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Asymptotic behavior and classical limit of the solutions to quantum hydrodynamic model for semiconductors

東京工業大学 鈴木 政尋 (Masahiro Suzuki)
東京工業大学 西畑 伸也 (Shinya Nishibata)

Department of Mathematical and Computing Sciences
Tokyo Institute of Technology, Tokyo 152-8552, Japan

1 Introduction

The main purpose of this short paper is to show the existence and the asymptotic stability of a stationary solution to the initial boundary value problem for a one-dimensional quantum hydrodynamic model of semiconductors. We also study a singular limit from this model to the classical hydrodynamic model. This limit is called a classical limit. In this paper, we briefly discuss the results in our paper [14].

A quantum effect, depending on particle resonant tunneling through potential barriers and charge density built-up in quantum wells, is not negligible in analysis on the behavior of electron flow through semiconductor devices as they become truly minute. The quantum hydrodynamic model is one of several models including quantum effect and derived from the moment expansion of the Wigner-Boltzmann equation (see [1, 4] for details).

It is formulated as the system of equations, corresponding to the conservation law of mass, the balance law of momentum and the Poisson equation

$$\rho_t + j_x = 0, \tag{1.1a}$$

$$j_t + \left(\frac{j^2}{\rho} + p(\rho)\right)_x - \varepsilon^2\rho\left(\frac{(\sqrt{\rho})_{xx}}{\sqrt{\rho}}\right)_x = \rho\phi_x - j, \tag{1.1b}$$

$$\phi_{xx} = \rho - D. \tag{1.1c}$$

The equation (1.1b) contains a momentum relaxation term, standing for the momentum change due to collisions of electrons with atoms in the semiconductor crystal, and a dispersion term based on the quantum (Bohm) potential. The unknown functions ρ, j and ϕ denote the electron density, the electric current and the electrostatic potential, respectively. The scaled Planck constant ε is equivalent to the Planck constant $\hbar$, that is, $\varepsilon = C\hbar$, where C is a positive constant. The pressure p is supposed to be

$$p = p(\rho) = K\rho, \tag{1.2}$$

where K is the positive constant. Moreover, $D = D(x) \in \mathcal{B}^0(\overline{\Omega})$ is a given function, called doping profile (distribution of the density of positively ionized impurities in semiconductor devices) and satisfies

$$\inf_{x\in\overline{\Omega}} D(x) > 0. \tag{1.3}$$

The system (1.1) is studied over the bounded domain $\Omega := (0,1)$. We prescribe the initial and the boundary conditions to the system (1.1) as

$$(\rho, j)(0,x) = (\rho_0, j_0)(x), \tag{1.4}$$
$$\rho(t,0) = \rho_l > 0, \quad \rho(t,1) = \rho_r > 0, \tag{1.5}$$
$$(\sqrt{\rho})_{xx}(t,0) = (\sqrt{\rho})_{xx}(t,1) = 0, \tag{1.6}$$
$$\phi(t,0) = 0, \quad \phi(t,1) = \phi_r > 0, \tag{1.7}$$

where ρ_l, ρ_r and ϕ_r are given constants. Here let us mention about boundary conditions on the quantum effect. Engineers study two kinds of boundary conditions for the quantum effect (see [4, 15]). One boundary condition is (1.6), which means the quantum (Bohm) potential vanishes on the boundary. Another is $\rho_x = 0$ on the boundary. A controversy, which boundary condition is reasonable for the quantum effect, still continues between researches in physics and engineerings.

In order to construct a classical solution, assume that the compatibility condition hold at $(t,x) = (0,0)$ and $(t,x) = (0,1)$, i.e.,

$$\begin{aligned} \rho(0,0) = \rho_l, \quad \rho(0,1) = \rho_r, \quad j_x(0,0) = j_x(0,1) = 0, \\ (\sqrt{\rho})_{xx}(0,0) = (\sqrt{\rho})_{xx}(0,1) = 0. \end{aligned} \tag{1.8}$$

Moreover the initial data are supposed to satisfy a subsonic condition and a positivity of the density

$$\inf_{x\in\Omega}\left(p'(\rho_0) - \frac{j_0^2}{\rho_0^2}\right)(x) > 0, \quad \inf_{x\in\Omega} \rho_0(x) > 0. \tag{1.9}$$

We construct the solution to problem (1.1) and (1.4)–(1.7) around the above initial data (ρ_0, j_0) to satisfy same condition: the subsonic condition and the positivity of the density

$$\inf_{x\in\Omega}\left(p'(\rho) - \frac{j^2}{\rho^2}\right) > 0, \tag{1.10a}$$
$$\inf_{x\in\Omega} \rho > 0. \tag{1.10b}$$

An explicit formula of the electrostatic potential

$$\begin{aligned} \phi(t,x) &= \Phi[\rho](t,x) \\ &:= \int_0^x \int_0^y (\rho - D)(t,z)\, dz dy + \left(\phi_r - \int_0^1 \int_0^y (\rho - D)(t,z)\, dz dy\right) x \end{aligned} \tag{1.11}$$

is given by integrating (1.1c) with the aid of the boundary condition (1.7).

The researchers in semiconductors pay more attentions to the quantum model recently as they become minute. The pioneering works in mathematics are given by Jüngel and Li [6, 7]. Both of papers adopt the boundary condition $\rho_x(t,0) = \rho_x(t,1) = 0$ for the quantum effect, instead of (1.6). They establish the existence of the stationary solution in [6]. Precisely, it is proved that: *for a given electric current $\tilde{j}$, there exists a certain value of the boundary potential ϕ_r such that the stationary solution exists.* However, the engineering experiments measure the electric current $\tilde{j}$ for the given potential ϕ_r on the boundary. Therefore, it is necessary to reconsider this problem to cover the problem in physics and engineerings. The stability of the stationary solution is shown in [7] under the flatness assumption of the doping profile, i.e., $|D(x) - \rho_l| \ll 1$. This assumption is too narrow to cover actual semiconductor devices. For instance the typical example of the doping profile, drawn in [4], does not satisfies this assumption. The asymptotic stability of the stationary solution for the non-flat doping profile had been an open problem, which is solved by the authors in [14].

Notation. For a nonnegative integer $l \geq 0$, $H^l(\Omega)$ denotes the l-th order Sobolev space in the L^2 sense, equipped with the norm $\|\cdot\|_l$. We note $H^0 = L^2$ and $\|\cdot\| := \|\cdot\|_0$. $C^k([0,T];H^l(\Omega))$ denotes the space of the k-times continuously differentiable functions on the interval $[0,T]$ with values in $H^l(\Omega)$. For a nonnegative integer $k \geq 0$, $\mathcal{B}^k(\overline{\Omega})$ denotes the space of the functions whose derivatives up to k-th order are continuous and bounded over $\overline{\Omega}$, equipped with the norm

$$|f|_k := \sum_{i=0}^{k} \sup_{x\in\overline{\Omega}} |\partial_x^i f(x)|.$$

Throughout the present paper C and c denote various generic positive constants.

2 Asymptotic stability of stationary solution

This section is devoted to considering the unique existence and the asymptotic stability of a stationary solution $(\tilde{\rho}, \tilde{j}, \tilde{\phi})$, which is a solution to (1.1) independent of a time variable t, satisfying a system of equations

$$\tilde{j}_x = 0, \tag{2.1a}$$

$$\left(K - \frac{\tilde{j}^2}{\tilde{\rho}^2}\right)\tilde{\rho}_x - \varepsilon^2\tilde{\rho}\left(\frac{(\sqrt{\tilde{\rho}})_{xx}}{\sqrt{\tilde{\rho}}}\right)_x = \tilde{\rho}\tilde{\phi}_x - \tilde{j}, \tag{2.1b}$$

$$\tilde{\phi}_{xx} = \tilde{\rho} - D \tag{2.1c}$$

and boundary conditions

$$\tilde{\rho}(0) = \rho_l > 0, \quad \tilde{\rho}(1) = \rho_r > 0, \tag{2.2}$$

$$(\sqrt{\tilde{\rho}})_{xx}(0) = (\sqrt{\tilde{\rho}})_{xx}(1) = 0, \tag{2.3}$$

$$\tilde{\phi}(0) = 0, \quad \tilde{\phi}(1) = \phi_r > 0. \tag{2.4}$$

Dividing the equation (2.1b) by $\tilde{\rho}$ and integrating the resultant equality over the domain Ω give the current-voltage relationship

$$\phi_r = F(\rho_r, \tilde{j}) - F(\rho_l, \tilde{j}) + \tilde{j} \int_0^1 \frac{1}{\tilde{\rho}}\, dx. \tag{2.5}$$

The strength of the boundary data

$$\delta := |\rho_r - \rho_l| + |\phi_r| \tag{2.6}$$

plays a crucial role in the following analysis.

Lemma 2.1. *Let the doping profile and the boundary data satisfy conditions* (1.3), (1.5) *and* (1.7). *For an arbitrary ρ_l, there exist positive constants δ_1 and ε_1 such that if $\delta \leq \delta_1$ and $\varepsilon \leq \varepsilon_1$, then the stationary problem* (2.1)–(2.4) *has a unique solution $(\tilde{\rho}, \tilde{j}, \tilde{\phi}) \in \mathcal{B}^4(\overline{\Omega}) \times \mathcal{B}^4(\overline{\Omega}) \times \mathcal{B}^2(\overline{\Omega})$ satisfying the conditions* (1.10). *In addition, the electric current $\tilde{j}$ is written by the formula,*

$$\tilde{j} = \mathcal{J}[\tilde{\rho}] := 2B_b \left\{ \int_0^1 \tilde{\rho}^{-1}\, dx + \sqrt{\left(\int_0^1 \tilde{\rho}^{-1}\, dx \right)^2 + 2B_b \left(\rho_r^{-2} - \rho_l^{-2} \right)} \right\}^{-1}, \tag{2.7}$$

$$B_b := \phi_r - \{\log \rho_r - \log \rho_l\}.$$

The proof of the existence of the stationary solution $(\tilde{\rho}, \tilde{j}, \tilde{\phi})$ is given by the Leray–Schauder fixed point theorem. The uniqueness is proved by the elementary energy method. At last, we get the formula (2.7) by solving the current-voltage relationship (2.5) with respect to $\tilde{j}$.

In order to state the stability theorem of the stationary solution, we give a definition of the function spaces as

$$\mathfrak{X}_i^l([0,T]) := \bigcap_{k=0}^{[i/2]} C^k([0,T]; H^{l+i-2k}(\Omega)) \quad \text{for} \quad i, l = 0, 1, 2, \cdots,$$

$$\mathfrak{X}_i([0,T]) := \mathfrak{X}_i^0([0,T]) \quad \text{for} \quad i = 0, 1, 2, \cdots,$$

$$\mathfrak{Y}([0,T]) := C^2([0,T]; H^2(\Omega)),$$

where $[\mu]$ denotes the largest integer which is less than or equal to μ.

Theorem 2.2. *Let $(\tilde{\rho}, \tilde{j}, \tilde{\phi})$ be the stationary solution of* (2.1)–(2.4). *Suppose that the initial data $(\rho_0, j_0) \in H^4(\Omega) \times H^3(\Omega)$ and the boundary data ρ_l, ρ_r and ϕ_r satisfy* (1.5), (1.7), (1.8) *and* (1.9). *Then there exists a positive constant δ_2 such that if $\delta + \varepsilon + \|(\rho_0 - \tilde{\rho}, j_0 - \tilde{j})\|_2 + \left\|\left(\varepsilon\partial_x^3\{\rho_0 - \tilde{\rho}\}, \varepsilon\partial_x^3\{j_0 - \tilde{j}\}, \varepsilon^2\partial_x^4\{\rho_0 - \tilde{\rho}\}\right)\right\| \le \delta_2$, the initial boundary value problem* (1.1) *and* (1.4)–(1.7) *has a unique solution (ρ, j, ϕ) in the space $\overline{\mathfrak{X}}_4([0,\infty)) \times \overline{\mathfrak{X}}_3([0,\infty)) \times \mathfrak{Y}([0,\infty))$. Moreover, the solution (ρ, j, ϕ) verifies the additional regularity $\phi - \tilde{\phi} \in \overline{\mathfrak{X}}_4^2([0,\infty))$ and the decay estimate*

$$\begin{aligned}\|(\rho - \tilde{\rho}, j - \tilde{j})(t)\|_2 &+ \left\|\left(\varepsilon\partial_x^3\{\rho - \tilde{\rho}\}, \varepsilon\partial_x^3\{j - \tilde{j}\}, \varepsilon^2\partial_x^4\{\rho - \tilde{\rho}\}\right)(t)\right\| + \|(\phi - \tilde{\phi})(t)\|_4 \\ &\le C\left(\|(\rho_0 - \tilde{\rho}, j_0 - \tilde{j})\|_2 + \left\|\left(\varepsilon\partial_x^3\{\rho_0 - \tilde{\rho}\}, \varepsilon\partial_x^3\{j_0 - \tilde{j}\}, \varepsilon^2\partial_x^4\{\rho_0 - \tilde{\rho}\}\right)\right\|\right) e^{-\alpha_1 t},\end{aligned} \tag{2.8}$$

where C and α_1 are positive constants, independent of t and ε.

In the proof of Theorem 2.2, we first obtain the elliptic estimate from the formula (1.11), and then we construct the unique existence of the time local solution by using an similar iteration method as in [8, 9, 13]. Next, an energy form is introduced in order to obtain the basic estimate. Moreover, apply the energy method to the system of the equations for the perturbation from the stationary solution to get the higher order estimates. Then the existence of the time global solution follows from the combination of the existence of the time local solution and an a-priori estimate. Finally, the decay estimate (2.8) is shown by the uniform estimates thus obtained.

Remark 2.3. *In the above theorem, we do not need the flatness assumption of doping profile. Moreover, the condition $\varepsilon \ll 1$ is reasonable since the system* (1.1) *is derived under this condition* (see [1, 4] in details).

3 Classical limit

In this section, we consider the singular limit of the solution (ρ, j, ϕ) to the problem (1.1) and (1.4)–(1.7) as the parameter ε tends to zero. This problem is called a classical limit. Hereafter, solutions to (1.1) and (1.4)–(1.7) are written with the suffix ε as $(\rho^\varepsilon, j^\varepsilon, \phi^\varepsilon)$. On the other hand, (ρ^0, j^0, ϕ^0) stands for a solution to the hydrodynamic model

$$\rho_t^0 + j_x^0 = 0, \tag{3.1a}$$

$$j_t^0 + \left(\frac{(j^0)^2}{\rho^0} + p(\rho^0)\right)_x = \rho^0\phi_x^0 - j^0, \tag{3.1b}$$

$$\phi_{xx}^0 = \rho^0 - D, \tag{3.1c}$$

which is obtain by substituting $\varepsilon = 0$ in (1.1). For the derivation of (3.1), see Bløtekjær [2]. The initial and the boundary data to (3.1) are same as those to (1.1) except the boundary data (1.6) for the quantum effect, i.e., (1.4), (1.5) and (1.7). The unique existence and the asymptotic stability of a stationary solution to (3.1), verifying the subsonic condition (1.10a) and the positivity of the density (1.10b), are shown in [5, 13]. These results are stated in Lemmas 3.1 and 3.2 below. Note that the stationary solution $(\tilde{\rho}^0, \tilde{j}^0, \tilde{\phi}^0)$ to (3.1), independent of time value t, satisfies the system of equations

$$\tilde{j}^0_x = 0, \tag{3.2a}$$

$$\left\{K - \left(\tilde{j}^0/\tilde{\rho}^0\right)^2\right\}\tilde{\rho}^0_x = \tilde{\rho}^0\tilde{\phi}^0_x - \tilde{j}^0, \tag{3.2b}$$

$$\tilde{\phi}^0_{xx} = \tilde{\rho}^0 - D \tag{3.2c}$$

with the boundary conditions (2.2) and (2.4).

Lemma 3.1. *Let the doping profile and the boundary data satisfy conditions* (1.3), (1.5) *and* (1.7). *For an arbitrary ρ_l, there exists a positive constant δ_3 such that if $\delta \leq \delta_3$, then the stationary problem* (2.2), (2.4) *and* (3.2) *has a unique solution $(\tilde{\rho}^0, \tilde{j}^0, \tilde{\phi}^0)(x)$ satisfying the conditions* (1.10) *in the space $\mathcal{B}^2(\overline{\Omega})$. Moreover the stationary solution satisfies the estimates*

$$0 < c \leq \tilde{\rho}^0 \leq C, \quad |\tilde{j}^0|_0 \leq C\delta, \quad |\tilde{\rho}^0|_2 + |\tilde{\phi}^0|_2 \leq C, \tag{3.3}$$

where c and C are positive constants independent of ρ_r and ϕ_r.

Lemma 3.2. *Let $(\tilde{\rho}^0, \tilde{j}^0, \tilde{\phi}^0)$ be the stationary solution of* (2.2), (2.4) *and* (3.2). *Suppose that the boundary data ρ_l, ρ_r and ϕ_r satisfy* (1.5) *and* (1.7) *In addition, assume that the initial data $(\rho_0, j_0) \in H^2(\Omega)$ satisfy the condition* (1.10) *and the compatibility condition $\rho_0(0) = \rho_l$, $\rho_0(1) = \rho_r$, $j_{0x}(0) = j_{0x}(1) = 0$. Then there exists a positive constant δ_4 such that if $\delta + \|(\rho_0 - \tilde{\rho}^0, j_0 - \tilde{j}^0)\|_2 \leq \delta_4$, the initial boundary value problem* (1.4), (1.5), (1.7) *and* (3.1) *has a unique solution $(\rho^0, j^0, \phi^0)(t,x) \in \mathfrak{X}_2([0,\infty))$. Moreover, the solution (ρ^0, j^0, ϕ^0) verifies the additional regularity $\phi - \tilde{\phi} \in \mathfrak{X}_2^2([0,\infty))$ and the decay estimate*

$$\|(\rho^0 - \tilde{\rho}^0, j^0 - \tilde{j}^0)(t)\|_2 + \|(\phi^0 - \tilde{\phi}^0)(t)\|_4 \leq C\|(\rho_0 - \tilde{\rho}^0, j_0 - \tilde{j}^0)\|_2 e^{-\alpha_2 t}, \tag{3.4}$$

where C and α_2 are positive constants independent of t.

In Lemma 3.2, the function spaces $\mathfrak{X}_2$ and $\mathfrak{X}_2^2$ are defined by

$$\mathfrak{X}_2([0,T]) := \bigcap_{k=0}^{2} C^k([0,T]; H^{2-k}(\Omega)), \quad \mathfrak{X}_2^2([0,T]) := \bigcap_{k=0}^{2} C^k([0,T]; H^{4-k}(\Omega)),$$

respectively.

Here we mention several results on the non-quantum model (3.1). Degond and Markowich [3] show the unique existence of the stationary solution, satisfying the subsonic condition (1.10a), to the one-dimensional non-quantum model with the Dirichlet boundary condition. Li, Markowich and Mei [10] study the asymptotic stability of the stationary solution. In their result, it is assumed that the Doping profile is flat. For the non-flat doping profile, the asymptotic stability of the stationary solution is considered under the periodic boundary condition by Matsumura and Murakami [12]. In the recent result [5], Guo and Strauss have shown the asymptotic stability of the stationary solution for the Dirichlet boundary condition with the non-flat doping profile. Concerning this, also see [13].

We can expect that the solution to (1.1) converges that to (3.1) as ε tends to zero. In order to prove this expectation, it is firstly studied that the stationary solution $(\tilde{\rho}^{\varepsilon}, \tilde{j}^{\varepsilon}, \tilde{\phi}^{\varepsilon})$ to the problem (2.1)–(2.4) approaches the stationary solution $(\tilde{\rho}^{0}, \tilde{j}^{0}, \tilde{\phi}^{0})$ to the problem (2.2), (2.4) and (3.2) as ε tends to zero. After that, we investigate the convergence of non-stationary solutions. The former result follows from the standard energy method.

Lemma 3.3. *Suppose that the same assumptions in Lemmas 2.1 and 3.1 hold. Let $(\tilde{\rho}^{0}, \tilde{j}^{0}, \tilde{\phi}^{0})$ be the stationary solution to* (2.2), (2.4) *and* (3.2), *and $(\tilde{\rho}^{\varepsilon}, \tilde{j}^{\varepsilon}, \tilde{\phi}^{\varepsilon})$ be the stationary solution to* (2.1)–(2.4). *For an arbitrary ρ_l, there exists a positive constant δ_5 such that if $\delta + \varepsilon \leq \delta_5$, then the stationary solution $(\tilde{\rho}^{\varepsilon}, \tilde{j}^{\varepsilon}, \tilde{\phi}^{\varepsilon})$ to* (2.1)–(2.4) *converges the stationary solution $(\tilde{\rho}^{0}, \tilde{j}^{0}, \tilde{\phi}^{0})$ to* (2.2), (2.4) *and* (3.2) *as ε tends to zero. Precisely,*

$$\|\tilde{\rho}^{\varepsilon} - \tilde{\rho}^{0}\|_1 + |\tilde{j}^{\varepsilon} - \tilde{j}^{0}| + \|\tilde{\phi}^{\varepsilon} - \tilde{\phi}^{0}\|_3 \leq C\varepsilon, \tag{3.5}$$

$$\left\| \left(\partial_x^2 \{\tilde{\rho}^{\varepsilon} - \tilde{\rho}^{0}\}, \partial_x^4 \{\tilde{\phi}^{\varepsilon} - \tilde{\phi}^{0}\}, \varepsilon \partial_x^3 \tilde{\rho}^{\varepsilon}, \varepsilon^2 \partial_x^4 \tilde{\rho}^{\varepsilon} \right) \right\| \to 0 \quad as \quad \varepsilon \to 0, \tag{3.6}$$

where the positive constant C is independent of ε.

The classical limit of the non-stationary problem is summarized in the next theorem.

Theorem 3.4. *Assume that the same conditions in Theorem 2.2 and Lemma 3.2 hold. Then there exists a positive constant δ_6 such that if*

$$\begin{aligned} \delta + \varepsilon + \|(\rho_0 - \tilde{\rho}^{0}, j_0 - \tilde{j}^{0})\|_2 + \|(\rho_0 - \tilde{\rho}^{\varepsilon}, j_0 - \tilde{j}^{\varepsilon})\|_2 \\ + \left\| \left(\varepsilon \partial_x^3 \{\rho_0 - \tilde{\rho}^{\varepsilon}\}, \varepsilon \partial_x^3 \{j_0 - \tilde{j}^{\varepsilon}\}, \varepsilon^2 \partial_x^4 \{\rho_0 - \tilde{\rho}^{\varepsilon}\} \right) \right\| \leq \delta_6, \end{aligned} \tag{3.7}$$

then the time global solution $(\rho^{\varepsilon}, j^{\varepsilon}, \phi^{\varepsilon})$ to (1.1), (1.4)–(1.7) *approaches the solution $(\rho^{0}, j^{0}, \phi^{0})$ to* (1.4), (1.5), (1.7) *and* (3.1) *as ε tends to zero. Precisely,*

$$\|(\rho^{\varepsilon} - \rho^{0}, j^{\varepsilon} - j^{0})(t)\|_1 + \|(\phi^{\varepsilon} - \phi^{0})(t)\|_3 \leq \sqrt{\varepsilon} C e^{\beta t} \quad for \quad t \in [0, \infty), \tag{3.8}$$

$$\sup_{t \in [0,\infty)} \left\{ \|(\rho^{\varepsilon} - \rho^{0}, j^{\varepsilon} - j^{0})(t)\|_1 + \|(\phi^{\varepsilon} - \phi^{0})(t)\|_3 \right\} \to 0 \quad as \quad \varepsilon \to 0, \tag{3.9}$$

where β and C are positive constants independent of ε and t.

The outline of the proof is as follow. First, the estimate (3.8) is obtain by the energy method and the Gronwall inequality. Next, we show the convergence of the density ρ^ε in (3.9) only since the others are similarly shown. Let $\gamma := 1/4$ and $T_1 := \{\log 1/\varepsilon^{(1/4)}\}/\beta$. For $t \le T_1$,

$$\|(\rho^\varepsilon - \rho^0)(t)\|_1 \le \sqrt{\varepsilon} C e^{\beta T_1} \le C\varepsilon^{1/4} \tag{3.10}$$

holds by substituting $t = T_1$ in the estimate (3.8). For $T_1 \le t$, use the estimates (2.8), (3.4) and (3.5) to obtain

$$\begin{aligned} \|(\rho^\varepsilon - \rho^0)(t)\|_1 &\le C\|(\rho^\varepsilon - \tilde{\rho}^\varepsilon, \rho^0 - \tilde{\rho}^0, \tilde{\rho}^\varepsilon - \tilde{\rho}^0)(t)\|_1 \\ &\le C\left(e^{-\alpha_1 T_1} + e^{-\alpha_2 T_1} + \varepsilon\right) \le C\left(\varepsilon^{\alpha_1/4\beta} + \varepsilon^{\alpha_2/4\beta} + \varepsilon\right). \end{aligned} \tag{3.11}$$

These estimates mean $\sup \|(\rho^\varepsilon - \rho^0)(t)\|_1$ converges to zero as ε tends to zero.

Remark 3.5. *The convergence of the stationary solution in Lemma 3.3 ensures that we can take the initial data (ρ_0, j_0) verifying the condition* (3.7) *in Theorem 3.4 if the constant ε is sufficient small.*

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