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EQUIVARIANT SURGERY UNDER THE WEAK GAP CONDITION

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Abstract. In this article we explain the equivariant surgery obstruction group with middle dimensional singular set and hypercomputability of this group.

1. Introduction

In this article we describe an equivaraint surgery theory under the 'weak gap condition' obtained in joint research with Anthony Bak.

Throughout this paper, let G be a finite group and let $\mathcal{S}(G)$ denote the set of all subgroups of G. Let X be a compact smooth G-manifold X of dimension n. The *singular set* X_{sing} of X is the subset $\bigcup_{g\in G\smallsetminus\{e\}} X^g$ of X and the free part X_{free} of X is the complement of X_{sing} in X. Let $\widehat{\Pi}(G,X)$ denote the set of all connected components of the fixed point manifolds X^H, where H runs over the set $\mathcal{S}(G)$. A precise definition of $\widehat{\Pi}(G,X)$ will be given in the next section. The underlying manifold of an element t in $\widehat{\Pi}(G,X)$ is denoted by X_t. The map $\rho_X : \widehat{\Pi}(G,X) \to \mathcal{S}(G)$ is defined by

$$\rho_X(t) = \bigcap_{x\in X_t} G_x$$

where G_x is the isotropy subgroup at x in the G-manifold X. Clearly $\widehat{\Pi}(G,X)$ inherits a G-action from X. For an integer i, let $\widehat{\Pi}(G,X,i)$ denote the subset of $\widehat{\Pi}(G,X)$ consisting of all t such that $\dim X_t = i$.

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If X is a connected, oriented smooth G-manifold then we have the *orientation homomorphism* $w_X : G \longrightarrow \{\pm 1\}$ associated with X, namely $w_X(g) = 1$ if and only if $g : X \to X$ is orientation preserving. The group ring $\mathbb{Z}[G]$ has the (anti-)involution $-$ defined by

$$\left(\sum_{g\in G} a_g g\right)^{-} = \sum_{g\in G} w_X(g) a_g g^{-1},$$

where a_g are integers.

We say that X satisfies the *weak gap condition* (resp. *gap condition*, *strong gap condition*) for $\{e\}$ if $2\dim X^g \le n$ (resp. $2\dim X^g + 1 \le n$, $2\dim X^g + 2 \le n$) for any $g \in G$ with $g \ne e$. Equivariant surgery theories under the strong gap condition and the gap condition are discussed in, for example, [18] and [9, 10], respectively. In [9, 10], the set

$$Q_X = \{g \in G \mid g^2 = e\ g \ne e,\ \dim X^g = [(\dim X - 1)/2]\},$$

where $[r]$ denotes the greatest integer not exceeding r, has the role of generating a quadratic form parameter: namely

$$\Lambda(Q_X) = \{x - (-1)^k \overline{x} \mid x \in \mathbb{Z}[G]\} + \mathbb{Z}[Q_X]$$

is a form parameter in the sense of [1]. It was found in [9] that the surgery obstruction group under the gap condition $\{e\}$ depends on $\Lambda(Q_X)$.

To discuss the essential part of our equivariant surgery theory under the weak gap condition for $\{e\}$, let us assume $\dim X = n = 2k$ (even) ≥ 6. Under this assumption, we set $\Theta_X = \widehat{\Pi}(G, X, k)$. Then Θ_X is the disjoint union of $\Theta_{X,+} = \Theta_+(G, X)$ and $\Theta_{X,-} = \Theta_-(G, X)$ such that

$$\Theta_{X,+} = \{t \in \Theta_X \mid X_t \text{ is orientable}\},$$
$$\Theta_{X,-} = \{t \in \Theta_X \mid X_t \text{ is nonorientable}\}.$$

If X is a compact smooth G-manifold, we denote by $\widetilde{\Theta}_{X,+} = \widetilde{\Theta}_+(G, X)$ the set of all generators in $H_k(X_t, \partial X_t; \mathbb{Z}) \cong \mathbb{Z}$, where t runs over $\Theta_{X,+}$. If ω is a generator in $H_k(X_t, \partial X_t; \mathbb{Z})$ then so is $-\omega$. Thus we have a bijection from $\widetilde{\Theta}_{X,+}$ to $\Theta_{X,+} \times \{\pm 1\}$. In addition, $\widetilde{\Theta}_{X,+}$ has a canonical $\{\pm 1\}$-action. Furthermore, we can give a G-action to $\widetilde{\Theta}_{X,+}$ so that the projection map $\pi_X : \widetilde{\Theta}_{X,+} \to \Theta_{X,+}$ is G-equivariant. Set

$$S_X = \{g \in G \mid g^2 = e,\ g \ne e,\ \dim X^g = k\}.$$

Note that this set is also determinded by $\rho_X : \Theta_X \to \mathcal{S}(G)$. We assign the datum

$$\mathcal{D}_X = (G, (-1)^k, w_X, \pi_X : \widetilde{\Theta}_{X,+} \to \Theta_X, \rho_X : \Theta_X \to \mathcal{S}(G), Q_X)$$

to a compact, connected, oriented, smooth G-manifold of dimension $n = 2k \geq 6$ satisfying the weak gap condition for $\{e\}$. Note that if X satisfies the strong gap condition or the gap condition for $\{e\}$ then $\mathcal{D}_X$ is essesentially the datum

$$(G, (-1)^k, w_X) \ \text{ or } \ (G, (-1)^k, w_X, Q_X), \text{ respectively.}$$

By [4], the datum $\mathcal{D}_X$ provides a Witt group $\nabla W(\mathcal{D}_X)_{\text{proj}}$ and this group is the surgery obstruction group under the weak gap condition for $\{e\}$. In the case where X fulfills the strong gap condition or the gap condition for $\{e\}$ then the group $\nabla W(\mathcal{D}_X)_{\text{proj}}$ coincides with a Wall group $L_n^h(\mathbb{Z}[G], w)_{\text{proj}}$ or a Bak group $W_n(\mathbb{Z}[G], \Lambda(Q_X), w)_{\text{proj}}$, respectively. The group $\nabla W(\mathcal{D}_X)_{\text{proj}}$ consists of equivalence classes of tuples $(M, B, q, \boldsymbol{\alpha})$ such that M is a finitely generated projectve $\mathbb{Z}[G]$-module, B is a nonsingular $(-1)^k$-Hermitian form $M \times M \to \mathbb{Z}[G]$, q is a generalized quadratic form $M \to \mathbb{Z}[G]/(\Lambda(Q_X) + \mathbb{Z}[S_X])$, and $\boldsymbol{\alpha}$ is a pair consisting of a $G \times \{\pm 1\}$-map $\widetilde{\alpha}_+ : \widetilde{\Theta}_{X,+} \to M$ and G-map $\alpha : \Theta_X \to M/2M$ making the diagram

$$\begin{array}{ccc} \widetilde{\Theta}_{X,+} & \xrightarrow{\widetilde{\alpha}_+} & M \\ {\scriptstyle \pi_X}\downarrow & & \downarrow \\ \Theta_X & \xrightarrow[\alpha]{} & M/2M \end{array}$$

commutative. We will give the precise definition of $\nabla W(\mathcal{D}_X)_{\text{proj}}$ in Section 3.

A *G-framed map $\boldsymbol{f} = (f, b)$ of degree one* is a pair consisting of a degree one G-map $f : (X, \partial X) \to (Y, \partial Y)$, where X and Y are compact, connected, oriented, smooth G-manifolds and a G-vetor bundle isomorphism $b : T(X) \oplus f^*\eta \to f^*\xi$, where $T(X)$ is the tangent bundle of X and ξ, η are real G-vector bundles over Y.

Theorem 1.1 ([4]). *Let $\boldsymbol{f} = (f, b)$ be a G-framed map of degree one as above. Suppose the following coniditons are satisfied.*

(1) $\dim X = \dim Y = n = 2k \geq 6$ *is even.*
(2) *X satisfies the weak gap conditon for $\{e\}$.*
(3) $\dim(X_t \cap X_{t'}) \leq k - 2$ *for all $t \in \widehat{\Pi}(G, X, k)$ and $t' \in \widehat{\Pi}(G, X, k-1)$.*
(4) *Y is simply connected.*

(5) $\partial f : \partial X \to \partial Y$ *is* $\mathbb{Z}$*-homology equivalence.*

(6) $f^P : X^P \to Y^P$ *is a* $\mathbb{Z}_p$*-homology equivalence for any prime* p *and any* $P \in \mathcal{S}(G)$ *of* p*-power order* $\neq 1$.

Then an element $\sigma(\boldsymbol{f})$ *in* $\nabla W(\mathcal{D}_X)_{\text{proj}}$ *is assigned to* $\boldsymbol{f} = (f, b)$ *so that* $\sigma(\boldsymbol{f}) = 0$ *if and only if* $\boldsymbol{f}$ *is G-framed cobordant, by G-surgeries on* $\boldsymbol{f}$ *relative to the singular set and the boundary of* X, *to a G-framed map* $\boldsymbol{f}' = (f', b')$ *such that the ambient map* $f' : X' \to Y$ *is a homotopy equivalence.*

Let X be a smooth G-manifold as in the theorem above. For $H \in \mathcal{S}(G)$, we obtain the datum

$$\mathcal{D}_H = (H, (-1)^k, w_H, \pi_H : \widetilde{\Theta}_H \to \Theta_H, \rho_H : \Theta_H \to \mathcal{S}(H), Q_H) \tag{1.1}$$

by setting $\mathcal{D}_H = \mathcal{D}_{\text{res}^G_H X}$, $w_H = w_{\text{res}^G_H X}$, $\widetilde{\Theta}_{H,+} = \widetilde{\Theta}_{\text{res}^G_H X,+}$, $\Theta_H = \Theta_{\text{res}^G_H X}$, $\pi_H = \pi_{\text{res}^G_H X}$, $\rho_H = \rho_{\text{res}^G_H X}$, and $Q_H = Q_{\text{res}^G_H X}$. By definition, we have

$$\Theta_H = \{t \in \Theta_G \mid \rho_G(t) \cap H \neq \{e\}\},$$
$$\widetilde{\Theta}_{H,+} = \{\omega \in \widetilde{\Theta}_{G,+} \mid \rho_G \circ \pi_G(\omega) \neq \{e\}\},$$
$$Q_H = Q_X \cap H.$$

Set $S_H = S_{\text{res}^G_H X}$. Then we have

$$S_H = S_X \cap H.$$

Lemma 1.2. *In the above setting and notation, if*

(C1) $\rho_G(t)$ *has prime order for each* $t \in \Theta_G$

then the equality

$$\Theta_{H \cap K} = \Theta_H \cap \Theta_K$$

holds for all H, $K \in \mathcal{S}(G)$.

The next theorem is proved by using results in [19], [6], [1], [2].

Theorem 1.3. *Let* X *be as in Theorem 1.1 and* $\mathcal{F}$ *a subset of* $\mathcal{S}(G)$ *closed with respect to conjugation and intersections. Suppose* $\mathcal{F}$ *contains all maximal cyclic subgroups of* G. *Further suppose* X *satisfies the following.*

(C1) $\rho_G(t)$ *has prime order for each* $t \in \Theta_G$.

(C2) $\Theta_G \times \Theta_G = \bigcup_{H \in \mathcal{F}} \Theta_H \times \Theta_H$.

Then $\nabla W(\mathcal{D}_G)_{\mathrm{proj}}$ *is* $\mathcal{F}$*-hypercomputable; in particular,*

$$\mathrm{Ind} : \lim_{\to \widetilde{\mathcal{F}}} \nabla W(\mathcal{D}_-)_{\mathrm{proj}} \to \nabla W(\mathcal{D}_G)_{\mathrm{proj}} \quad \textit{and} \quad \mathrm{Res} : \nabla W(\mathcal{D}_G)_{\mathrm{proj}} \to \lim_{\leftarrow \widetilde{\mathcal{F}}} \nabla W(\mathcal{D}_-)_{\mathrm{proj}}$$

are isomorphisms, where

$$\widetilde{F} = \{K \in \mathcal{S}(G) \mid \exists\, H \in \mathcal{F} :\ H \trianglelefteq K, K/H \textit{ has prime power order}\}.$$

The reader can obtain basic knowledge of the Burnside ring from [6], [5], [12]. A finite group G is called an *Oliver group* if G admits a smooth G-action on a disk without G-fixed points, cf. [16, 15], [8].

Theorem 1.4. *Let G be an Oliver group and let X be as in the above theorem. Let D be an acyclic finite G-CW complex such that*

(C3) *the Euler characteristics $\chi(D^K)$ are equal to 1 for all subgroups K of the group $\langle \rho_G(t), \rho_G(t') \rangle$, where t, t' range over Θ_G.*

Then the vanishing property

$$([G/G] - [D])^{2m+2} \nabla W(\mathcal{D}_G)_{\mathrm{proj}} = 0$$

holds for the integer m defined by $|G| = 2^m m'$ with odd m', and where $[G/G]$ and $[D]$ are the elements in the Burnside ring determined respectively by the finite G-CW complexes G/G and D.

2. Connected components of fixed point sets

Let X be a finite G-CW complex or a compact smooth G-manifold. According to [17], we define the G-poset $\Pi(G, X)$ associated with X by

$$\Pi(G, X) = \coprod_{H \in \mathcal{S}(G)} \pi_0(X^H),$$

where $\pi_0(X^H)$ is the set of all connected components of the H-fixed point set X^H of X. The map $\rho : \Pi(G, X) \to \mathcal{S}(G)$ is defined so that for $t \in \Pi(G, H)$, $\rho(t) = H$ holds if and only if $t \in \pi_0(X^H)$. The underlying space of $t \in \Pi(G, X)$ is denoted by X_t. The set $\Pi(G, X)$ inherits a G-action from X, namely for $g \in G$ and $t \in \Pi(G, H)$,

gt is the element having the property $\rho(gt) = g\rho(t)g^{-1}$ and $X_{gt} = gX_t$. The set $\mathcal{S}(G)$ has the G-action induced by conjugation. It is easy to check that ρ is a G-map. For two elements t, $t' \in \Pi(G, X)$, we say $t \leq t'$ if and only if $X_t \subseteq X_{t'}$ and $\rho(t) \supseteq \rho(t')$. For $t \in \Pi(G, H)$, we define $\widehat{t}$ to be the minimal element in $\Pi(G, H)$ such that $X_t = X_{\widehat{t}}$. Then we define the set $\widehat{\Pi}(G, X)$ by

$$\widehat{\Pi}(G, X) = \{\widehat{t} \mid t \in \Pi(G, X)\}.$$

3. Definition of $\nabla W(\mathcal{D}_X)_{\mathrm{proj}}$

Let X be a compact, connected, oriented smooth G-manifold of dimension $n = 2k \geq 6$ satisfying the weak gap condition for $\{e\}$ and

$$\mathcal{D}_X = (G, (-1)^k, w_X, \pi_X : \widetilde{\Theta}_{X,+} \to \Theta_X, \rho_X : \Theta_X \to \mathcal{S}(G), \mathcal{Q}_X)$$

the datum associated with X described in Section 1. We call a tuple $\boldsymbol{M} = (M, B, q, \boldsymbol{\alpha})$ a *$\mathcal{D}_X$-quadratic module* if it satisfies the following.

(1) M is a finitely generated projective $\mathbb{Z}[G]$-module.

(2) $B : M \times M \to \mathbb{Z}[G]$ is a nonsingular $(-1)^k$-Hermitan form. Thus,

  (i) B is bilinear over $\mathbb{Z}$,

  (ii) $B(x, y) = (-1)^k \overline{B(y, x)}$ for x, $y \in M$,

  (iii) $B(x, ay) = aB(x, y)$ for $a \in \mathbb{Z}[G]$, x, $y \in M$,

(3) $q : M \to \mathbb{Z}[G]/(\Lambda(\mathcal{Q}_X)+\mathbb{Z}[S_X])$ is a quadratic form associated with B. Thus,

  (iv) $q(ax) = aq(x)\bar{a}$ for $a \in \mathbb{Z}[G]$, $x \in M$,

  (v) $B(x, x) = \widetilde{q(x)} + (-1)^k \overline{\widetilde{q(x)}}$ in $\mathbb{Z}[G]/\mathbb{Z}[S_X]$ for $x \in M$, where $\widetilde{q(x)} \in \mathbb{Z}[G]$ is a lifting of $q(x)$.

  (vi) $q(x+y) - q(x) - q(y) = B(x, y)$ in $\mathbb{Z}[G]/(\Lambda(\mathcal{Q}_X) + \mathbb{Z}[S_X])$ for x, $y \in M$.

(4) $\boldsymbol{\alpha}$ is a pair consisting of a $(G \times \{\pm 1\})$-map $\widetilde{\alpha}_+ : \widetilde{\Theta}_{X,+} \to M$ and a G-map $\alpha : \Theta_X \to M/2M$ such that the diagram

$$\begin{array}{ccc} \widetilde{\Theta}_{X,+} & \xrightarrow{\widetilde{\alpha}_+} & M \\ {\scriptstyle \pi_X}\downarrow & & \downarrow \\ \Theta_X & \xrightarrow[\alpha]{} & M/2M \end{array}$$

commutes.

If $\boldsymbol{M} = (M, B, q, \boldsymbol{\alpha})$ possesses a stably free $\mathbb{Z}[G]$-submodule $L \subset M$ such that $L = L^{\perp}$, $q(L) = 0$, $\mathrm{Image}(\widetilde{\alpha}_{+}) \subset L$ and $\mathrm{Image}(\alpha) \subset L/2L$ ($\subset M/2M$), then L is called a *Lagrangian* of $\boldsymbol{M}$ and $\boldsymbol{M}$ is called a *null module*, where

$$L^{\perp} = \{x \in M \mid B(x,y) = 0 \ (y \in L)\}.$$

Let $\mathfrak{Q}(\mathcal{D}_X)$ denote the category of $\mathcal{D}_X$-quadratic modules, where morphisms are isomorphisms. Then the notion of *direct sum* on $\mathfrak{Q}(\mathcal{D}_X)$ is clear. Thus we have the Grothendieck group $K_0(\mathfrak{Q}(\mathcal{D}_X))$ and the Witt group

$$W(\mathfrak{Q}(\mathcal{D}_X))_{\mathrm{proj}} = K_0(\mathfrak{Q}(\mathcal{D}_X))/\langle \text{null modules} \rangle.$$

Let $\nabla\mathfrak{Q}(\mathcal{D}_X)$ be the full subcategory of $\mathfrak{Q}(\mathcal{D}_X)$ consisting of all objects $\boldsymbol{M} = (M, B, q, \boldsymbol{\alpha})$ such that

$$B(\Sigma_s - sx, x) = 0 \ \text{ in } \mathbb{Z}_2[G]/\mathbb{Z}_2[G \smallsetminus \{e\}] \text{ for } s \in S_X,\ x \in M,$$

where

$$\Sigma_s = \sum \{\alpha(t) \mid t \in \Theta_X : \ \rho_X(t) \ni s\}.$$

We obtain the Grothendieck group $K_0(\nabla\mathfrak{Q}(\mathcal{D}_X))$ and the Witt group

$$\nabla W(\mathcal{D}_X)_{\mathrm{proj}} = K_0(\nabla\mathfrak{Q}(\mathcal{D}_X))/\langle \text{null modules in } \nabla\mathfrak{Q}(\mathcal{D}_X) \rangle.$$

4. Computability property

Let $\mathfrak{S}(G)$ be the subgroup category defined by J. A. Green [7]: namely its objects are subgroups of G and its morphisms are triples (H, g, K) such that H, $K \in \mathcal{S}(G)$ and $g \in G$ such that $gHg^{-1} \subset K$. Let $\mathfrak{Ab}$ be the category of abelian groups: namely its objects are abelian groups and its morphisms are homomorphisms of groups. Let $w : G \to \{\pm 1\}$ be a homomorphism and $\mathcal{G}$ a family of subgroups of G. The notion of $(w, \mathcal{G})$-Mackey functor is similar to that of Mackey functor (cf. [13], [11]). A *$(w, \mathcal{G})$-Mackey functor* $M = (M^*, M_*)$ is a bifunctor from $\mathfrak{S}(G)$ to $\mathfrak{Ab}$ such that $M_*(H) = M^*(H)$ $(= M(H))$ for $H \in \mathcal{S}(G)$ and the following is satisfied.

(1) $c_{(H,g)*} = c^*_{(gHg^{-1},g^{-1})}$ for $H \in \mathcal{S}(G)$ and $g \in G$.
(2) $c_{(H,h)*} = w(h) id_{M(H)}$ for $H \in \mathcal{S}(G)$ and $h \in H$.

(3) $\mathrm{res}^L_K \circ \mathrm{ind}^L_H$ coincides with

$$\bigoplus_{KgH \in K\backslash L/H} \mathrm{ind}^K_{H\cap gHg^{-1}} \circ \left(w(g)c_{(H\cap g^{-1}Kg,g)*}\right) \circ \mathrm{res}^H_{H\cap g^{-1}Kg}$$

for $L \in \mathcal{G}$, H, $K \in \mathcal{S}(L)$,

where $c_{(H,g)*} = M_*(H, g, gHg^{-1})$, $c^*_{(H,g)} = M^*(H, g, gHg^{-1})$, $\mathrm{ind}^K_H = M_*(H, e, K)$, and $\mathrm{res}^K_H = M^*(H, e, K)$. In the case w is trivial, a $(w, \mathcal{G})$-Mackey functor is called a *$\mathcal{G}$-Mackey functor.* Moreover in the case $\mathcal{G} = \mathcal{S}(G)$, a $\mathcal{G}$-Mackey functor is called a *Mackey functor.*

Theorem 4.1. *Let X be as in Theorem 1.3. Then $\nabla W(-)_{\mathrm{proj}}$ canonically has the structure of a Mackey functor: namely there exists a Mackey functor $M = (M_*, M^*) : \mathfrak{S}(G) \to \mathfrak{Ab}$ such that $M_*(H) = M_*(H) = \nabla W(\mathcal{D}_H)_{\mathrm{proj}}$.*

Let $\mathcal{F}$ be a family of subgroups of G closed under conjugation by all elements in G and under arbitrary intersections: namely $gHg^{-1} \in \mathcal{F}$ for all $H \in \mathcal{F}$ and $g \in G$, and $H \cap K \in \mathcal{F}$ holds for all H, $K \in \mathcal{F}$. A $(w, \mathcal{G})$-Mackey functor M is called *$\mathcal{F}$-computable* if the induction homomorphism

$$\mathrm{Ind} : \varinjlim_{\mathcal{F}} M(-) \to M(G)$$

and the restriction homomorphism

$$\mathrm{Res} : M(G) \to \varprojlim_{\mathcal{F}} M(-)$$

are both isomorphisms. If $\mathcal{U}$ is a set of prime integers then we denote by $\mathcal{U}'$ the multplicatively closed subset of integers generated by 1 and all prime integers $q \notin \mathcal{U}$ dividing $|G|$. A $(w, \mathcal{G})$-Mackey functor M is called *$\mathcal{F}$-hypercomputable* if $\mathcal{U}'^{-1}M$ is $\mathcal{F}^{\mathcal{U}}$-computable for all $\mathcal{U}$ as above, where

$$\mathcal{F}^{\mathcal{U}} = \{K \in \mathcal{S}(G) \mid K \trianglerighteq H,\ H \in \mathcal{F},\ K/H \text{ has } p\text{-power order for some } p \in \mathcal{U}\}.$$

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