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Some problems of amalgamation bases *

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In this paper, we shall pose some open problems concerned with semigroup amalgamation bases.

Let $\mathcal{A}$ be the class of semigroups or the class of finite semigroups. A triple of semigroups S, T, U with $U = S \cap T$ being a subsemigroup of S and T is called an *amalgam* of S and T with a *core* U in $\mathcal{A}$ and denoted by $[S, T; U]$.

An amalgam $[S, T; U]$ of $\mathcal{A}$ is *embeddable* in $\mathcal{A}$ if $\xi_1(S) \cap \xi_2(T) = \xi_1(U)$.

Let $\mathcal{A}$ be the class of semigroups [res. the class of finite semigroups]. A semigroup U in $\mathcal{A}$ is an *amalgamation base for all semigroups* [res. *for finite semigroups*] if any amalgam with a core U in $\mathcal{A}$ is embeddable in $\mathcal{A}$.

Let $\mathcal{T}(X)$ be the full transformation semigroups on the set X with the right to left composition. In the case that X is a finite set, it follows from Corollary C of the paper[4] that $\mathcal{T}(X)$ is an amalgamation base for all semigroups.

Hence we pose the following problem.

Open problem I Is the full transformation semigroups $\mathcal{T}(X)$ on any infinite set X is an amalgamation base for all semigroups?

T.E. Hall[1] showed that every semigroup that is an amalgamation base for all semigroups has the representation extension property. In fact, we say that a subsemigroup U of a semigroup S has *the representation extension property* in S if for any set X and any representation $\rho : U \to \mathcal{T}(X)$, there exists a set Y disjoint from X and a representation $\alpha : S \to \mathcal{T}(X \cup Y)$ such that $\alpha(u)|_X = \rho(u)$ for all $u \in U$.

Also we say that U has the *representation extension property* if U does so in S for any semigroup S containing U as a subsemigroup.

However the following problem is left open.

*This is an absrtact and the paper will appear elsewhere.

Open problem II Does the full transformation semigroups $\mathcal{T}(X)$ on any infinite set X have the repsentation extension property?

K. Shoji[3] showed that every semigroup that is an amalgamation base for finite semigroups has both the representation extension property and the anti-representation extension property[1]

Open problem III If U is an amalgamation base for finite semigorups then is it an amalgamation bas for all semigorups?

T.E.Hall and M.S. Putch showed that if a finite semigroup U is an amalgamation base for finite semigorup, then all $\mathcal{J}$-classes 0f U form a chain.

Open problem IV If a finite semigroup U whose all $\mathcal{J}$-classes form a chain is an amalgamation base for all semigorups then is it an amalgamation bas for finite semigorups?

References

[1] T. E. Hall. *Representation extension and amalgamation for semigroups.* Quart. J. Math. Oxford (2) **29**(1978), 309-334.

[2] T. E. Hall and M.S. Putch, *The potenstial $\mathcal{J}$-relation and amalgamation bases for finite semigroups*, Proc. Amer. Math. Soc. **95**(1985), 361-364.

[3] T. E. Hall and K. Shoji, *Finite bands and amalgamation bases for finite semigroups*, Communications in algebra **30**(2) (2002), 911-933.

[4] K. Shoji, *Absolute flatness of the full transfomation semigroups*, Journal of algebra **118**(1988) 245-254.

[1] Let $\mathcal{T}^{op}(X)$ be the full transformation semigroups on the set X with the left to right composition. A representation of a semigroup to $\mathcal{T}^{op}(X)$ is called *anti-representation.* The anti-representation extension property is defined by substituting "representation" by "anti-representation" in the definition of the representation extension property.