Constructions of a knot whose n shadows are all trees

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1 Introduction

A subspace of $\mathbb{R}^3$ that is ambient isotopic to a piecewise linear spatial graph is said to be **a spatial graph**.

Let $P_1, P_2, \cdots, P_{n-2}$ and P_{n-1} be planes in the xyz-space containing the z-axis such that the angle between P_i and P_{i+1} is $\frac{\pi}{n-1}$ for each i in $\{1, 2, \cdots, n-2\}$.

Let $G \subseteq \mathbb{R}^3$ be a spatial graph. Then G is said to be T**-**n if there exists a spatial graph that is ambient isotopic to G whose orthogonal shadows on the xy-plane, $P_1, P_2, \cdots, P_{n-2}$ and P_{n-1} are all trees.

We note that a trivial knot lying on a plane has at least one shadow that is not a tree. See for example, Figure 1.

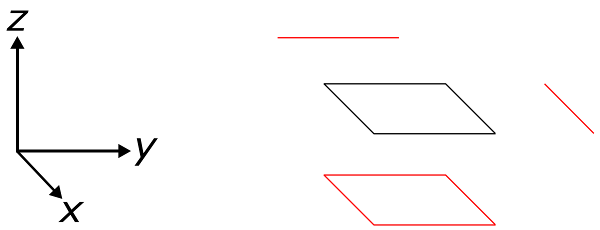

Figure 1 : Orthogonal shadow on the xy-plane is not a tree.

The following example, which is called T-3 trivial A in [4], illustrated in Figure 2 is found by John Rickard and described in [6].

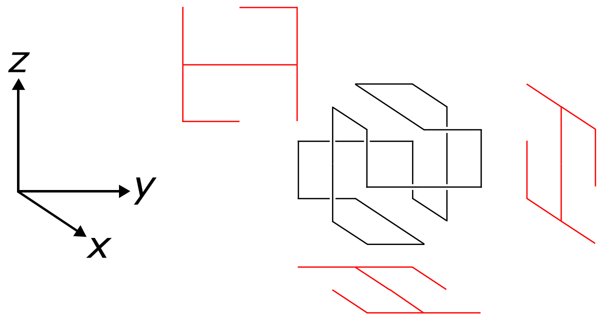

Figure 2 : A simple closed curve whose three shadows are all trees

Namely we have the following theorem.

Theorem 1.1. A trivial knot is T-3.

Then the following theorem is shown.

Theorem 1.2.[4] Every two bridge torus knot or link is T-3.

See also [1], [2] and [3].

As far as the author knows, no other knots or links are shown to be T-3. We note that T-n for $n > 3$ is defined by us and therefore nothing is known about it. The following are our main theorems.

Main theorem 1.1.
Let $G \subseteq \mathbb{R}^3$ be a spatial graph. Then G is T-3 if and only if G is not a spatial graph consisting of only two or more isolated vertices.

Main theorem 1.2.
Let n be a positive integer greater than 3. Then a trivial knot is T-n.

2 Construction of a figure-eight knot whose three shadows are all trees

See Figure 3.

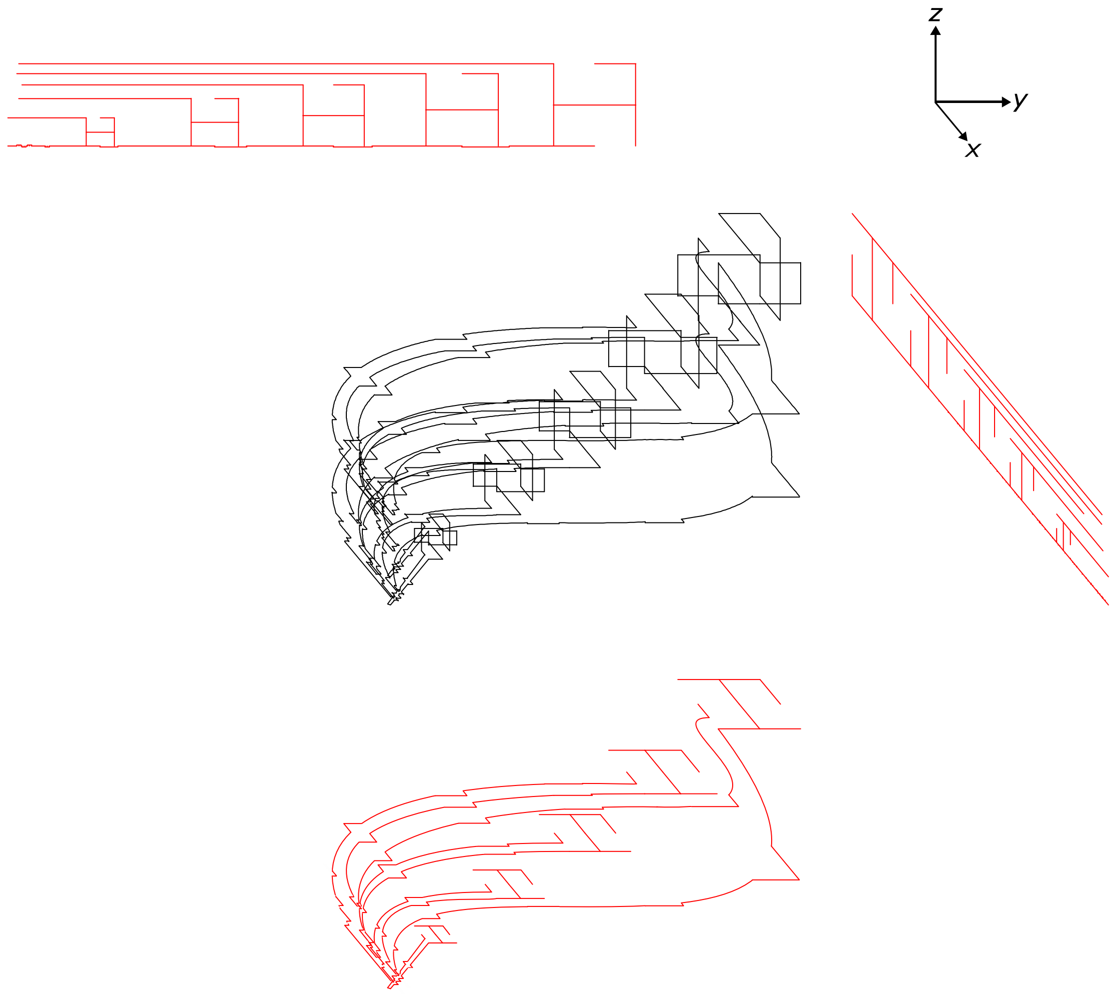

Figure 3 : Figure-eight knot whose three shadows are all trees

3 Construction of a trivial knot whose five shadows are all trees

See Figure 4.

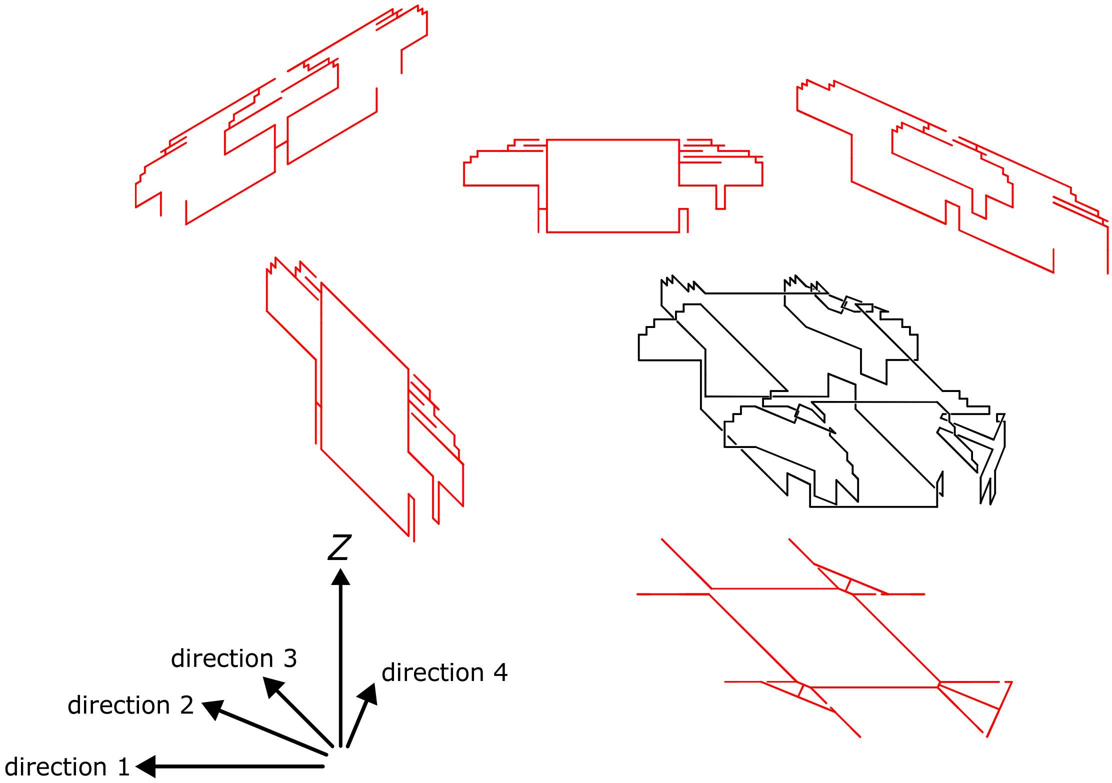

Figure 4 : A trivial knot whose five shadows are all trees

References

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