Harmonic analysis in new 2-microlocal Besov and Triebel-Lizorkin spaces

秋田大学　坂　光一

Koichi Saka

Akita university

数理研　2024. 11.25 -11.27

I. Definition

Let $\mathcal{S} = \mathcal{S}(\mathbb{R}^n)$ be the space of all Schwartz functions on $\mathbb{R}^n$ and $\mathcal{S}'$ its dual.

Let ϕ be a Schwartz function satisfying

(1.1) supp $\hat{\phi} \subset \{\xi \in \mathbb{R}^n : \frac{1}{2} \leq |\xi| \leq 2\}$,

(1.2) $|\hat{\phi}(\xi)| \geq C > 0$ if $\frac{3}{5} \leq |\xi| \leq \frac{5}{3}$.

Let $\phi_j(x) = 2^{nj}\phi(2^j x),\ j \in \mathbb{N}$.

We select a function $\phi_0 \in \mathcal{S}$ satisfying

(1.3) supp $\hat{\phi_0} \subset \{\xi \in \mathbb{R}^n :\ |\xi| \leq 2\}$,

(1.4) $|\hat{\phi_0}(\xi)| \geq C > 0$ if $|\xi| \leq \frac{5}{3}$.

For $f \in \mathcal{S}'$ we define some sequences indexed by dyadic cubes P:

$B^s_{pq}(P)$

$= (\sum_{j \geq (-\log_2 l(P)) \vee 0} ||2^{js}\phi_j * f||^q_{L^p(P)})^{1/q}$,

$0 < p, q \leq \infty$, where $a \vee b = \max\{a, b\}$

$F^s_{pq}(P) =$

$||(\sum_{j \geq (-\log_2 l(P)) \vee 0}(2^{js}|\phi_j * f|)^q)^{1/q}||_{L^p(P)}$,

$0 < p < \infty$, $0 < q \leq \infty$

<u>Defiition 1.1</u>

Let $s,\ s',\ \in \mathbb{R},\ \sigma \geq 0$ and let $x_0 \in \mathbb{R}^n$.

$$A^s(E^{s'}_{pq})^{\sigma}_{x_0} = \{f \in \mathcal{S}' : ||f||_{A^s(E^{s'}_{pq})^{\sigma}_{x_0}} \equiv \sup_P (l(P)+|x_P - x_0|)^{-\sigma} l(P)^{-s} E^{s'}_{pq}(P) < \infty\}$$

where E^s_{pq} to denote either B^s_{pq} or F^s_{pq}.

II. Example.

Examples.

(i) $A^0(E^{s'}_{pq})^0_{x_0} = E^{s'}_{pq}(\mathbb{R}^n)$ is the classical Besov space, or Triebel-Lizorkin space , [Triebel 10,11].

(ii) The Besov type spaces $B^{s,\tau}_{pq}(\mathbb{R}^n)$ and the Triebel-Lizorkin type spaces $F^{s,\tau}_{pq}(\mathbb{R}^n)$ introduced by [D. Yang and W. Yuan 12] and [D. Yang , W. Sickel and W. Yuan 13] , are contained in our definition as special cases that

$\boldsymbol{E^{s',s}_{pq}(\mathbb{R}^n) = A^{ns}(E^{s'}_{pq})^0_{x_0}}$.

(iii)

The Morrey space is $\boldsymbol{\mathcal{M}^u_p = A^{n(\frac{1}{p}-\frac{1}{u})}(F^0_{p2})^0_{x_0}}$ **if** $\boldsymbol{1 < p \leq u < \infty}$.

(iv) The $\boldsymbol{\dot{B}_\sigma}$**-Morrey spaces studied by [Y. Komori-Furuya et al. 3], are contained in our definition as special cases, that is,** $\boldsymbol{\dot{B}_\sigma(L_{p,\lambda}) = A^{\lambda+\frac{n}{p}}(F^0_{p2})^\sigma_0}$, $\boldsymbol{1 < p < \infty}$.

(v) The local Morrey spaces $\boldsymbol{LM_{p,\lambda}}$ **studied by [Ts. Batbold and Y. Sawano, 1] are realied in our cases as**

$$\boldsymbol{LM_{p\lambda} = A^0(F^0_{p2})^{\lambda/p}_0, \quad 1 < p < \infty.}$$

(vi) $\boldsymbol{C^{s,s'}_{x_0}}$ **studied in [Y. Meyer , 4], is in our cases,**

$$C_{x_0}^{s,s'} = A^0(B_{\infty\infty}^{s+s'})_{x_0}^{-s'}.$$

III. Harmonic analysis in new 2-microlocal Besov and Triebel-Lizorkin spaces

1. Characterization[K.Saka, 5,7]
   (a) ϕ-transform in the sense of Fraizer-Jawerth[2]
   (b) wavelet trnsform
   (c) atomic and molecular decomposition
   (d) defference characterization
   (e) oscillation characterization
2. Boundedness [K. Saka, 5,7]
   (a) Calderón-Zygmond operators
   (b) psudodifferential operators
3. Local theory [K. Saka, 6]
4. Predual[K. Saka, 8]
   (a) Zoro type

(b) Choquet integral

5. Pointwise multiplier[K. Saka, 9]

6. Interpolation

References

[1] Ts. Batbold and Y. Sawano, *Decomposition for local Morrey spaces*, Eurasian Math. J., 5 (2014), 9-45.

[2] Frazier M. and Jawerth B. *A discrete transform and decompositions of distribution spaces*. J. Func. Anal. 93(1990), 34-171.

[3] Komori-Furuya Y et al. *Applications of Littlewood-Paley theory for $\dot{B}_\sigma$- Morrey spaces to the*

boundedness of integral operators. J. Funct. spaces Appli. (2013), 1-21.

[4] Meyer Y. *Wavelets, Vibrations and Scalings.*CRM Monograph Series. AMS vol. 9 (1998), 393-401.

[5] Saka, K. *2-microlocal spaces associated with Besov type and Triebel-Lizorkin type spaces*, Rev. Mat. Compl., 35 (2022), 923-962.

[6] Saka, K. *Local theory for 2-microlocal Besov and Triebel-Lizorkin spaces of Jaffard type*, Math. Narch., 296 (1) (2023), 4920-4947.

[7] Saka, K. *New 2-microlocal Besov and Triebel-Lizorkin spaces via the Littlewood-Paley decomposi-*

tion, Eurasian Math. J., 14 (3)(2023), 75-111.

[8] Saka, K. *A predual of 2-microlocal Besov and Triebel-Lizorkin spaces of Jaffard type.* to be prepared

[9] Saka, K. *A characterizations of new 2-microlocal Besov and Triebel-Lizorkin spaces and its application to pointwise multipliers.* to be prepared

[10] Triebel H. *Theory of function spaces*. Birkhaeuser Verlag, 1983.

[11] Triebel H. *Theory of function spaces II*. Birkhaeuser Verlag, 1992.

[12] Yang D. and Yuan W. *New Besov-type spaces and Triebel-Lizorkin-type spaces*. Math. Z. 265(2010), 451-480.

[13] Yang D., Sickel W. and Yuan W. *Morrey and Campanato meet Besov, Lizorkin and Triebel*. Lect. Notes Math. 2005(2010), Springer.