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Matrix representation of the (time zero) field operators on $P(\phi)_4$ Euclidean QFT by using *Hida distributions*

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Abstract

In the Nelson's approach to the Euclidean QFT, **sharp time free field operator (time zero field operator)** ϕ_0 can be expresed by the random variables which plays both the part of **vectors** and **operators**. Here, we give a clear distinction of these two parts of the random variables. It is shown that the **Hida distribution** : ϕ_{d-1}^4 : defined for $d \geq 3$, which is not a random variable anymore, defines an unboundes **operator** on the Euclidean QFT. Moreover, an expression of such operator by means of a matrix is given.

0 Preliminaries

Throughout this paper, we set $d \in \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers, the space-time dimension, and take that $d-1$ is the space dimension and 1 is the dimension of time. Correspondingly, we use the notations

$$\mathbf{x} \equiv (t, \vec{x}) \in \mathbb{R} \times \mathbb{R}^{d-1}.$$

Let $\mathcal{S}(\mathbb{R}^d)$ (resp. $\mathcal{S}(\mathbb{R}^{d-1})$) be the Schwartz space of rapidly decreasing test functions on the d dimensional Euclidean space $\mathbb{R}^d$ (resp. $d-1$ dimensional Euclidean space $\mathbb{R}^{d-1}$), equipped with the usual topology by which it is a Fréchet nuclear space. Let $\mathcal{S}'(\mathbb{R}^d)$ (resp. $\mathcal{S}'(\mathbb{R}^{d-1})$) be the topological dual space of $\mathcal{S}(\mathbb{R}^d)$ (resp. $\mathcal{S}(\mathbb{R}^{d-1})$).

Now, suppose that on a complete probability space $(\Omega, \mathcal{F}, P)$ we are given an isonormal Gaussian process $B^{d-1} = \{B^{d-1}(h), h \in L^2(\mathbb{R}^{d-1}; \lambda^{d-1})\}$, where λ^{d-1} denotes the Lebesgue measure on $\mathbb{R}^{d-1}$ (cf., e.g., [HKPS], [SiSi], [AY1,2] and references therein). Precisely, B^{d-1} is a centered

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Gaussian family of random variables such that

$$E[B^{d-1}(h)\,B^{d-1}(g)] = \int_{\mathbb{R}^{d-1}} h(\vec{x})\,g(\vec{x})\lambda^{d-1}(d\mathbf{x}), \quad h,\ g \in L^2(\mathbb{R}^{d-1};\lambda^{d-1}). \tag{0.1}$$

We write

$$B^{d-1}_\omega(h) = \int_{\mathbb{R}^{d-1}} h(\vec{y})\dot{B}^{d-1}_\omega(\vec{y})\,d\vec{y}, \qquad \omega \in \Omega.$$

Namely, $\dot{B}^{d-1}_\omega(\cdot)$ is the Gaussian **white noise** on $\mathbb{R}^{d-1}$.

We are considering a massive scalar field. Suppose that we are given a mass $m > 0$. Let Δ_d and resp. Δ_{d-1} be the d, resp. $d-1$, dimensional Laplace operator, and define the pseudo differential operators $L_{-\frac{1}{2}}$ and $H_{-\frac{1}{4}}$ as follows:

$$L_{-\frac{1}{2}} = (-\Delta_d + m^2)^{-\frac{1}{2}}. \tag{0.2}$$

$$H_{-\frac{1}{4}} = (-\Delta_{d-1} + m^2)^{-\frac{1}{4}}, \tag{0.3}$$

By the same symbols as $L_{-\frac{1}{2}}$ and $H_{-\frac{1}{4}}$, we also denote the integral kernels of the corresponding pseudo differential operators, i.e., the Fourier inverse transforms of the corresponding symbols of the pseudo differential operators.

By making use of stochastic integral expressions, we define two fundamental random fields ϕ_N, the **Nelson's Euclidean free field**, and ϕ_0, the **sharp time free field**, as follows:
For $d \geq 2$,

$$\phi_N(\cdot) \equiv \int_{\mathbb{R}^d} L_{-\frac{1}{2}}(\mathbf{x} - \cdot)\dot{B}^d(\mathbf{x})\,d\mathbf{x}, \tag{0.4}$$

$$\phi_0(\cdot) \equiv \int_{\mathbb{R}^{d-1}} H_{-\frac{1}{4}}(\vec{x} - \cdot)\dot{B}^{d-1}(\vec{x})\,d\vec{x}. \tag{0.5}$$

Here $\dot{B}^d(\mathbf{x})$ is the Gaussian **white noise** on $\mathbb{R}^d$. These definitions of ϕ_N and resp. ϕ_0 seems formal, but they are rigorously defined as $\mathcal{S}'(\mathbb{R}^d)$ and resp. $\mathcal{S}'(\mathbb{R}^{d-1})$ valued random variables through a limiting procedure (cf. [AY1,2]).

Our main concern here is to consider the properties of ϕ_0 as the **multiplicative operator** on $L^2(\Omega, P)$ (more presisely, on $L^2(\mu_0)$ defined below).

Let μ_0 be the probability measure on $\mathcal{S}'(\mathbb{R}^{d-1} \to \mathbb{R})$ which is the probability law of the sharp time free field ϕ_0 on $(\Omega, \mathcal{F}, P)$ (cf. (0.5)).

We denote

$$\phi_0(f) \equiv < \phi_0, f > \equiv \int_{\mathbb{R}^{d-1}} \left(H_{-\frac{1}{4}} f\right)(\vec{x})\, \dot{B}^{d-1}(\vec{x})\, d\vec{x},$$

and

$$\begin{aligned} &: \phi_0(f_1) \cdots \phi_0(f_n) : \\ &\quad = \int_{\mathbb{R}^{k(d-1)}} H_{-\frac{1}{4}} f_1(\vec{x}_1) \cdots H_{-\frac{1}{4}} f_1(\vec{x}_k) : \dot{B}^{d-1}(\vec{x}_1) \cdots \dot{B}^{d-1}(\vec{x}_k) : \\ &\quad \times d\vec{x}_1 \cdots d\vec{x}_k \in \cap_{q \geq 1} L^q(\mu_0) \\ &\qquad \text{for} \quad f,\ f_j \in \mathcal{S}(\mathbb{R}^{d-1} \to \mathbb{R}), \quad j = 1, \cdots, k, \quad k \in \mathbb{N}, \end{aligned} \tag{0.6}$$

where (0.6) is the k-th multiple stochastic integral with respect to the Gaussian white noise $\dot{B}^{d-1}$ on $\mathbb{R}^{d-1}$.

Technical Remark 1. From the view point of the notational rigorousness, ϕ_0 is the distribution valued random variables on the probability space $(\Omega, \mathcal{F}, P)$, hence the notation (cf. (0.6)) such as

$$: \phi_0(\varphi_1) \cdots \phi_0(\varphi_n) :\in \bigcap_{q \geq 1} L^q(\mu_0)$$

is incorrect. However in the above and in the sequel, since there is no ambiguity, for the simplicity of the notations we use the notation ϕ_0 (with an obvious interpretation) to indicate the measurable function X on the measure spaces $(\mathcal{S}'(\mathbb{R}^{d-1}), \mu_0, \mathcal{B}(\mathcal{S}'(\mathbb{R}^{d-1})))$ such that

$$P\Big(\{\omega : \phi_0(\omega) \in A\}\Big) = \mu_0\Big(\{\phi : X(\phi) \in A\}\Big), \quad A \in \mathcal{B}(\mathcal{S}'(\mathbb{R}^{d-1})),$$

where $\mathcal{B}(S)$ denotes the Borel σ-field of the topological space S.

$\square$

Since, $: \phi_0(\varphi_1) \cdots \phi_0(\varphi_n) :$ is nothing more than an element of the n-th Wiener chaos of $L^2(\mu_0)$, it also adomits an expression by means of the **Hermite polynomial** of $\phi_0(\varphi_j)$, $j = 1, \cdots, k$ (cf., e.g., [AY1,2] and references therein).

Conceptual Remark 2. We recall that the multiplication of $:\phi_0(\varphi_1)\cdots\phi_0(\varphi_n):$ with $\phi_0(\varphi_j)$ defines a new random variable (a **vector**) such that $:\phi_0(\varphi_1)\cdots\phi_0(\varphi_n):\ \phi_0(\varphi_j)$, and hence, the random variable $:\phi_0(\varphi_1)\cdots\phi_0(\varphi_n):$ is not only a vector itself on $L^2(\mu_0)$ but also an **oprtator** on $L^2(\mu_0)$.

□

From the above **Remark**, we have to carefully distinguish the two roles of "**vectors**" and "**operators**" that are played by one **random variable** $:\phi_0(\varphi_1)\cdots\phi_0(\varphi_n):$. In order to do so, we give the additional notations in the next section.

1 Interpretation of the operator $(:\phi^4:)_4$ by Hida distribution

Let $\mathcal{H}$ be the Hilbert space such that

$$\mathcal{H} \equiv L^2(\mu_0). \tag{1.1}$$

We use the notation

$$\overrightarrow{:\phi_0(f_1)\cdots\phi_0(f_n):} \tag{1.2}$$

when the random variable $:\phi_0(f_1)\cdots\phi_0(f_n):\in\mathcal{H}$ defined by (0.6) plays the **role** of the **vector** in $\mathcal{H}$. On the otherhand, we use the original notation

$$:\phi_0(f_1)\cdots\phi_0(f_n): \tag{1.3}$$

when the random variable takes the part of an **operator** on a subspace of $\mathcal{H}$. Let $\mathcal{D}$ be a linear subspace of $\mathcal{H}$ such that

$$\mathcal{D} \equiv linear\ hull\{\overrightarrow{:\phi_0(f_1)\cdots\phi_0(f_n):} \mid f_i \in \mathcal{S},\ i=1,\ldots,n.\, n\in N\cup\{0\}\} \tag{1.4}$$

for $f,\ g \in \mathcal{S}(\mathbb{R}^{d-1}\to\mathbb{R})$ we define an innerproduct $<f,g>$ such that

$$\int_{\mathbb{R}^{d-1}} H_{-\frac{1}{4}}f(\vec{x})\cdot H_{-\frac{1}{4}}g(\vec{x})\,d\vec{x} \tag{1.5}$$

Let

$$\{f_i^0\}_{i\in N}, \qquad f_i^0 \in \mathcal{S}(\mathbb{R}^{d-1}\to\mathbb{R}) \tag{1.6}$$

be an O.N.B. of the Hilbert space with the inner product $<\cdot,\cdot>$ defined by (1.5), and define an O.N.B. $\mathcal{D}_0$ of $\mathcal{H}$ as follows:

$$\mathcal{D}_0 \equiv \{K_{i_1,\dots,i_n} \overrightarrow{:\phi_0(f^0_{i_1})\cdots\phi_0(f^0_{i_n}):} \mid i_k \in N,\ k=1,\dots,n,\ n \in N\cup\{0\}\}, \tag{1.7}$$

where $K_{i_1,\dots,i_n}$ is the normarizing constant.

We denote

$$K_{i_1,\dots,i_n} \overrightarrow{:\phi_0(f^0_{i_1})\cdots\phi_0(f^0_{i_n}):} = \overrightarrow{\phi_{i_1,\dots,i_n}}. \tag{1.8}$$

Finally, let $\vec{1}$ be the constant 1 in $\mathcal{H}$. By using these notations we have the characterization of the **operator**

$$\phi(f)\ :\mathcal{D}\longrightarrow\mathcal{D}$$

as follows:

$$\phi(f)\,\vec{1} = \overrightarrow{\phi(f)}, \tag{1.9}$$

$$\phi(f)\,[\overrightarrow{\phi(g)}] = \overrightarrow{:\phi(f)\,\phi(g):} + <f,g>, \tag{1.10}$$

$$\phi(f)\,[\overrightarrow{:\phi(g)\,\phi(h):}] = \overrightarrow{:\phi(f)\,\phi(g)\,\phi(h):} + <f,g> \overrightarrow{\phi(h)} + <f,h> \overrightarrow{\phi(g)}, \tag{1.11}$$

$$\text{etc.} \tag{1.12}$$

Next, we give an expression of the O.N.B. of $\mathcal{H}$ (cf. (1.7)) by means of the vectors having infinitely many coordinates. These expressions are rather heuristic but by which we can understand the structure of $\mathcal{H}$ easily.

$$\vec{1} \cong \begin{pmatrix} 1 \\ \hline 0 \\ \vdots \\ \hline 0 \\ \vdots \\ \hline 0 \\ \vdots \end{pmatrix}, \quad \overrightarrow{\phi_1} \cong \begin{pmatrix} 0 \\ \hline 1 \\ 0 \\ \vdots \\ \hline 0 \\ \vdots \\ \hline 0 \\ \vdots \end{pmatrix}, \quad \overrightarrow{\phi_2} \cong \begin{pmatrix} 0 \\ \hline 0 \\ 1 \\ 0 \\ \vdots \\ \hline 0 \\ \vdots \\ \hline 0 \\ \vdots \end{pmatrix}, \cdots, \quad \overrightarrow{\phi_{1,1}} \cong \begin{pmatrix} 0 \\ \hline 0 \\ \vdots \\ \hline 1 \\ 0 \\ \vdots \\ \hline 0 \\ \vdots \end{pmatrix},$$

$$\overrightarrow{\phi_{1,2}} \cong \left(\begin{array}{c} 0 \\ \hline 0 \\ \vdots \\ \hline 0 \\ 1 \\ 0 \\ \vdots \\ \hline 0 \\ \vdots \end{array}\right), \quad \overrightarrow{\phi_{1,3}} \cong \left(\begin{array}{c} 0 \\ \hline 0 \\ \vdots \\ \hline 0 \\ 0 \\ 1 \\ 0 \\ \vdots \\ \hline 0 \\ \vdots \end{array}\right), \cdots \overrightarrow{\phi_{1,1,1}} \cong \left(\begin{array}{c} 0 \\ \hline 0 \\ \vdots \\ \hline 0 \\ \vdots \\ \hline 1 \\ 0 \\ \vdots \end{array}\right), \cdots$$

Here, each coordinate of the right hand side of the above formulas should be adequately understood as

$$\left(\begin{array}{c} \text{0th Wiener chaos} \\ \hline \text{1st Wiener chaos} \\ \hline \text{2nd Wiener chaos} \\ \hline \text{3rd Wiener chaos} \\ \hline \vdots \end{array}\right).$$

Note that the **Wiener chaos** is a probabilistic representation of the **Fock space** in QFT. Thus, by (1.11) and (1.12) we can identify the **operator** $\phi_0(f_1^0)$ as the matrix that satisfies the following:

$$\phi_0(f_1^0)[\overrightarrow{\phi_{1,1}}] \cong \left(\begin{array}{c} 0 \\ \hline \sqrt{2} \\ 0 \\ \vdots \\ \hline 0 \\ \vdots \\ \hline \sqrt{3} \\ 0 \\ \vdots \\ \hline 0 \\ \vdots \end{array}\right), \quad \phi_0(f_1^0)[\overrightarrow{\phi_{1,1,1}}] \cong \left(\begin{array}{c} 0 \\ \hline 0 \\ \vdots \\ \hline \sqrt{3} \\ 0 \\ \vdots \\ \hline 0 \\ \vdots \\ \hline \sqrt{4} \\ 0 \\ \vdots \end{array}\right), \cdots \tag{1.13}$$

Up to the above discussion, all the vectors and the operators are in the framework of $\mathcal{H}$, regardless of the dimension d. We now proceed to the considerations of the **operators** that have the restricted domain $\mathcal{D}$ (cf. (1.4)) and need not map the element of $\mathcal{D}$ to an element of $\mathcal{H}$.

For $r \in \mathbb{N}$, let

$$\Lambda_{r,d-1} \equiv \{\vec{x} \,:\, |\vec{x}| < r\}.$$

For $p \in \mathbb{N}$ define a **Hida distribution** $: \phi_0^p : (\Lambda_{r,d-1})$ as follows:

$$\begin{aligned} &: \phi_0^p : (\Lambda_{r,d-1}) \\ &\quad \equiv \int_{(\mathbb{R}^{d-1})^p} \left\{ \int_{\Lambda_{r,d-1}} \prod_{k=1}^{p} H_{-\frac{1}{4}}(\vec{x} - \vec{x}_k)\, d\vec{x} \right\} \\ &\quad \times : \dot{B}^{d-1}(\vec{x}_1) \cdots \dot{B}^{d-1}(\vec{x}_p) : d\vec{x}_1 \cdots d\vec{x}_p, \end{aligned} \tag{1.14}$$

here, all the way of using notations follow the rule given by Remark in section 1.

For $d = 2 \quad (d-1 = 1)$ we know that

$$: \phi_0^p : (\Lambda_{r,1}) \in \bigcap_{q \geq 1} L^q(\mu_0).$$

But our main interest is concentrated on the case where $d = 4 \quad (d - 1 = 3)$, and in this case $: \phi_0^p : (\Lambda_{r,3})$ is not a random variable any more for $p \geq 2$, but a **Hida distribution**. However, even for the case $d \geq 4$, it is possible to take $: \phi_0^p : (\Lambda_{r,3})$ as an **operator** on $\mathcal{D}$ which need not map the elements in $\mathcal{D}$ to $\mathcal{H}$.

In fact, through the analogous discussions by wich we derived the formula (1.13), we can give an expression of $: \phi_0^p : (\Lambda_{r,3})$ by means of a **matrix**.

Namely, the **operator** $: \phi_0^4 : (\Lambda_{r,d-1}) \quad (d \geq 1)$ is the **matrix** that satisfies the following:

$$
:\phi_0^4:(\Lambda_{r,d-1})\vec{1}\cong\left(\begin{array}{c}0\\ \hline 0\\ \vdots\\ \hline 0\\ \vdots\\ \hline 0\\ \vdots\\ \hline \alpha_{1,1,1,1}\\ \vdots\\ \alpha_{i,j,k,l}\\ \vdots\\ \hline 0\\ \vdots\end{array}\right), \quad :\phi_0^4:(\Lambda_{r,d-1})\overrightarrow{\phi_m}\cong\left(\begin{array}{c}0\\ \hline 0\\ \vdots\\ \hline 0\\ \vdots\\ \hline \beta_{1,1,1}\\ \vdots\\ \beta_{i,j,k}\\ \vdots\\ \hline 0\\ \vdots\\ \hline \beta_{1,1,1,1,1}\\ \vdots\\ \beta_{i,j,k,l,m}\\ \vdots\\ \hline 0\\ \vdots\end{array}\right),
$$

$$
:\phi_0^4:(\Lambda_{r,d-1})\overrightarrow{\phi_{j,k}} \cong \left(\begin{array}{c} 0 \\ \hline 0 \\ \vdots \\ \hline \gamma_{1,1} \\ \vdots \\ \gamma_{i_1,i_2} \\ \vdots \\ \hline 0 \\ \vdots \\ \hline \gamma_{1,1,1,1} \\ \vdots \\ \gamma_{i_1,i_2 i_3,i_4} \\ \vdots \\ \hline 0 \\ \vdots \\ \hline \gamma_{1,1,1,1,1,1} \\ \vdots \\ \gamma_{i_1,\ldots,i_6} \\ \vdots \\ \hline 0 \\ \vdots \end{array}\right), \cdots, \tag{1.15}
$$

where

$$
\alpha_{i,j,k,l} = K_{i,j,k,l} \int_{\Lambda_{r,d-1}} (Lf_i)(\vec{x})(Lf_j)(\vec{x})(Lf_k)(\vec{x})(Lf_l)(\vec{x})\, d\vec{x},
$$

$$
\beta_{i,j,k} = 4! K_{i,j,k} \int_{\Lambda_{r,d-1}} (Lf_m)(\vec{x})(Lf_i)(\vec{x})(Lf_j)(\vec{x})(Lf_k)(\vec{x})\, d\vec{x},
$$

$$
\begin{aligned}
\beta_{i,j,k,l,n} &= 4! K_{i,j,k,l,n} \Big\{ < f_m, f_i > (\int_{\Lambda_{r,d-1}} (Lf_j)(\vec{x})(Lf_k)(\vec{x})(Lf_l)(\vec{x})(Lf_n)(\vec{x})\, d\vec{x}) \\
&+ \cdots + < f_m, f_n > (\int_{\Lambda_{r,d-1}} (Lf_i)(\vec{x})(Lf_j)(\vec{x})(Lf_k)(\vec{x})(Lf_l)(\vec{x})\, d\vec{x}) \Big\},
\end{aligned}
$$

with

$$
L \equiv (-\Delta_{d-1} + m^2)^{-\frac{1}{2}}.
$$

To derive these numbers, we used the fact that the multiplication $:\phi_0^4:(\Lambda_{r,d-1})\circ\phi_0(f_m^0)$ is given by

$$\begin{aligned}
&:\phi_0^4:(\Lambda_{r,d-1})\circ\phi_0(f_m^0)\\
&\equiv 4\int_{(\mathbb{R}^{d-1})^3}\left\{\int_{\Lambda_{r,d-1}}(Lf_m^0)(\vec{x})\prod_{k=1}^{3}H_{-\frac{1}{4}}(\vec{x}-\vec{x}_k)\,d\vec{x}\right\}\\
&\times :\dot{B}^{d-1}(\vec{x}_1)\dot{B}^{d-1}(\vec{x}_2)\dot{B}^{d-1}(\vec{x}_3):\,d\vec{x}_1\,d\vec{x}_2\,d\vec{x}_3.
\end{aligned}\tag{1.16}$$

And $\gamma's$ are also defined by the similar way.

Note that for $d\geq 3$,

the **vectors** defined by (1.15) $\notin l^2\cong\mathcal{H}$.

But $:\phi_0^4:(\Lambda_{r,d-1})$ surely defines an unbounded **operator** (of which region is the outside of $\mathcal{H}$).

Through the analogous discussions as above, we have the following result:

Theorem 1.1 *Let $d\geq 3$, and $0<r<\infty$. For each $:\phi_0(f_1)\cdots\phi_0(f_n):$, $f_i\in\mathcal{S}$, $i=1,\ldots,n$, $n\in N\cup\{0\}$, the multiplication $:\phi_0^4:(\Lambda_{r,d-1})\circ :\phi_0(f_1)\cdots\phi_0(f_n):$ defines an unbounded* **operator** *on $\mathcal{D}$ which admits an expression by means of a* **matrix** *(cf. (1.15)). Such* **operators** *map the elements of $\mathcal{D}$ to a region that is wider than $\mathcal{H}$.*

Concluding Remark 3. In the construction of $P(\phi)_d$ Euclidean QFT, in particular the $(:\phi^4:)_d$ Euclidean QFT (with a truncation), for the corresponding sharp time field we have to prepare the term such that

$$\exp\{:\phi_0^4:(\Lambda_{r,d-1})\}.\tag{1.17}$$

It is well known and also we pointed out that (1.17) does not exist as a random variable. However, by using Theorem1.1 and the earlier results (**Hida product**) reported by the same authors in the previous seminars in RIMS, we may have some substitute of (1.17). Such a consideration will be announced in the next seminar.

□

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