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Financial Integration and Aggregate Stability[1]

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Abstract. This paper explores a dynamic two-country model with production externalities in which capital goods are not traded and international lending and borrowing are allowed. Unlike the integrated world economy model based on the Heckscher-Ohlin setting, our model yields indeterminacy of equilibrium under a wider set of parameter values than in the corresponding closed economy model. Our finding demonstrates that the assumption on trade structure would be a relevant determinant in considering the relation between globalization and economic volatility. *Keywords*: two-country model, non-traded goods, equilibrium indeterminacy, social constant returns. *JEL classification:* F43, O41

1 Introduction

The relation between globalization and economic volatility has been one of the major research concerns in the field of international macroeconomics. So far, the theoretical as well as empirical studies have shown diverse conclusions: internationalization of an economy may or may not enhance economic volatility. Even though we restrict our attention to the equilibrium business cycle theory based on indeterminacy and sunspots, we find that the theory has presented two different results. On the one hand, several authors such as Meng (2003), Meng and Velasco (2003 and 2004) and Weder (2001) show that small-open economies with

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production externalities produce indeterminacy of equilibrium under a wider set of parameter values than in the corresponding closed economy model. Hence, according to these studies, globalization of an economy may increase economic volatility.[2] Nishimura and Shimomura (2002), on the other hand, demonstrate that a world economy consisting of two symmetric countries with production externalities holds the same stability conditions as those for a closed economy counterpart. In addition, Sim and Ho (2007) find that if one of the two counties has no production externalities in Nishimura and Shimomura's model, then the equilibrium path of the world economy would be determinate even though the country with production externalities exhibits autarkic indeterminacy. These studies indicate that globalization does not necessarily enhance economic fluctuations.

The opposite results shown above seemingly stem from the difference in the analytical frameworks used by the foregoing studies. The small-open economy models are based on the partial equilibrium analysis in which behavior of the rest of the world is exogenously given. In contrast, the models of world economy employ the general equilibrium approach that treats the world economic system as a closed economy consisting of multiple countries. The world economy models thus consider more complex interdependency between the countries than that assumed in the small-open economy models. One may conjecture that such a difference would generate the contrasting views as to the destabilizing effect of globalization.

The purpose of this paper is to reveal that the difference in conclusions mentioned above mainly comes from the assumptions on trade structures rather than from the modelling strategies. To confirm this fact, we introduce non-traded goods and financial capital mobility into the model studied by Nishimura and Shimomura (2002) (in what follows, we call it the NS model). The NS model employs the standard Heckscher-Ohlin framework where both investment and consumption goods are freely traded but there is no intertemporal trade between the two countries. We assume that consumption goods are internationally traded but investment goods are non-tradables. We also assume that international lending and borrowing are possible. Unlike the Heckscher-Ohlin setting, in the presence of non-traded

[2] Lahiri (2001) also examines indeterminacy in a small-open economy model. Since he uses a framework different from the one used by Meng (2003) and others, his model needs a relatively high degree of external increasing returns to yield indeterminacy. Yong and Meng (2004) and Zhang (2008) also discuss equilibrium indeterminacy in small-open economies.

investment goods, the factor price equalization fails to hold in our model. As a result, in our modified framework the relative price between consumption and investment goods in the home country may diverge from that in the foreign country at least out of the steady state. This means that the dynamic behavior of our model out of the steady state will not be the same as that of a corresponding closed economy. Such a difference in transition dynamics generates the divergence of determinacy conditions between the closed economy and the integrated world economy with symmetric countries.

Our main finding is that the equilibrium indeterminacy conditions for the world economy with non-traded investment goods and financial transactions are similar to the stability conditions for the small-open economy models that have the same trade structure. More specifically, we show that our model may exhibit indeterminacy regardless of the restrictions on the preference structure. The closed-economy version of our model, which is essentially the same as the NS model, needs a high elasticity of intertemporal substitution in consumption to hold indeterminacy. Our study, therefore, demonstrates that even though the countries in the world economy have identical technologies and preferences, the presence of non-traded investment goods and financial capital mobility would generate a divergence in dynamic behavior of the integrated world economy and a closed, single country. In this sense, the structure of international trade would be a relevant determinant for the relation between globalization and volatility.

2 Model

Consider a world economy consisting of two countries, home and foreign. Both countries have the same production technologies. In each country there is a continuum of identical, infinitely-lived households. All the agents in both countries have an identical time discount rate and the same form of instantaneous felicity function. The only difference between the two countries is the initial stock of wealth held by the households in each country.

2.1 Production

The home country has two production sectors. The first sector ($i = 1$) produces investment goods and the second sector ($i = 2$) produces pure consumption goods. The production function of i-th sector is specified as

$$Y_i = A_i K_i^{a_i} L_i^{b_i} \bar{X}_i, \qquad a_i > 0,\ b_i > 0,\ \ 0 < a_i + b_i < 1, \qquad i = 1,2$$

where Y_i, K_i and L_i are i-th sector's output, capital and labor input, respectively. Here, $\bar{X}_i$ denotes the sector and country-specific production externalities. We define:

$$\bar{X}_i = \bar{K}_i^{\alpha_i - a_i} \bar{L}_i^{1-\alpha_i-b_i}, \quad a_i < \alpha_i < 1, \quad 1 - \alpha_i > b_i \quad i = 1,2.$$

If we normalizes the number of producers to one, then it holds that $\bar{K}_i = K_i$ and $\bar{L}_i = L_i$ $(i = 1,2)$ in equilibrium. This means that the i-th sector's social production technology that internalizes the external effects is:

$$Y_i = A_i K_i^{\alpha_i} L_i^{1-\alpha_i}, \quad i = 1,2. \tag{1}$$

Hence, the social technology satisfies constant returns to scale, while the private technology exhibits decreasing returns to scale.[3]

The factor and product markets are competitive, so that the private marginal product of each production factor equals its real factor price. These conditions are given by the following:

$$r = pa_1 \frac{Y_1}{K_1} = a_2 \frac{Y_2}{K_2}, \tag{2a}$$

$$w = pb_1 \frac{Y_1}{L_1} = b_2 \frac{Y_2}{L_2}, \tag{2b}$$

where w is the real wage rate, r is the rental rate of capital and p denotes the price of investment good in terms of the consumption good. The production technologies of the foreign country are the same as these of the home country where variables with an asterisk denote foreign variables.

2.2 Households

We assume that the households in the home country access the international financial market where foreign bonds are freely traded. By trading bonds, the households in the home country

[3]This specification of production technology was first introduced by Behbhabib and Nishimura (1998). Benhabib et al. (2000), Meng (2003), Meng and Velasco (2003, 2004), Mino (2001) and Nishimura and Shimomura (2002) utilize the same functional forms.

can borrow from or lend to the foreign households. The representative household in the home country maximizes

$$U = \int_0^{\infty} \frac{C^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt, \quad \sigma > 0, \quad \rho > 0$$

subject to the flow budget constraint

$$\dot{\Omega} = R\Omega + w + \pi_1 + \pi_2 - C, \tag{3}$$

together with the no-Ponzi-game condition

$$\lim_{t \to \infty} \exp\left(-\int_0^t R_s ds\right) \Omega_t \geq 0$$

and the initial value of Ω_0. In the above, C is consumption, R denotes interest rate, π_i is the excess profits in the i-th sector[4] and Ω is the net wealth (in terms of the consumption goods). The net wealth of held by the household consists of domestic capital and foreign bonds:

$$\Omega = pK + B,$$

where B denotes the stock of foreign bonds (in terms of the consumption goods). When selecting its optimal consumption plan, the household take the sequences of $\{R_t, w_t, \ \pi_{1,t}, \pi_{2,t}, p_t\}_{t=0}^{\infty}$ as given.

The definition of net wealth yields $\dot{\Omega} = p\dot{K} + \dot{p}K + \dot{B}$. Thus, the flow budget constraint (3) can be rewritten as

$$\dot{B} = RB + \left(R - \frac{\dot{p}}{p}\right) pK + w + \pi_1 + \pi_2 - C - p\dot{K}.$$

We also assume that the financial markets are perfect in the sense that domestic capital and foreign bonds are perfectly substitute each other. This means that arbitrage is excluded in each moment, so that the net rate of return to capital equals the real interest on bonds:

$$\frac{r}{p} - \delta = R - \frac{\dot{p}}{p}, \tag{4}$$

where $\delta \in [0, 1)$ denotes the rate of capital depreciation. As a consequence, the optimization problem for the representative household in the home country is to maximize U by controlling C and I subject to the following constraints:

$$\dot{B} = RB + rK + w + \pi_1 + \pi_2 - C - pI, \tag{5}$$

[4]Remember that the private technology exhinits decreasing returns to scale with respect to capital and labor.

$$\dot{K} = I - \delta K, \tag{6}$$

together with the initial holdings of K_0 and B_0. In this reformulation, the no-Ponzi-game condition is given by

$$\lim_{t \to \infty} \exp\left(-\int_0^t R_s ds\right) B_t \geq 0. \tag{7}$$

Set up the Hamiltonian function for the optimization problem:

$$H = \frac{C^{1-\sigma} - 1}{1 - \sigma} + \lambda\left[RB + rK + w + \pi_1 + \pi_2 - C - pI\right] + q\left(I - \delta K\right),$$

where λ and q respectively denote the implicit price of the foreign bonds and domestic capital. Focusing on an interior solution, we see that the necessary conditions for an optimum are:

$$C^{-\sigma} = \lambda \tag{8a}$$

$$p\lambda = q, \tag{8b}$$

$$\dot{\lambda} = \lambda\left(\rho - R\right), \tag{8c}$$

$$\dot{q} = q\left(\rho + \delta\right) - \lambda r = q\left(\rho + \delta - \frac{r}{p}\right). \tag{8d}$$

The optimization conditions also involve the transverslity conditions on holding B and K: $\lim_{t \to \infty} \lambda e^{-\rho t} B = 0$ and $\lim_{t \to \infty} q e^{-\rho t} K = 0$.

Since the foreign households have the same preference structure, their optimization conditions corresponding to $(8a)$, $(8b)$, $(8c)$ and $(8d)$ are as follows:

$$C^{*-\sigma} = \lambda^*, \tag{9a}$$

$$p^* \lambda^* = q^*, \tag{9b}$$

$$\dot{\lambda}^* = \lambda^*\left(\rho - R\right), \tag{9c}$$

$$\dot{q}^* = q^*\left(\rho + \delta - \frac{r^*}{p^*}\right). \tag{9d}$$

It is to be noted that while the interest rate, R, is common for both countries, the real rate of return to capital in the foreign country, r^*/p^*, may differ from r/p, because in our framework the factor-price equalization fails to hold out of the steady state.

2.3 Market Equilibrium Conditions

We assume that consumption goods are internationally traded but investment goods are non-tradables.[5] Although such an assumption is restrictive one, it helps to elucidate the role of trade structure in a dynamic world economy. Moreover, a large portion of investment goods are construction and structures, so that the investment goods sector shares a larger part of non-tradables than the consumption good sector.[6] Since investment goods are traded in the domestic markets alone and consumption goods are internationally traded, the market equilibrium conditions for investment and consumption goods are respectively given by

$$Y_1 = I, \qquad Y_1^* = I^*, \tag{10}$$

$$Y_2 + Y_2^* = C + C^*, \tag{11}$$

where I and I^* are gross investment expenditures in the home and foreign countries, respectively. Physical capital in each country accumulates according to

$$\dot{K} = I - \delta K, \qquad \dot{K}^* = I^* - \delta K^*. \tag{12}$$

As for the factor markets, we follow the standard Heckscher-Ohlin modelling: it is assumed that capital and labor are perfectly shiftable between the production sectors within a country, but they cannot move across the borders. Therefore, the full-employment conditions for production factors in each country are the following:

$$K = K_1 + K_2, \qquad 1 = L_1 + L_2, \tag{13a}$$

$$K^* = K_1^* + K_2^*, \qquad 1 = L_1^* + L_2^*. \tag{13b}$$

We assume that labor supply in each country is fixed and normalized to one.

Finally, the equilibrium condition for the bond market is

$$B + B^* = 0,$$

[5]The structure of our model is one of the dependent economy models discussed in open-economy macroeconomics literature. Turnovsky and Sen (1995) treat a small-open economy model with non-tradable capital and Turnovsky (1997, Chapter 7) studies a neoclassical two-country, two-sector model in which capital goods are not traded. Mino (2008) also discusses the similar two-country model with external increasing returns. See also Chapter 5 in Turnousky (2009) for a brief literature review.

[6]Bems (2008) finds that the share of investment expenditure on non-traded goods is about 60% and that this figure has been considerably stable over the last 50 years both in developed and developing countries.

which means that $\Omega+\Omega^* = K+K^*$. Bonds are IOUs between the home and foreign households and, hence, the aggregate stock of bonds is zero in the world financial market at large.

3 Volatility of the World Economy

3.1 Dynamic System

In equilibrium it holds that $\bar{K}_i = K_i$, $\bar{L}_i = L_i$, $\bar{K}_i^* = K_i^*$ and $\bar{L}_i^* = L_i$ $(i = 1, 2)$. From $(2a)$, $(2b)$ and the counterparts in the foreign country the factor prices in each country satisfy the following:

$$r = pa_1 A_1 k_1^{\alpha_1 - 1} = a_2 A_2 k_2^{\alpha_2 - 1}, \tag{14a}$$

$$w = pb_1 A_1 k_1^{\alpha_1} = b_2 A_2 k_2^{\alpha_2}, \tag{14b}$$

$$r^* = p^* a_1 A_1 k_1^{*\alpha_1 - 1} = a_2 A_2 k_2^{*\alpha_2 - 1}, \tag{14c}$$

$$w^* = p^* b_1 A_1 k_1^{*\alpha_1} = b_2 A_2 k_2^{*\alpha_2}, \tag{14d}$$

where $k_i = K_i/L_i$ and $k_i^* = K_i^*/L_i^*$ $(i = 1, 2)$. By use of $(14a)$, $(14b)$, $(14c)$ and $(14d)$, we can express the optimal factor intensity in each production sector as a function of relative price:

$$k_1 = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2 - \alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_2}{\alpha_2 - \alpha_1}} \left(\frac{b_1}{b_2}\right)^{\frac{\alpha_2 - 1}{\alpha_1 - \alpha_2}} p^{\frac{1}{\alpha_2 - \alpha_1}} \equiv k_1(p),$$

$$k_1^* = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2 - \alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_2}{\alpha_2 - \alpha_1}} \left(\frac{b_1}{b_2}\right)^{\frac{\alpha_2 - 1}{\alpha_1 - \alpha_2}} p^{*\frac{1}{\alpha_2 - \alpha_1}} \equiv k_1(p^*),$$

$$k_2 = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2 - \alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_1}{\alpha_2 - \alpha_1}} \left(\frac{b_1}{b_2}\right)^{\frac{\alpha_1 - 1}{\alpha_1 - \alpha_2}} p^{\frac{1}{\alpha_2 - \alpha_1}} \equiv k_2(p),$$

$$k_2^* = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2 - \alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_1}{\alpha_2 - \alpha_1}} \left(\frac{b_1}{b_2}\right)^{\frac{\alpha_1 - 1}{\alpha_1 - \alpha_2}} p^{*\frac{1}{\alpha_2 - \alpha_1}} \equiv k_2(p^*).$$

These expressions show that

$$\text{sign } k_i'(p) = \text{sign } k_i'(p^*) = \text{sign } (\alpha_2 - \alpha_1), \quad i = 1, 2. \tag{16}$$

In the above, the sign of $\alpha_1 - \alpha_2$ represents the factor intensity ranking from the social perspective. When $\alpha_1 - \alpha_2$ is positive (negative), the investment good sector is more (less) capital intensive than the consumption good sector from the social perspective.

In this paper we restrict our attention to the interior equilibrium in which both countries imperfectly specialize. To ensure this restriction, we assume that relative price in each country satisfies the following condition:

$$L_1 = \frac{K - k_2(p)}{k_1(p) - k_2(p)} \in (0,1), \qquad L_1^* = \frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} \in (0,1). \tag{17}$$

Using functions $k_1(p)$ and $k_2(p)$, we see that capital accumulation equation in each country is written as

$$\dot{K} = y^1(K,p) - \delta K, \tag{18a}$$

$$\dot{K}^* = y^1(K^*,p^*) - \delta K^*, \tag{18b}$$

where $y^1(K,p)$ and $y^1(K^*,p^*)$ express the supply functions of investment goods given by

$$y^1(K,p) \equiv \frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1},$$

$$y^1(K^*,p^*) \equiv \frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} A_1 k_1(p^*)^{\alpha_1}.$$

It is easy to see that these supply functions satisfy:

$$\text{sign } y_K^1(K,p) = \text{sign } y_{K^*}^1(K^*,p^*) = \text{sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right), \tag{19a}$$

$$\text{sign } y_p^1(K,p) = \text{sign } y_{p^*}^1(K^*,p^*) = \text{sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(\alpha_1 - \alpha_2). \tag{19b}$$

Notice that the sign of $\frac{a_1}{b_1} - \frac{a_2}{b_2}$ shows the factor intensity ranking from the private perspective.

The shadow values of capital in both countries change according to

$$\dot{q} = q[\rho + \delta - \hat{r}(p)], \tag{20a}$$

$$\dot{q}^* = q^*[\rho + \delta - \hat{r}(p^*)], \tag{20b}$$

where $\hat{r}(p) \equiv r/p = a_1 A_1 k_1(p)^{\alpha_1 - 1}$ and $\hat{r}(p^*) \equiv r^*/p^* = a_1 A_1 k_1(p^*)^{\alpha_1 - 1}$. Dynamic equations $(18a)$, $(18b)$, $(20a)$ and $(20b)$ depict behaviors of capital stocks and their implicit prices in the home and foreign countries.

The optimization conditions $(8c)$ and $(9c)$ mean that λ/λ^* stays constant over time and, therefore, from $(8a)$ and $(9a)$ the relative consumption, C/C^*, also stays constant even out of the steady state. Let us denote $C^*/C = (\lambda^*/\lambda)^{-1/\sigma} = \bar{m}\,(>0)$. Then the world market equilibrium condition for consumption goods given by (11) becomes

$$(1+\bar{m})\,\lambda^{-\frac{1}{\sigma}} = y^2(K,p) + y^2(K^*,p^*)\,, \tag{21}$$

where

$$y^2(K,p) = \frac{k_1(p) - K}{k_1(p) - k_2(p)} A_2 k_2(p)^{\alpha_2}\,,$$
$$y^2(K^*,p^*) = \frac{k_1(p^*) - K^*}{k_1(p^*) - k_2(p^*)} A_2 k_2(p^*)^{\alpha_2}\,.$$

The supply functions of consumption goods satisfy the following:

$$\text{sign } y^2_K(K,p) = \text{sign } y^2_{K^*}(K^*,p^*) = -\text{ sign }\left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right), \tag{22a}$$

$$\text{sign } y^2_p(K,p) = \text{sign } y^2_{p^*}(K^*,p^*) = -\text{ sign }\left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(\alpha_1 - \alpha_2)\,. \tag{22b}$$

In view of (21), we see that λ is expressed as a function of capital stocks, prices and $\bar{m}$:

$$\begin{aligned}\lambda &= (1+\bar{m})^{\sigma}\left[y^2(K,p) + y^2(K^*,p^*)\right]^{-\sigma} \\ &\equiv \lambda(K,K^*,p,p^*;\bar{m})\,. \end{aligned} \tag{23}$$

Thus optimization conditions (8b) and (9b) yield

$$p = \frac{q}{\lambda(K,K^*,p,p^*;\bar{m})}, \quad p^* = \frac{q^*}{\lambda(K,K^*,p,p^*;\bar{m})}.$$

Solving these equations with respect to p and p^* presents the following expressions:

$$p = \pi(K,K^*,q,q^*;\bar{m})\,, \quad p^* = \pi^*(K,K^*,q,q^*;\bar{m})\,. \tag{24}$$

Substituting (24) into (18a), (18b), (20a) and (20b), we obtain a complete dynamic system of K, K^*, q and q^*.

3.2 Conditions for Equilibrium Indeterminacy

We first characterize the stationary equilibrium of the world economy. The steady state of the dynamic system derived above is established when $\dot{K} = \dot{K}^* = \dot{q} = \dot{q}^* = 0$. From (24) the relative price in the home and foreign countries, p and p^*, also stay constant in the steady-state equilibrium. As for the existence of a feasible steady state, we can confirm the following:

Proposition 1 *Suppose that both countries imperfectly specialize in the steady state. Then the steady-state values of K, K^*, p and p^* are uniquely determined. Additionally, if $\bar{m}$ is fixed, the steady-state levels of q and q^* are uniquely given as well.*

Proof. Omitted. ■

It is worth noting that while the steady-state levels of K, K^*, p and p^* are independent of $\bar{m}$, the steady-state values of q and q^* depend on $\bar{m}$. Therefore, the presence of a unique set of steady state levels of q and q^* critically depends upon our assumption that the value of $\bar{m}$ is exogenously given. To complete our analysis on the steady-state equilibrium, we should consider how $\bar{m}$ is determined. Before discussing this problem, let us explore the local determinacy of the steady-state equilibrium under a given level of $\bar{m}$.

Proposition 2 *Under a given level of $\bar{m}$, the steady-state equilibrium of the world economy is locally indeterminate, if the investment good sector is more capital intensive than the consumption good sector from the social perspective but it is less capital intensive from the private perspective.*[7]

Proof. Omitted. ■

Proposition 2 claims that in our model equilibrium indeterminacy may emerge regardless of the magnitude of σ. This is in contrast to the conclusion of the NS model where, in addition to the conditions given in Proposition 2, the elasticity of intertemporal substitution in consumption, $1/\sigma$, should be high to hold intermediacy.[8] Since the closed economy version of our model is the same as the NS model, we need the same condition for holding indeterminacy if our model economy is closed. Hence, our result shows that the financially integrated world with non-tradable capital goods may produce indeterminacy under a wider range of parameter spaces than in the closed economy counterpart. In this sense, our model indicates that globalization may enhance the possibility of sunspot-driven economic fluctuations.[9]

[7] We can also show that, as well as in the NS model, our model holds equilibrium determinacy, if the factor-intensity rankings are the same both from private and social perspectives.

[8] More precisely, the indeterminacy conditions in the NS model include the following:

$$\frac{1}{\sigma} > \max\{1, \frac{(1-\alpha_1)a_2b_1(\rho+\delta)+\alpha_1 a_1\left[\rho b_2+\delta b_1 a_2+(1-a_1)b_2\delta\right]}{(a_2b_1-a_1b_2)(\alpha_1-\alpha_2)\left[\rho+\delta(1-a_1)\right]}\}.$$

[9] The indeterminacy conditions in Proposition 2 require that constant returns prevail in each production sector and that the external effects associated with capital are larger in the investment good sector than in the

We now consider how to determine $\bar{m}$. Using the market equilibrium condition for the investment goods in (10) and the factor income distribution relation such that $pY_1 + Y_2 = rpK + w + \pi_1 + \pi_2$ and $p^*Y_1^* + Y_2^* = r^*p^*K^* + w^* + \pi_1^* + \pi_2^*$, we find that the dynamic equation of foreign bonds are expressed as

$$\dot{B} = RB + Y_2 - C, \quad \dot{B}^* = RB^* + Y_2^* - C^*.$$

These equations represent the current accounts of both countries. In view of the no-Ponzi game and the transversality conditions, the intertemporal constraint for the current account of each country is respectively given by the following:

$$\int_0^\infty \exp\left(-\int_0^t R_s ds\right) C_t dt = \int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2 (K_t, p_t)\, dt + B_0,$$

$$\int_0^\infty \exp\left(-\int_0^t R_s ds\right) C_t^* dt = \int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2 (K_t^*, p_t^*)\, dt + B_0^*.$$

Since it holds that $C_t^* = \bar{m} C_t$ for all $t \geq 0$, the above equations yield

$$\bar{m} = \frac{\int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2 (K_t^*, p_t^*)\, dt + B_0^*}{\int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2 (K_t, p_t)\, dt + B_0}. \tag{25}$$

Equation (25) demonstrates that $\bar{m}$ depends on the initial holdings of bonds, B_0 and B_0^*, as well as on the discounted present value of consumption goods produced in each country. It is to be noticed that the discounted present values of consumption goods are independent of $\bar{m}$. To see this, we differentiate both sides of (23) logarithmically with respect to time, which yields

$$\frac{\dot{\lambda}}{\lambda} = -\sigma \left[\frac{Y_K^2 K}{Y^2} \frac{\dot{K}}{K} + \frac{Y_{K^*}^2 K^*}{Y^2} \frac{\dot{K}^*}{K^*} + \frac{Y_p^2 p}{Y^2} \frac{\dot{p}}{p} + \frac{Y^2 p^*}{Y^2} \frac{\dot{p}^*}{p^*} \right], \tag{26}$$

consumption good sector. Several investigations on scale economies have suggested that our indeterminacy conditions are not unrealistic. For example, the well-cited study by Basu and Fernald (1997) find that most industries in the US approximately exhibit constant returns to scale, which may support our assumption of social constant returns. Using the US data, Harrison (2003) claims that returns to scale of the consumption goods sector are close to be constant, while the investment goods sector exhibits weak increasing returns. In addition, she reveals that external effects may be larger in the investment good sector than in the consumption good sector. However, the existing studies do not present direct empirical evidence for our discussion. Since the indeterminacy conditions in Proposition 2 are frequently used in the literature, it is a relevant task to find more convincing empirical support.

where $Y^2 \equiv y^2(K,p) + y^2(K^*,p^*)$ denotes the aggregate supply of consumption goods in the world market. Note that from $(8b)$, $(8c)$, $(8d)$, $(9b)$, $(9c)$ and $(9d)$, we obtain:

$$\frac{\dot{p}}{p} = \frac{\dot{q}}{q} - \frac{\dot{\lambda}}{\lambda} = R + \delta - \hat{r}(p), \tag{27a}$$

$$\frac{\dot{p}^*}{p^*} = \frac{\dot{q}^*}{q^*} - \frac{\dot{\lambda}^*}{\lambda^*} = R + \delta - \hat{r}(p^*). \tag{27b}$$

Substituting $(18a)$, $(18b)$, $(27a)$, and $(27b)$ into (26) yields the following:

$$\begin{aligned}\rho - R \;\; = \;\; & -\sigma\left[\frac{Y_K^2 K}{Y^2}\left(\frac{y^1(K,p) - \delta K}{K}\right) + \frac{Y_{K^*}^2 K^*}{Y^2}\left(\frac{y^2(K^*,p) - \delta K^*}{K^*}\right)\right. \\ & \left. + \frac{Y_p^2 p}{Y^2}(R + \delta - \hat{r}(p)) + \frac{Y_{p^*}^2 p^*}{Y^2}(R + \delta - \hat{r}(p^*))\right].\end{aligned}$$

Observe that each side of the above equation does not involve $\bar{m}$. Solving the above with respect to R, we find that the equilibrium level of the world interest rate can be expressed as a function of K, K^*, p and p^* :

$$R = R(K, K^*, p, p^*). \tag{28}$$

Consequently, by use of $(18a)$, $(18b)$, $(27a)$, $(27b)$ and (28), we obtain an alternative expression of the complete dynamic system of (K, K^*, p, p^*) in such a way that

$$\left.\begin{aligned} \dot{K} &= y^1(K,p) - \delta K, \\ \dot{K}^* &= y^1(K^*,p^*) - \delta K^*, \\ \dot{p} &= p\left[R(K,K^*,p,p^*) + \delta - \hat{r}(p)\right], \\ \dot{p}^* &= p^*\left[R(K,K^*,p,p^*) + \delta - \hat{r}(p^*)\right]. \end{aligned}\right\} \tag{29}$$

From $(8c)$ the steady-state level of interest rate satisfies $R = \rho$.[10] Since the dynamic system (29) does not involve $\bar{m}$, if the steady state is locally determinate (i.e. the linearized dynamic system has two stable roots), then the equilibrium path of p_t and p_t^* are uniquely expressed as functions of K_t and K_t^* on the two-dimensional stable manifold. When we denote the relation between the relative prices and capital stocks on the stable saddle path as $p = \phi(K, K^*)$ and $p^* = \phi^*(K, K^*)$, the behaviors of capital stocks on the saddle path are expressed as

$$\begin{aligned}\dot{K} \;\; &= \;\; y^1(K, \phi(K,K^*)) - \delta K, \\ \dot{K}^* \;\; &= \;\; y^1(K^*, \phi^*(K,K^*)) - \delta K^*.\end{aligned}$$

[10] We can show that dynamic analysis of (29) presents the same conclusion as that stated in Proposition 2. However, since function (28) is rather complex, stability analysis is more cumbersome than that shown in Appendix 2.

These differential equations show that once the initial capital stocks, K_0 and K_0^*, are specified, the paths of $\{K_t, K_t^*\}_{t=0}^{\infty}$ are uniquely determined. As a result, the paths of $\{p_t, p_t^*, R_t\}_{t=0}^{\infty}$ are also uniquely given under the specified levels of K_0 and K_0^*. This means that when equilibrium determinacy holds, the level of $\bar{m}$ given by (25) is also uniquely selected under the given initial levels of K_0, K_0^*, B_0 and B_0^*.

In contrast, if the converging path of (29) is indeterminate (that is, the linearly approximated dynamic system of (29) has three or four stable roots), then the given initial levels of K_0 and K_0^* alone cannot pin down the equilibrium paths of p_t and p_t^*. Therefore, the level of $\bar{m}$ determined by (25) becomes indeterminate as well. In this situation, we should specify expectations formation of agents to select a particular path leading to the steady state. Once we specify a particular trajectory of the world economy with self-fulfilling expectations, we can determine the value of $\bar{m}$ that satisfy (25). However, such an equilibrium path may fluctuate if a sunspot shock hits the world economy, so that $\bar{m}$ is also affected by expectations-driven fluctuations.

In the steady state it holds that $\dot{B} = \dot{B}^* = 0$ and $R = \rho$. Thus the steady-state level of bond holdings in both countries are given by

$$B = \frac{y^2(K,p) - C}{\rho} = \frac{\bar{m} - 1}{\rho(1 + \bar{m})} y^2(K,p), \tag{30a}$$

$$B^* = \frac{y^2(K,p) - \bar{m}C}{\rho} = \frac{1 - \bar{m}}{\rho(1 + \bar{m})} y^2(K,p). \tag{30b}$$

The above expressions show that when $\bar{m}$ is selected, the long-run asset position of each country is also determined. It is obvious that whether the home country becomes a creditor or a debtor in the long run depends solely on whether or not $\bar{m}$ exceeds one. As (25) demonstrates, if the equilibrium path is determinate and if the initial stocks of capital and bonds held by the home households are relatively large, then the home country tends to be a creditor in the long-run equilibrium. However, if there is a continuum of covering path around the steady state, the value of $\bar{m}$ determined by (25) is affected by the expectations formation of agents. This implies that in the presence of equilibrium indeterminacy, the initial holding of wealth in each country does not necessarily determine the asset position of that country in the long-run equilibrium.

To sum up, we have shown:

Proposition 3 *If the steady-state equilibrium of the world economy is locally determinate (indeterminate), then the steady-state level of asset position of each country is determinate (indeterminate).*

4 Discussion

As emphasized in the previous section, the prominent difference in indeterminacy conditions between our formulation and the NS model is that our model may produce indeterminacy regardless of the form of instantaneous utility function, while the NS model need a high elasticity of intertemporal substation in consumption. In this section we consider the key factors that generate such a difference in indeterminacy conditions. We first summarize dynamic behavior of the NS model and give an intuitive implication of its indeterminacy conditions. We then examine the indeterminacy conditions for a small-open economy with non-traded investment goods and financial capital mobility, which reveals that the indeterminacy conditions for our model ares close to these for the small-open economy. Finally, we explore the case where consumption goods are non-tradables and investment goods are traded. This consideration demonstrates that the absence of investment goods trade plays a pivotal role for producing the difference in indeterminacy conditions between our model and the NS model.

4.1 Indeterminacy in the NS Model

In the NS model, both investment and consumption goods are freely traded, but there is no intertemporal trade.[11] Therefore, firms in both countries face a common relative price. This means that under our assumption of symmetric technologies between the two countries, both home and foreign firms in each production sector select the same capital intensity as long as both countries imperfectly specialize. Hence, it holds that $k_i(p) = k_i^*(p) \quad (i = 1, 2)$ for all

[11] If we introduce international lending and borrowing into the NS model, the households' portfolio choice between capital and bonds becomes indeterminate. Hence, intertemporal trade is redundant in the NS model that uses the standard Heckscher-Ohlin framework. This result is a reconfirmation of Mundel's (1957) well known finding in the static Heckscher-Ohlin model. As to the discussion of this issue in the dynamic context, see Claustre and Kehoe (2010) and Cremers (1997). Ono and Shibata (2010) point out that lending and borrowing can be introduced into the standard dynamic Heckscher-Ohlin model if we assume the presence of investment adjustment costs.

$t \geq 0$. Thus the world market equilibrium condition for investment goods, $\dot{K} + \delta K + \dot{K}^* + \delta K^* = Y_1 + Y_1^*$, yields the capital formation equation such that

$$\dot{K}_w = \frac{K_w - 2k_2(p)}{k_1(p) - k_2(p)} - \delta K_w, \tag{31}$$

where $K_w = K + K^*$ denotes the world level of capital stock. In addition, since firms in both countries choose the same capital intensities, the factor prices are also equalized between the two countries, that is, $\hat{r}^*(p) = \hat{r}(p)$ and $w^*(p) = w(p)$, implying that the dynamic equation of the shadow value of capital in the foreign country (20b) is replaced with $\dot{q}^* = q^*(\rho + \delta - \hat{r}(p))$. Tuss equation (20$a$) shows that q^*/q stays constant over time and that the complete dynamic system of the world economy is given by (20a), (31) and the world market equilibrium condition for consumption goods:

$$Y_2 + Y_2^* = C + C^* \Rightarrow (1 + \bar{m})\, q = \frac{2k_1(p) - K_w}{k_1(p) - k_2(p)}, \tag{32}$$

where $\bar{m}$ is a positive constant defined as $\bar{m} = C/C^* = (q/q^*)^{-1/\sigma}$. Given $\bar{m}$, the equilibrium relative price can be expressed as $p = \pi(K_w, q; \bar{m})$. Substituting this function into (20a) and (31), we obtain a complete dynamic system with respect to K_w and q. This aggregate dynamic system is essentially the same as the closed economy model in Benhabib and Nishimura (1998).[12] As a consequence, the intuitive implication of the indeterminacy conditions for the NS model is the same as that for the case of a closed economy.

Now suppose that the indeterminacy conditions for the NS model are satisfied, that is, $1/\sigma$ has a large value and the indeterminacy conditions in Proposition 2 hold. Suppose further that a sunspot shock hits the economy so that agents anticipate the rate of return to capital will rise. Then the households increase investment to accumulate a larger amount of capital, but in the absence of international borrowing, they should reduce their current consumption to raise savings. Since $1/\sigma$ is assumed to be high, the substitution effect dominates the income effect so that an anticipated rise in the rate of return actually increases the households' current savings, which leads to a higher level of capital stock. As we have assumed that

[12]Nishimura and Shimomura (2002) show that the steady-state levels of K_w and p are uniquely given. Distribution of capital stock between the two countries depends on the initial holdings of capital if the equilibrium path of the world economy is determinate. They also confirm that the long-run distribution of capital (so the long-run trade patterns) becomes indeterminate, if the steady state of the dynamic system of the world economy is a sink.

the consumption good sector is more capital intensive from the private perspective, (17) shows that an increase in capital stock raises consumption goods production relative to the investment goods. This increases the price of investment good p, and the firms selects a lower capital intensity because the social technology of the capital goods is more capital intensive than that of the consumption good sector (see (16)). Consequently, the rate of return to capital will increase and the sunspot-driven expectations will be self-fulfilled.

4.2 Indeterminacy in the Small-Open Economy

Next, consider a small-open economy that has the same trade structure as ours. Since the world interest rate is exogenously given for the small country, its dynamic behavior is described by (18a) and $\dot{\lambda} = \lambda\left(\bar{R} - \rho\right)$, where $\bar{R}$ is a given world interest rate. The conventional assumption is that the time discount rate is set to satisfy $\rho = \bar{R}$ to obtain a feasible steady state, which means that the shadow value of foreign bonds, λ, stays constant over time. As a reslut, from (8b) the shadow value of capital is proportional to the relative price even out of the steady state. This means that the price dynamics is given by

$$\dot{p} = p\left(\bar{R} + \delta - \hat{r}(p)\right) = p\left(\rho + \delta - \hat{r}(p)\right). \tag{33}$$

The complete dynamic system consists of (18a) and (33). It is easy to confirm that the conditions displayed in Proposition 2 are necessary and sufficient for indeterminacy in the small-open economy.

Again assume that a sunspot-driven expectation change makes the households raise their investment. Unlike the NS model, in a small country with financial capital mobility, the households may rise their investment by borrowing from the foreign households without reducing current consumption. Therefore, the elasticity intertermporal substitution in consumption will play no role for equilibrium determinacy/indeterminacy.

It is now obvious that there is a strong similarity between the dynamic behavior of our model and that of the small-open economy with non-traded investment goods and financial capital mobility. In fact, if we ignore the foreign country and set $R = \bar{R}$ in (29), then we obtain a dynamic system in which the home country is assumed to be a small-open economy. In the general equilibrium model of the world economy, the interest rate, R, is an endogenous variable and, hence, the conditions in Proposition 2 are sufficient but not necessary for indeterminacy in our model. However, under our assumption that both countries have the same technologies and the identical homothetic preferences, the dynamic belabor of an individual country in the world economy is close to that of the small-open economy at least near the steady state.

5 A Final Remark

The world economy as a whole is a closed economy in which there are heterogeneous countries. Therefore, its model structure is similar to that of a closed, single economy model with heterogeneous agents. In particular, if consumption and saving decisions are made by the representative household in each country, the world economy model is closely connected to the closed economy model with heterogeneous households. There is, however, an important difference between the world economy models and the single country setting: when dealing with the world economy model, we should specify the trade structure between the countries. This paper has revealed that the assumption on trade structure may be critical for the presence of equilibrium indeterminacy even if there is no international heterogeneity in technologies and preferences. Several authors have explored recently how the presence of heterogeneous preferences and technologies alter the determinacy/indeterminacy conditions in the equilibrium business cycle models with market distortions. These studies have shown that the heterogeneity in preferences and technologies often affects stability condition in a critical manner.[13] In a similar vein, Sim and Ho (2007) find that introducing technological heterogeneity into the NS model may produce a substantial change in equilibrium indeterminacy results. Those existing findings suggest that it is worth extending our model by considering further heterogeneity between the two countries.

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