+a)\}, \forall (d_I, d_{II}).$$

By the equality (1) and Lemmas 1A-1E, we prove this lemma case by case. The remaining part of the proof is omitted.

From Lemma 1F we see the strategy $(S1)$ is optimal if $2(\frac{m}{m+1})^2 + \frac{2m+1}{m+1}(c+a) > 2+c+a-\frac{2}{m+1}$, i.e., $c + a > \frac{2}{m+1}$. The strategy $(S2)$ is optimal if $c + a < \frac{2}{m+1}$. This completes the proof of the theorem. ∎

4. Analysis of the Problem on a Star Graph with 3 Nodes.

In this section we solve the rendezvous search in the case of 3 nodes, that is, $m = 2$. We assume

$$a_o = a, \text{ and } a_1 = a_2 = b.$$

Letting $m = 2$ in (1), we get:

$$\begin{aligned} f(d_I, d_{II}) &= \sum_{n_I(0), n_{II}(0)} f(d_I, d_{II}; n_I(0), n_{II}(0)) P(n_I(0), n_{II}(0)) \\ &= \frac{2}{9} f(d_I, d_{II}; ter, mid) + \frac{2}{9} f(d_I, d_{II}; mid, ter) + \frac{2}{9} f(d_I, d_{II}; ter, \overline{ter}) \\ &\quad + \frac{2}{9} f(d_I, d_{II}; ter, ter) + \frac{1}{9} f(d_I, d_{II}; mid, mid), \end{aligned} \tag{2}$$

where $\overline{ter}$ means a different terminal from ter. The problem is to find a strategy pair (d_I, d_{II}) which minimizes the expected cost $f(d_I, d_{II})$.

Theorem 2. Assume either $a > 1 + c + 2b$ or $a < c + 2b$.
If $a > max\{\frac{3}{2}b + \frac{1}{3} + c, \frac{5}{6}b + \frac{7}{9} + \frac{c}{3}\}$ then an optimal strategy is

$(T1)$ $\quad d_I(1) = (SE, ME), d_I(2) = (MN, MN), d_I(3) = (ME, ME)$ and $d_{II}(t) = (SN, MN), \forall t = 1, \ldots.$

The expected cost for $(T1)$ is $\frac{5}{3} + \frac{3}{2}b + 2c$.
If $a < \frac{3}{2}b + \frac{1}{3} + c$ and $b > \frac{2}{3} - c$ then an optimal strategy is

$(T2)$ $\quad d_I(t) = (ME, SE),$ and $d_{II}(t) = (MN, SN), \forall t = 1, \ldots.$

The expected cost for $(T2)$ is $\frac{4}{3} + a + c$.
If $a < \frac{5}{6}b + \frac{7}{9} + \frac{c}{3}$ and $b < \frac{2}{3} - c$ then an optimal strategy is

$(T3)$ $\quad d_I(1) = (SE, SE), d_I(t) = (ME, SE)$ and $d_{II}(1) = (SN, SN), d_{II}(t) = (MN, SN), \forall t = 2, \ldots.$

The expected cost for $(T3)$ is $\frac{8}{9} + a + \frac{2}{3}b + \frac{5}{3}c$.

Roughly speaking, the theorem says that if a is relatively large, then Agents must move to the terminal nodes first and then examine, while if a is relatively small and b is relatively large, Agents must move to the middle node first and examine. If both of a and b is small, then Agents must stay first.

Proof: We calculate the expected cost for the strategy $(T1)$.

$$f(d_I, d_{II}; ter, mid) = 2 + \frac{3}{2}b + 2c, \text{ since } d_I(ter, 1) = SE, \text{ and } d_{II}(mid, 1) = MN,$$

$$f(d_I, d_{II}; mid, ter) = 2 + \frac{3}{2}b + 2c, \text{ since } d_I(mid, 1) = ME, \text{ and } d_{II}(ter, 1) = SN,$$

$$f(d_I, d_{II}; ter, \overline{ter}) = 2 + 2b + 3c, \text{ since } d_I(ter, 1) = SE, \text{ and } d_{II}(ter, 1) = SN,$$

$$f(d_I, d_{II}; ter, ter) = b + c, \text{ since } d_I(ter, 1) = SE, \text{ and } d_{II}(ter, 1) = SN,$$

$$f(d_I, d_{II}; mid, mid) = 3 + \frac{3}{2}b + 2c, \text{ since } d_I(mid, 1) = ME, \text{ and } d_{II}(mid, 1) = MN.$$

Hence by the equality (2), we have $f(d_I, d_{II}) = \frac{5}{3} + \frac{3}{2}b + 2c$.

We calculate the expected cost for the strategy $(T2)$.

$$f(d_I, d_{II}; ter, mid) = 1 + a + c, \text{ since } d_I(ter, 1) = ME, \text{ and } d_{II}(mid, 1) = SN,$$

$$f(d_I, d_{II}; mid, ter) = 1 + a + c, \text{ since } d_I(mid, 1) = SE, \text{ and } d_{II}(ter, 1) = MN,$$

$$f(d_I, d_{II}; ter, \overline{ter}) = 2 + a + c, \text{ since } d_I(ter, 1) = ME, \text{ and } d_{II}(ter, 1) = MN,$$

$$f(d_I, d_{II}; ter, ter) = 2 + a + c, \text{ since } d_I(ter, 1) = ME, \text{ and } d_{II}(ter, 1) = MN,$$

$$f(d_I, d_{II}; mid, mid) = a + c, \text{ since } d_I(mid, 1) = SE, \text{ and } d_{II}(mid, 1) = SN.$$

Hence by the equality (2), we have $f(d_I, d_{II}) = \frac{4}{3} + a + c$.

We calculate the expected cost for the strategy $(T3)$.

$$f(d_I, d_{II}; ter, mid) = 1 + a + b + 2c, \text{ since } d_I(ter, 1) = SE, \text{ and } d_{II}(mid, 1) = SN,$$

$$f(d_I, d_{II}; mid, ter) = 1 + 2a + 2c, \text{ since } d_I(mid, 1) = SE, \text{ and } d_{II}(ter, 1) = SN,$$

$$f(d_I, d_{II}; ter, \overline{ter}) = 2 + a + b + 2c, \text{ since } d_I(ter, 1) = SE, \text{ and } d_{II}(ter, 1) = SN,$$

$$f(d_I, d_{II}; ter, ter) = b + c, \text{ since } d_I(ter, 1) = SE, \text{ and } d_{II}(ter, 1) = SN,$$

$$f(d_I, d_{II}; mid, mid) = a + c, \text{ since } d_I(mid, 1) = SE, \text{ and } d_{II}(mid, 1) = SN.$$

Hence by the equality (2), we have $f(d_I, d_{II}) = \frac{8}{9} + a + \frac{2}{3}b + \frac{5}{3}c$.

So, by considering the strategies $(T1)$,$(T2)$ and $(T3)$, the expected cost is at most $min\{\frac{5}{3} + \frac{3}{2}b + 2c, \frac{4}{3} + a + c, \frac{8}{9} + a + \frac{2}{3}b + \frac{5}{3}c\}$. Next before seeing that the expected cost is at least $min\{\frac{5}{3} + \frac{3}{2}b + 2c, \frac{4}{3} + a + c, \frac{8}{9} + a + \frac{2}{3}b + \frac{5}{3}c\}$, we need lemmas. The remaining part of the proof is omitted. See Ruckle/Kikuta (2004).

5. A Search Game on a Finite Graph

Assuming the examination cost makes a model more realistic and interesting, but the analysis becomes difficult. In this section we see how it is when the model is a search game on a cyclic graph.

Let (N, E) be a cyclic graph, that is,

$$E = \{(0, 1), (1, 2), \ldots, (m - 1, m), (m, 0)\}.$$

There are two playsers, called the hider and the seeker. The hider hides among one of all nodes except for the node 0, and stays there. The seeker examines each node until he finds the hider, traveling along edges. We asume that at the beginning of the search the seeker is at the node 0, and he chooses a path which minimizes the length between i and j when $(i,j) \notin E$ and examines i after having examined j. Associated with the examination of $i(1 \leq i \leq m)$ is the cost that consists of two parts: (i) a traveling cost $d(i,j) > 0$ of examining i after having examined j, and (ii) an examination cost a_i. Assume

$$d(i,j) = 1 \text{ for all } (i,j) \in E.$$

There is not a probability of overlooking the hider, given that the right node is examined. Before searching (hiding resp.), the seeker (the hider) must determine a strategy so as to make the cost of finding the hider as small (large resp.) as possible. A pure strategy for the hider is expressed by an element in $N \setminus \{0\}$. A pure strategy for the seeker is a permutation σ on $N \setminus \{0\}$, which means the seeker examines $\sigma(1), \ldots, \sigma(m)$ in this order.

It is easy to solve this game for the case where $a_i = a$ for every $i \in N \setminus \{0\}$. But It is very difficult to solve this game in general. It is solved for a very special case.

Theorem 3. (Kikuta (2004)) Assume for $k > 0$, $\frac{1+a_i}{1+a_1} = k^{i-1}$ for every $i \in N \setminus \{0\}$. The value of the game is

$$\frac{1+a_1}{1+k} \sum_{x=1}^{m+1} k^{x-1}.$$

An optimal strategy for the hider is to hide at the node i with probability $\frac{k^{i-1}}{\sum_{x=1}^{x-1} k^{x-1}}$ for $i \in N \setminus \{0\}$. An optimal strategy for the seeker is to examine $1, \ldots, m$ in this order with probability $\frac{1}{k+1}$ and $m, m-1, \ldots, 1$ in this order with the remaining probability.

By letting $k = 1$, Theorem 3 is applicable to the case where $a_i = a$ for every $i \in N \setminus \{0\}$. The hider must hide at every node at random, and the seeker examines $1, \ldots, m$ and $m, m-1, \ldots, 1$ with probability $1/2$ respectively. When $m = 3$, this game is solved completely (See Kikuta (2004)). It may be difficult but interesting to solve this game when m is small.

6. Comments.

This is the first study for rendezvous search on a finite graph with examination cost, and the problems are solved for very special cases. The followings are left for the future studies:

(i) Rendezvous search on a star graph where the examination costs are different.

(ii) Rendezvous search on a linear graph with examination cost where the meeting takes place when agents are at adjacent nodes.

(iii) After solving unmarkable cases, apply the results to find upper bounds for the markable case.

(iv) Symmetric rendezvous search on a star graph.

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