

RIMS workshop “Nonlinear and Random Waves”

Preface

This publication is the proceedings of the hybrid format workshop entitled “Nonlinear and Random Waves”, which was held during October 3-5, 2022, at the Research Institute for Mathematical Sciences (RIMS), Kyoto University, Japan.

The main scientific subject of the workshop was nonlinear singular stochastic dispersive equations. Studies of nonlinear wave propagation in a random environment, or forced by a stochastic perturbation are of great importance in engineering and physics: nonlinear optics, condensed matter physics, fluid mechanics, turbulence analysis. Since Martin Hairer’s contribution in the field of stochastic PDEs, singular stochastic parabolic equations were very competitive research subjects these last five years, but applications of Hairer’s theory are limited for nonlinear singular dispersive equations due to the lack of smoothing properties, although the wave equations can be accessible somehow by case. On the other hand, Bourgain’s almost-everywhere approach by the use of Gibbs measure was followed by numerous developments, and simultaneously the study of the propagation of randomness under Hamiltonian flows, like wave equations, attracts now many researchers in the world. And both topics are closely related.

The purpose of this workshop was to broaden such arguments for random nonlinear dispersive equations from different and various aspects, for example, the propagation of randomness under nonlinear dispersive equations, asymptotic properties like large deviation principle and scaling limits, qualitative properties of nonlinear dispersive-type equations, scattering and stability of special solutions.

The content of this proceedings consists of short notes by Anne-Sophie de Suzzoni, Aurélien Deya, Hirotsu Nagoji, Itsuko Hashimoto, Laurent Thomann, Mamoru Okamoto, Masaya Maeda, Minami Watanabe and Yukimi Goto. Those notes are related to the topics of their talks in the workshop and cast a spotlight on recent developpements in nonlinear and random wave problems : The note by A-S. de Suzzoni is concerned with the propagation of chaos in wave turbulence modeled by the incompressible Euler equation on the torus with size L , and explains in which sense a solution corresponding to a random initial data with independent Gaussian Fourier coefficients sees its Fourier coefficients at a later fixed time remain independent for sufficiently large L . The note by A. Deya summarizes the well-posedness issues for the linear wave equation driven by a multiplicative space-time fractional noise, proposing two approaches to give sense to the stochastic product: Skorohod approach and pathwise approach. H. Nagoji offers in his note a new result as an application of the I -method for the global existence of solutions for two-dimensional nonlinear wave equations driven by subordinate cylindrical noises, following the local existence result given in his talk. I. Hashimoto explains her result on the asymptotic stability of the stationary solution on the outflow problem described by the compressible Navier-Stokes equation in the exterior domain of a ball. L. Thomann gives an overview of his results on the almost sure global existence and the almost sure scattering of the solution of the one-dimensional nonlinear Schrödinger equation in the whole space with random initial data. The new idea is the use of the absolute continuity between the linear flow measure and the nonlinear flow measure, originated from the quasi-invariance of the initial measure in the Hamiltonian structure, and the monotonicity of the Gibbs type measure. This method seems more applicable to various situations than Bourgain’s original arguments since it does not make use of the ‘invariance’ of the measure. M. Okamoto, in his note, focuses on the phenomenon of phase transition in terms of non normalizability/normalizability for Φ_3^2 -measure and Gibbs measure under Hartree type interactions. M. Maeda’s note, as a brief résumé of the works by Kowalczyk, Martel, Muñoz, Van den Bosch, introduces a virial method to prove the decay estimate of the solution of Schrödinger equations, and develop some ideas on how to remove the eigenvalues which are an obstacle for such decay properties. M. Watanabe considers the classification problem in the nonlinear Schrödinger equation, in case of double power nonlinearities. Due to the double nonlinearity, the estimate of the distance between the stationary solution and the general evolution seems not immediate by the method used in Duycaerts and Merle for the case of pure power nonlinearity, but in place the application of the one pass Lemma by Nakanishi

and Schlag gets over this problem. Y. Goto's note addresses a partial resolution of Solovej's conjecture for the approximation of Born-Oppenheimer potential energy in the case of Kohn-Sham model.

As another objective of the workshop was the promotion of women, the speakers were equally matched in gender. This "parity" structure between men and women allowed equitable discussions, and interactions with a large part of the researchers concerned with the thematic; this would certainly not have been possible in a workshop with female-only audience. We believe that the concern about diversity of speakers should be taken into account in all conferences, as our workshop shows that such a concern does not impact the quality of the meeting. On the other hand, we had in addition an interesting opportunity which was called Cambridge INI-RIMS networking events. The idea was to share by zoom one conference on each side: from our workshop and from the organized semester at Newton Institute. In the present case, we had the opportunity to share a conference with the dispersive hydrodynamics program organized by Gennady El and Mark Hoefer; this opened us up to some different aspects of the analysis of dispersive hydrodynamic systems that was very much appreciated. We are grateful to Koji Ohkitani who proposed us this nice idea.

We would like to thank all the speakers for their cooperation. We also would like to thank the participants for the success of the workshop. The workshop clearly allowed to initiate discussions and collaborations among the participants. Last but not least, we greatly acknowledge the support from RIMS.

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