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A Note on Controlled Assembly Maps

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1. Introduction

In [4], I considered an $L^{-\infty}$-theory functor $\mathbb{L}(-;-)$. To avoid confusion with the ordinary L^h-theory functor, $\mathbb{L}(X;p)$ will be denoted $\mathbb{L}^{-\infty}(X;p)$ with decoration $-\infty$ in this paper. $\mathbb{L}^{-\infty}$ is functorial; there is a "forget-control" map

$$F : \mathbb{L}^{-\infty}(X; p : E \to X) \to \mathbb{L}^{-\infty}(*; E \to *) = \mathbb{L}^{-\infty}(E).$$

If E has the homotopy type of a connected CW-complex, then the homotopy groups of $\mathbb{L}^{-\infty}(E)$ are isomorphic to the groups $L^{-\infty}(\pi_1(E))$ of Wall and Ranicki.

The following is the key technical result of [4, Theorem 3.9]:

Theorem 1.1 (Characterization Theorem). *Suppose $p : E \to |K|$ is a simplicially stratified fibration on a finite polyhedron K. Then there is a homotopy equivalence ("controlled assembly map")*

$$A_j : \mathbb{H}_j(K; \mathbb{L}^{-\infty}(p)) \longrightarrow \mathbb{L}^{-\infty}_{-j}(|K|; p)$$

such that its composition with the foreget-control map F is the ordinary assembly map $a_j : \mathbb{H}_j(K; \mathbb{L}^{-\infty}(p)) \to \mathbb{L}^{-\infty}_{-j}(E)$.

Unfortunately there were erroros in the construction of the controlled assembly map A_j. In this paper, I will give an analysis of the errors. A correction will be given in a separate paper [6].

2. Errors

The proof of the Characterization Theorem is divided into two parts: the first part is the construction of the controlled assembly map A, and the second part is to check that A is a homotopy equivalence. In this section I identify the errors which occurs in the first part of the proof.

First of all, there is a problem concerning glueing and splitting problems over a triangulation as discussed in [5]. This problem can be overcome by using a different

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approach to homology theory using the idea of 'cycles'. I pretend that glueing and splitting can be done and analyze the proof of the characterization theorem.

On p. 589 of [4], I tried to construct a map

$$FS\Omega^{n-j}(|\mathbb{L}^{-\infty}_{-n}(p)|/s(X)) \to \mathbb{L}^{-\infty}_{-j}(X;p) ,$$

where $F : \mathbf{css} \to \Delta$ is the forgetful functor from the category of css-sets to the category of Δ-sets, and $S : \mathbf{CW} \to \mathbf{css}$ is the singular complex functor from the category of CW complexes to **css** [3]. First I represented a k-simplex of $FS\Omega^{n-j}(|\mathbb{L}^{-\infty}_{-n}(p)|/s(X))$ by a map $\rho : S^{n-j} \times \Delta^k \to |\mathbb{L}^{-\infty}_{-n}(p)|/s(X)$. Then I argued that there exist a codimension 0 submanifold V of $S^{n-j} \times \Delta^k$ and a cellular map $\rho' : V \to |\mathbb{L}^{-\infty}_{-n}(p)|$ such that ρ sends the complement of the interior of V to the basepoint $[s(X)]$ and $\rho|V$ factors through ρ'. I further modified ρ' so that it is a realization of a Δ-map, also denoted ρ', from V to a subcomplex $\left(\coprod_{\Delta \in X} \mathbb{L}^{-\infty}_{-n}(p^{-1}(\Delta)) \times FG(\Delta)\right) / \sim$ of $\mathbb{L}^{-\infty}_{-n}(p)$, where $G : \Delta \to \mathbf{css}$ is the left adjoint of F [2].

Now, for each top dimensional simplex σ of V, $\rho'(\sigma)$ is a pair of a geometric quadratic Poincaré $(n-j+k+2)$-ad c_σ on $\mathbb{R}^l \times p^{-1}(\Delta)$ (and hence on $\mathbb{R}^l \times E$) and a (possibly degenerate) $(n-j+k)$-face of Δ for some simplex Δ of X. In [4], I used the same symbol Δ instead of σ, and started to confuse the simplices of V with the simplices of X. The size of c_σ measured in X is smaller than or equal to the diameter of Δ. After stabilization we can assemble all these to get a geometric quadratic Poincaré special $(k+2)$-ad on $\mathbb{R}^m \times E$, for some m, and it defines a k-simplex of the uncontrolled $\mathbb{L}^{-\infty}$-theory spectrum $\mathbb{L}^{-\infty}_{-j}(E)$ (denoted $\mathbb{P}'_{-j}(X;p)$ in [4]). I claimed that its size is the maximum of the diameters of the simplices of V, and it is wrong because gluing operations increase size. When we glue things over a finite polyhedron, there is a constant $\lambda > 0$ which depends on the triangulation and the dimensions of objects involved such that the union has size $\leq \lambda \cdot$ (the maximum size of the pieces). So each piece must have vey small size compared with λ in order to get a union with very small size.

And I made another serious error immediately after this, in what I called the barycentric subdivision argument. In the theorem, I assumed that p is a simplicially stratified fibratiom.This means that X is a geometric realization of a finite ordered simplicial complex K and that p has an iterated mapping cylinder decomposition with respect to K in the sense of Hatcher [1, p.105] [2, p.457]. I tried to squeeze each piece

[2]This uses the fact that $\mathbb{L}^{-\infty}(p^{-1}(\Delta))$ is Kan. Recall that, for Kan complexes, the two products $\times$ and $\otimes$ are 'homotopy equivalent'.

using the iterated mapping cylinder structure of $p^{-1}(\Delta) \subset E$. But this may not be possible since the ad-structure of c_σ may not respect the iterated mapping cylinder structure at all! The iterated mapping cylinder structure restricts the directions for squeezing, and we need to be very careful how we squeeze things.

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