

数理解析研究所講究録 1798

RIMS 合宿型セミナー

Modern approach and developments to  
Onsager's theory on statistical vortices

京都大学数理解析研究所

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*Modern approach and developments to Onsager's theory  
on statistical vortices*

*August 28 ~31, 2011*

*edited by Hiroshi Ohtsuka*

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*Research Institute for Mathematical Sciences*

*Kyoto University, Kyoto, Japan*

This is a report of research done at the Research Institute for Mathematical Sciences, Kyoto University. The papers contained herein are in final form and will not be submitted for publication elsewhere.

# Preface

The RIMS Camp-style Seminar “Modern approach and developments to Onsager’s theory on statistical vortices” was held at Apical Inn Kyoto , Kyoto, Japan, during the period August 28-31, 2011.

The purpose of the seminar was to share the recent progress around the Onsager ’s theory of vortices for two-dimensional turbulence. We were mainly interested in

- Mathematical analysis of mean field equations of vortices,
- Euler equations of fluid mechanics,
- Relating physical background for the underlying phenomena.

Several main speakers from foreign countries as well as Japanese researchers of various research fields and various generations attended and 18 talks were presented by the participants. Most of the papers in this volume are based on the talks.

On behalf of the organizers, I would like to thank all the participants of the seminar and all the authors to this volume. I hope that the encounter in the seminar and this volume will bear many fruits in the near future. I also would like to thank the co-organizers Professor Taku Yanagisawa of Nara Women’s University and Professor Yuichi Yatsuyanagi of Shizuoka University for their kind support to me. Finally I would like to express my sincere gratitude to RIMS for support of the seminar and publication of the proceedings.

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RIMS Camp-style Seminar  
**Modern approach and developments to Onsager's theory  
on statistical vortices**

August 28 (Sun.) - 31 (Wed.), Apical Inn Kyoto , Kyoto, Japan  
[http://www.bea.hi-ho.ne.jp/pickles/2011RIMS\\_Onsager/](http://www.bea.hi-ho.ne.jp/pickles/2011RIMS_Onsager/)

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**Program**

Aug. 28 (Sun.)

(The seminar room is available from 14:00 to 21:00.)

15:30-16:00 (registration)

16:00-17:00 **Hiroshi Ohtsuka** (University of Miyazaki)

Opening Talk ; Purpose of this seminar

18:00- (Dinner)

Aug. 29 (Mon.)

(The seminar room is available from 9:00 to 21:00.)

- Recent Insights from Physics - (Chairman: Prof. Suzuki)

9:30-10:30 **Pierre-Henri Chavanis** (Université Paul Sabatier) : Plenary talk

Kinetic theory of two-dimensional point vortices

11:00-12:00 **Freddy Bouchet** (ENS de Lyon) : Plenary talk

Invariant measures of the 2D Euler equations and applications to equilibrium and non equilibrium phase transitions

12:00-14:30 (Lunch, free discussions)

- Session for Young Physicists - (Chairman: Prof. Yatsuyanagi)

14:30-15:00 **Hidetoshi Morita** (Kyoto University)

Non-“equilibrium” oscillations in two-dimensional Euler equations

15:00-15:30 **Akio Sanpei** (Kyoto Institute of Technology)

Experimental Study of Formation of Vortex Crystal Configuration in Pure Electron Plasma

15:30-16:00 **Makoto Hirota** (Japan Atomic Energy Agency)

Variational formulation of nonlinear hydrodynamic stability

- Developments of the theory of vortices - (Chairman: Prof. Fukumoto)

16:30-17:00 **Ken Sawada** (Meteorological College)

Mean field equation for vortex filament systems

17:00-17:30 **Yuichi Yatsuyanagi** (Shizuoka University)

Analytical derivation of diffusion coefficient of two-dimensional point vortex system with Klimontovich formalism

18:00- (Dinner)

Aug. 30 (Tues.)

(The seminar room is available from 9:00 to 21:00.)

- Leading-edge of the fluid equations - (Chairman: Prof. Yanagisawa)

9:30-10:30 **Zoran Grujic** (University of Virginia) : Plenary talk

Anomalous dissipation as a trigger for the energy cascade in 3D inviscid flows

11:00-12:00 **Dongho Chae** (Sungkyunkwan University) : Plenary talk

On the blow-up problem for the Euler equations and the Liouville type results in the fluid equations

12:00-14:00 (Lunch, free discussions)

- Session for Young Mathematicians I : Fluid equations - (Chairman: Prof. Chae)

14:00-14:30 **Ryo Takada** (Tohoku University)

Propagation of the analyticity for the solution to the Euler equations with non-decaying initial velocity

14:30-15:00 **Yasunori Maekawa** (Kobe University)

On vorticity concentration at the zero viscosity limit for the Navier-Stokes flows in the half plane

15:00-15:30 **Yasushi Taniuchi** (Shinshu University)

Uniqueness of almost periodic-in-time solutions to Navier-Stokes equations in unbounded domains

- Session for Young Mathematicians II : Elliptic equations - (Chairman: Prof. Ohtsuka)

16:00-16:30 **Toru Kan** (Tohoku University)

Bifurcation structure of the mean field equation for an annular domain

16:30-17:00 **Ryo Takahashi** (Osaka University)

Residual vanishing of concentration arising in the mean field equations

17:00-17:30 **Tonia Ricciardi** (FedericoII University of Naples)

Blow-up analysis and optimal Trudinger-Moser inequalities for some mean field equations in statistical hydrodynamics

19:00- (Banquet : Garden BBQ)

Aug. 31 (Wed.)

(The seminar room is available from 9:00 to 12:00.)

- Perspectives of futures - (Chairman: Prof. Yanagisawa)

10:00-10:45 **Yasuhide Fukumoto** (Kyushu University)

Kinematic variational principle for vortical structure of Euler flows and beyond

11:00-11:45 **Takashi Suzuki** (Osaka University)

From static to kinetic mean field theories - hierarchy and duality

11:45-12:00 (Closing)

12:00-13:00 (Lunch)

Modern approach and developments to Onsager's theory on statistical vortices

Onsager の点渦統計理論に対する現代的接近と広がり

RIMS 合宿型セミナー報告集

2011年8月28日～8月31日

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