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Strategic location model for oil spills response installations considering oil transportation

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1 Introduction

Although every oil & gas plant should avoid oil spills, it is impossible to have a guarantee that no incident will occur. For instance, in 2010, Deepwater Horizon oil spill occurred in Golf of Mexico, and, in 2011, another occurred in Campos Basin in Brazil. These events show the importance of studies related to the problems caused by oil spills on environment and means to make contingency.

To avoid impacts of an oil spill in offshore oil & gas production, it is necessary to maintain vessels in stand-by with protection equipment near oil production maritime units. Determination of location of these vessels is complex, as it is important to attend any oil spill in less than a critical time, according to regulations. The main objective of this work is similar to the one presented by Costa (2007), there is, implement an optimization model for strategic decision of the location of these facilities. It is a covering problem, where each vessel works like a warehouse and accident points are clients.

This work presents an implementation similar Iakovou, Douligeris, Ip e Korde (1996) model, which is. However, now this work also intends to consider oil transportation in the model. This work assumes that oil would have constant speed and would spread in a radius trajectory.

2 Literature review

Iakovou, Douligeris, Ip e Korde (1996) proposed a mixed linear integer mathematical programing model to define locations of oil response installations that is the base for this work. This model was also on experiments of Costa (2007). In this model, main objective is to determine the candidate locations for warehouse opened, the quantity of equipment stored at them and quantity of equipment that each of this warehouses would be responsible to send to maritime units in case of oil spill.

On model input, we should provide costs for opening a new facility, operational costs to maintain equipment in facility, transportation costs in case of emergency, demands in case of accident, duration of trips between each warehouse location candidate and clients, critical time to reach accident zone and demands in case of accident.

Model considers that probability of having two accidents at same time is very low, so response system capacity considers only one accident per scenario.

Model is as follows:

Sets

I	Candidate locations for vessel (warehouse)
J	Types of equipment

K	Client locations (maritime unit location)

Parameters

α_i	Cost to open warehouse on location i
β_{ij}	Cost to maintain equipment j on location i
γ_j	Cost to transport equipment j per unit of time
τ_{ik}	Travel time between warehouse i and location k
C_{ij}	Capacity of location i for equipment j
D_{jk}	Demands of equipment j on discretization l of client k
A_{ik}	1 if travel time of route between location i and point l of client k circumference takes less than critical time
M	Number sufficiently big.

Variables

x_{ij}	Quantity of equipment j stored at vessel i
y_i	1 if vessel i opens
z_{ijk}	Quantity of equipment j sent from vessel i to point l in client k, that is in position p in route

$$min \sum_i \alpha_i y_i + \sum_i \sum_j \beta_{ij} x_{ij} + \sum_i \sum_j \sum_k \gamma_j \tau_{ik} z_{ijk} \quad \textbf{(1)}$$

s.t.

$$x_{ij} \leq C_{ij} y_i \quad \forall\, i \in I, j \in J \quad \textbf{(2)}$$

$$\sum_i z_{ijk} \geq D_{jk} \quad \forall\, j \in J, k \in K \quad \textbf{(3)}$$

$$z_{ijkl} \leq \mathrm{x_{ij}} \quad \forall\, i \in I, j \in J, k \in K \quad \textbf{(4)}$$

$$z_{ijkl} \leq M\, A_{ikl} \quad \forall\, i \in I, j \in J, k \in K \quad \textbf{(5)}$$

$$x_{ij}, z_{ijk} \in \mathbb{Z}^+ \quad \forall\, i \in I, j \in J, k \in K \quad \textbf{(6)}$$

$$y_i \in \{0,1\} \quad \forall\, i \in I \quad \textbf{(7)}$$

Objective function minimizes opening costs, operational costs to maintain equipment on warehouse and transportation costs for scenarios concerning accidents on each of the risk points. Constraint (2) guarantees that only opened facility can store any product, until its capacity. Constraint (3) ensure attendance of demands of each client. Constraint (4) deals with capacity of warehouse. Constraint (5) makes possible only trips between warehouse and clients that take less than critical time. Rest of constraints deal with definition of variables.

3 Model

On this work, we propose modifications on model proposed by Iakovou, Douligeris, Ip e Korde (1996) to consider oil transportation. Therefore, now we does not need to attend not only original location of clients, but also all border of area affected by oil spill. We adopt two different approaches: in a simpler model, we discretize this border in points and treat them exactly as previous model; and in a more advanced model, we also define order of stops during attendance of points.

3.1 Discrete model considering oil transportation

In this case, oil suffers transportation after spill due to wind and maritime currents. Therefore, now it is necessary to reach not only oil spill origin, but also all border of affected area within critical time. For simplification, model considers that oil spreads is in a radial trajectory, with constant speed w, in all directions. In critical time (T_{crit}), oil would cover an area inside a circle with

$$r = wT_{crit}. \quad \textbf{(8)}$$

Therefore, now constraints of problem have to impose that all vessels could reach all points of this circumference within T_{crit}.

For example, for a point B in circumference, assuming that vessels operate with speed v, time required to reach oil is

$$t(\theta) = \frac{v}{dist}. \quad \textbf{(9)}$$

Where $dist$ is the distance between warehouse and any point in circumference, as in Figure 1. Warehouse is represented by point A, with coordinates (a, b), and client is represented by point C, with coordinates (c, d). We should attend all points of discretized circumference formed by oil spread, for example point B.

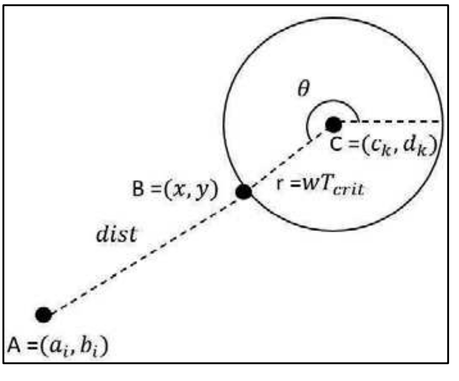

Figure 1. Distance traveled by vessel.

Expressing x and y in function of θ, we have:

$$x = c_k + r\cos(\theta) \tag{10}$$

$$y = d_k + r\sin(\theta) \tag{11}$$

To calculate distance between A and B, we proceed as following:

$$dist = \sqrt{(x - a_i)^2 + (y - b_i)^2} \tag{12}$$

Considering that vessel has a speed v, model only allows routes performed bellow critical time. Therefore, now matrix of allowed routes would have following values:

$$\begin{cases} A_{ikl} = 1, if \ \dfrac{dist}{v} \leq T_{crit} \\ A_{ikl} = 0, if \ \dfrac{dist}{v} > T_{crit} \end{cases} \tag{13}$$

For simplification, model discretizes circumference in a set of points, which vessel would have to attend. For N discretization, each points l would have following demands β_{jkl}:

$$\beta_{jkl} = \frac{\propto_{jk}}{N} \tag{14}$$

For example, if the quantity of discretization is four, clients considered in this model would be as in Figure 2.

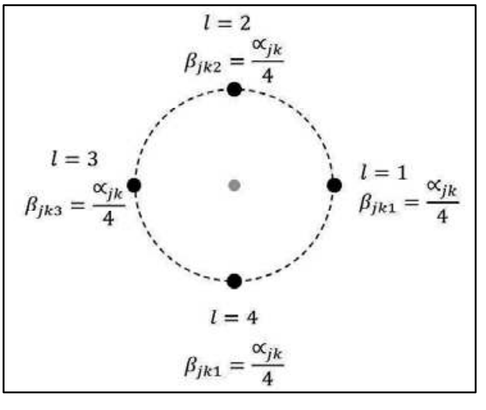

Figure 2. Example of clients for new model, considering circumference divided in 4 sections.

With these new assumptions, model is now as follows:

Sets

I	Candidate locations for vessel (warehouse)
J	Types of equipment
K	Client locations (maritime unit location)
L	Points of discretized circle

Parameters

α_i	Cost to open warehouse on location i
β_{ij}	Cost to maintain equipment j on location i
γ_j	Cost to transport equipment j per unit of time
τ_{ikl}	Travel time between warehouse i and discretization l of location k
C_{ij}	Capacity of location i for equipment j
D_{jk}	Demands of equipment j on discretization l of client k
A_{ikl}	1 if travel time of route between location i and point l of client k circumference takes less than critical time
M	Number sufficiently big. In this case, we consider it as $\max(C_{ij})$

Variables

x_{ij}	Quantity of equipment j stored at vessel i
y_i	1 if vessel i opens
z_{ijkl}	Quantity of equipment j sent from vessel i to point l in client k, that is in position p in route

$$min \sum_i \alpha_i y_i + \sum_i \sum_j \beta_{ij} x_{ij} + \sum_i \sum_j \sum_k \sum_l \gamma_j \tau_{ikl} z_{ijkl} \qquad \textbf{(15)}$$

s.t.

$$x_{ij} \le C_{ij} y_i \qquad \forall\, i \in I, j \in J \qquad \textbf{(16)}$$

$$\sum_i z_{ijkl} \ge D_{jkl} \qquad \forall\, j \in J, k \in K, l \in L \qquad \textbf{(17)}$$

$$\sum_l z_{ijkl} \le \mathrm{x_{ij}} \qquad \forall\, i \in I, j \in J, k \in K \qquad \textbf{(18)}$$

$$\sum_l z_{ijkl} \le M\, A_{ikl} \qquad \forall\, i \in I, j \in J, k \in K \qquad \textbf{(19)}$$

$$x_{ij}, z_{ijk} \in \mathbb{Z}^+ \qquad \forall\, i \in I, j \in J, k \in K \qquad \textbf{(20)}$$

$$y_i \in \{0,1\} \qquad \forall\, i \in I \qquad \textbf{(21)}$$

Objective function minimizes opening costs, operational costs to maintain equipment on warehouse and transportation costs for scenarios concerning accidents on each of the risk points. Constraint (16) guarantees that only opened facility can store any product, until its capacity. Constraint (17) ensure attendance of demands of each point l of each client circumference. Constraint (18) deals with capacity of warehouse, imposing that, for each scenario of oil spill, sum of deliveries from a warehouse to circumference points should be lower than this warehouse capacity. Constraint (19) makes possible only trips between warehouse and points of circumference that take less than critical time. Rest of constraints deal with definition of variables.

3.2 Discrete model considering oil transportation and vessel routing

In previous approach, we only allowed clients reached before critical time in a directly route from vessel original position. It does not take into account that vessel can attend several clients before arriving, resulting in much more time than expected. To deal with it, we also propose a more complex model, which can decide vessel routes and arrival times.

Sets

I	Candidate locations for vessel (warehouse)
J	Types of equipment
K	Client locations (maritime unit location)
L	Points of discretized circle, zero representing routes that come from origin

Parameters

α_i	Cost to open warehouse on location i
β_{ij}	Cost to maintain equipment j on location i
γ_j	Cost to transport equipment j per unit of time
$T_{kl_1l_2}$	Travel time between points in discretization l_1 and l_2 of location k
τ_{ikl}	Travel time between warehouse i and discretization l of location k
C_{ij}	Capacity of location i for equipment j
D_{jk}	Demands of equipment j on discretization l of client k
CT	Critical time

Variables

x_{ij}	Quantity of equipment j stored at vessel i
y_i	1 if vessel i opens
z_{ijkl}	Quantity of equipment j sent from vessel i to point l in client k, that is in position p in route
$a_{ikl_1l_2}$	Vessel i performs trip between points l_1 and l_2 of client k
t_{ikl}	Time when point l of client k is attended by vessel i

$q_{ijkl_1l_2}$	Quantity of equipment j onboard on vessel between points l_1 and l_2 of client k

$$\min \sum_{i} \alpha_i y_i + \sum_{i}\sum_{j} \beta_{ij} x_{ij} + \sum_{i}\sum_{j}\sum_{k}\sum_{l_1}\sum_{l_2} \gamma_j T_{kl_1l_2} q_{ijkl_1l_2} + \sum_{i}\sum_{j}\sum_{k}\sum_{l} \gamma_j \tau_{ikl} q_{ijk0l} \quad (22)$$

s.t.

$$x_{ij} \leq C_{ij} y_i \qquad \forall\, i \in I, j \in J \quad (23)$$

$$\sum_{i \in I} z_{ijkl} \geq D_{jkl} \qquad \forall\, j \in J, k \in K, l \in L \quad (24)$$

$$\sum_{l \in L} z_{ijkl} \leq \mathrm{x_{ij}} \qquad \forall\, i \in I, j \in J, k \in K \quad (25)$$

$$\sum_{l \in L} a_{ik0l} \leq 1 \qquad \forall\, i \in I, k \in K \quad (26)$$

$$\sum_{l_3 \in L} a_{ikl_2l_3} \leq \sum_{l_1 \in L'} a_{ikl_1l_2} \qquad \forall\, i \in I, k \in K, l_2 \in L \quad (27)$$

$$a_{ikll} \leq 0 \qquad \forall\, i \in I, k \in K, l \in L' \quad (28)$$

$$t_{ikl_1} + T_{kl_1l_2} - t_{ikl_2} = \mathrm{M}_1(1 - a_{ikl_1l_2}) \qquad \forall\, i \in I, k \in K, l_1 \in L, l_2 \in L \quad (29)$$

$$t_{ik0} + \tau_{ikl} - t_{ikl_2} = \mathrm{M}_1(1 - a_{ikl_1l_2}) \qquad \forall\, i \in I, k \in K, l_2 \in L \quad (30)$$

$$t_{ikl_2} \leq \mathrm{M}_1 \sum_{l_1 \in L'} (1 - a_{ikl_1l_2}) \qquad \forall\, i \in I, k \in K, l_2 \in L' \quad (31)$$

$$t_{ikl} \leq \mathrm{CT} \qquad \forall\, i \in I, k \in K, l \in L \quad (32)$$

$$z_{ijkl_2} \leq \mathrm{M}_2 \left(\sum_{l_1 \in L`} a_{ikl_1l_2} \right) \qquad \forall\, i \in I, j \in J, k \in K, l_2 \in L \quad (33)$$

$$a_{ikl_1l_2} \leq \sum_{j} z_{ijkl_1} \qquad \forall\, i \in I, k \in K, l_1 \in L', l_2 \in L \quad (34)$$

$$q_{ijkl_1l_2} - \mathrm{x_{ij}} \leq \mathrm{M}_3(1 - a_{ikl_1l_2}) \qquad \forall\, i \in I, j \in J, k \in K, l_1 \in L', l_2 \in L \quad (35)$$

$$x_{ij} \in \mathbb{Z}^+ \qquad \forall\, i \in I, j \in J, k \in K \quad (36)$$

$$z_{ijkl}, t_{ikl}, q_{ijkl_1l_2}, r_{ijkl} \in \mathbb{R}^+ \qquad \forall\, i \in I, k \in K, l_1 \in L, l_2 \in L, l \in L \quad (37)$$

$$y_i, a_{ikl_1l_2}, b_{ikl} \in \{0,1\} \qquad \forall\, i \in I, k \in K, l_1 \in L, l_2 \in L, l \in L \quad (38)$$

Objective function minimizes opening costs, operational costs to maintain equipment on warehouse and transportation costs for scenarios concerning accidents on each of the risk points. Constraint (23) guarantees that only opened facility can store any product, until its capacity. Constraint (24) ensures attendance of demands of each point l of each client circumference. Constraint (25) allows deliveries only in case of warehouse has enough equipment for it and it is opened.

Constraint (26) ensures that every trip have only one beginning. Constraint (27) ensures that trips can occur only in arcs where vessel already arrived at first client. Constraint (28) ensures that trips cannot occur in arcs beginning and ending on same point.

Constraints (29) and (30) calculate arrival time at each client and on first client of route, respectively. Constraint (31) makes time be equal to zero on departing time of vessel and on cases there is no route occurring. Constraint (32) ensures that only points reached before critical time can be attended.

Constraint (33) ensures that deliveries only occur in points attended by a route. Constraint (34) ensures that there is only trips in case of necessity of deliveries. Constraints (35) calculates quantity of equipment onboard on every stop of vessel, which we consider equal to the original quantity of goods in the vessel. Other constraints deal with definition of variables.

4 Experiments

Experimentation used same instance as Costa (2007). For experiments, we implemented model proposed by Iakovou, Douligeris, Ip e Korde (1996), as well as our two model considering oil transportation. For simple model considering oil transportation, experiments considered different choices for quantities of discretization, ranging from 1 to 500. For more complex model, we also did this, but with quantities of discretization in a range from 1 to 10. We considered instance with two sizes, one with only first two clients and another one with all 10 clients.

Implementation was in Python, with Pulp library. Optimal solution was obtained using Gurobi solver for all models and also COIN-OR for Iakovou, Douligeris, Ip e Korde (1996) model. Computer used has following specification: Processor Intel core I7 2.20GHz, 16GB memory RAM.

5 Results

5.1 Model without oil transportation

For instance with 10 clients, model constructed has 534 variables and 1088 constraints. According to optimal solution found by solvers, locations 4 and 5 will open. For this instance, GUROBI obtained solution in 0.42s and COIN-OR in 0.39s. Considering also time demanded to process results and make output in Excel, time required was 1.92s and 1.90s, respectively.

5.2 Model with oil transportation

Time required for solving problem, for different quantities of discretization, is in Figure 3. Model can achieve solution in acceptable computational time for any choices for discretization. Cases with more number of points lead to a better approximation of problem, but it makes problem more difficult to solve and it takes more time to reach optimal result. However, even in this cases, demanded time is not so big.

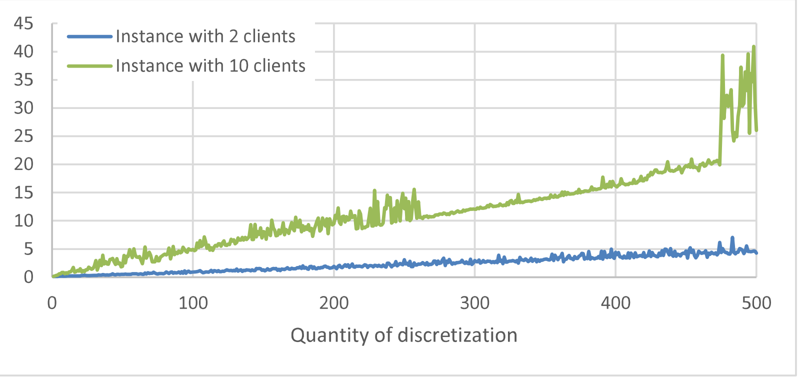

Figure 3. Computational time required for solve problem with oil transportation.

As seen on Figure 4 and Figure 5, quantity of variables and constraints of each problem have a great increase when number of divisions is bigger.

Figure 4. Number of variables for model with oil transportation.

Figure 5. Number of constraints for model with oil transportation.

5.3 Model with oil transportation and routing

Time required for solving problem, for different quantities of discretization, is in Figure 6. As model is much more complex than previously, time required is bigger, then we defined a time limit for solver of 5000s. In case this time is reached, solver stops processing and gives current results, which would be not optimal solution. In Figure 7, we present gap obtained in each of the results. It would be zero in cases we reached in optimal solution, and worse in cases it is near 1. This problem is much more complex than previous, resulting in bigger computational times and solution far from optimality as number of discretization increases.

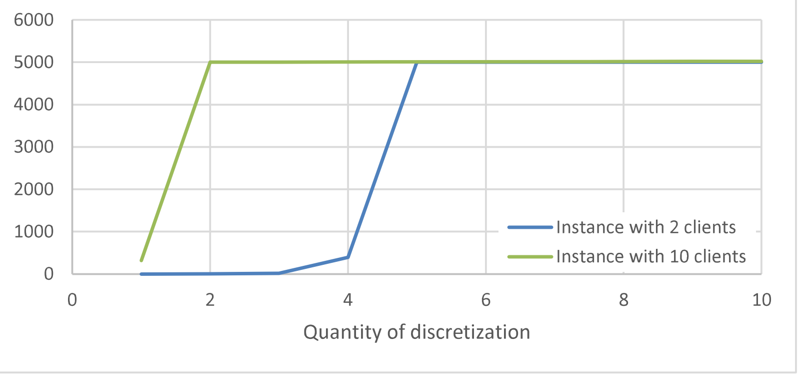

Figure 6. Computational time required for solve problem with oil transportation and routing.

Figure 7. Gap for problem with oil transportation and routing

Increase on time can be explained by dimension of problem, as can be seen on number of variables and constraints of on Figure 8 and Figure 9.

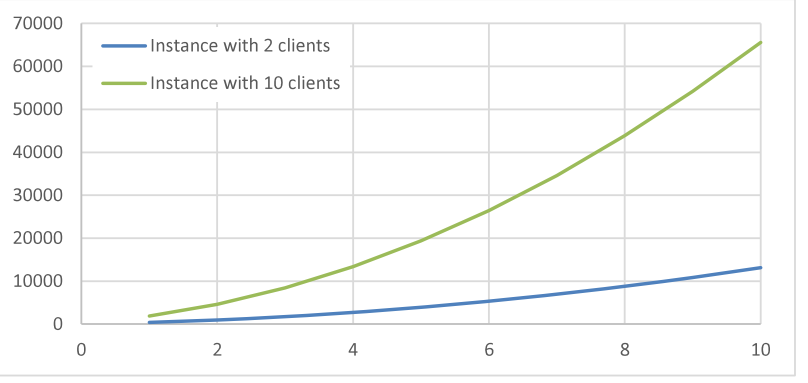

Figure 8. Number of variables for model with oil transportation and routing.

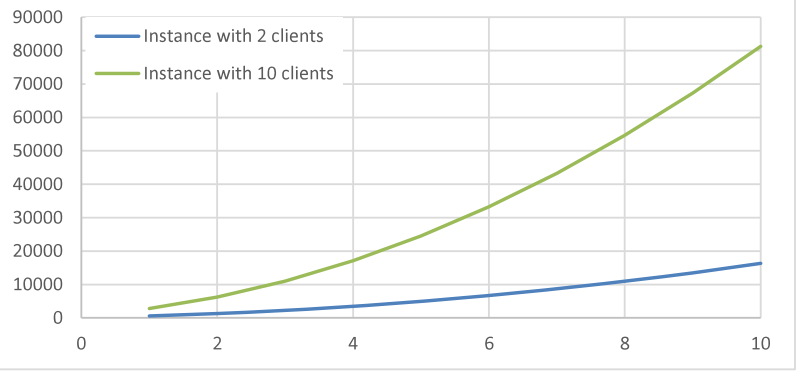

Figure 9. Number of constraints for model with oil transportation and routing.

6 Conclusion

This work presented a model to define location for oil spill response installations considering oil transportation in radial trajectory. We discretize border of oil spill regions in points and consider that all this points should be attended by vessel. With this model we can define quantities of supplies that each of locations would store and how many equipment would be sent for each risk point in case of accident.

In addition, we also presented a more complex model capable to define sequence of stops inside each route. With this, we can obtain time vessel reaches each client considering route travel time, leading to a result more similar to real world.

For simple model, we could obtain results in acceptable computational time. However, for more complex model, model demanded more time to reach optimal solution. Therefore, for future works, it is expected to have enhancement on already implemented models, leading to less computational time.

In addition, we expect to develop a new model considering continuous approach, without discretization.

7 Acknowledgments

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