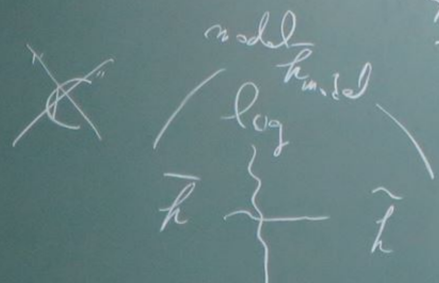
bi-anab. vs mono-anab.

cf. [AbsTopIII] Rem 3.7.3
3.7.4
3.7.5




# §9. Hodge theaters

## §9.0 Some Conventions

**Def 9.1** $\mathcal{C}$ : cat. A, B

(i) – a poly-morphism

– the full p

# §9. Hodge theaters — Arithmetic Upper Half Plane

## §9.0 Some Conventions

Def 9.1 $\mathcal{C}$: cat. $A, B \in \mathrm{Ob}(\mathcal{C})$

(1). – a poly-morphism $A \to B$ $\stackrel{\text{def}}{\Leftrightarrow}$ a subset of $\mathrm{Hom}_{\mathcal{C}}(A, B)$

– the full poly-morphism $A \to B$ $\stackrel{\text{def}}{\Leftrightarrow}$ the poly-morph given by $\mathrm{Hom}_{\mathcal{C}}(A, B)$

– poly-isom

– full poly-isom

(2). a capsul…

$\stackrel{\text{def}}{\Leftrightarrow}$ …

a morp…

Half Plane

$\cdots$ by $\mathrm{Hom}_{\mathcal{C}}(A,B)$

- poly-isom ← a subset of $\mathrm{Isom}_{\mathcal{C}}(A,B)$
- full poly-isom ← gain by $\mathrm{Isom}_{\mathcal{C}}(A,B)$

(2). a capsule of obj's of $\mathcal{C}$

$\overset{\text{def}}{\Longleftrightarrow}$ a finite collection of $\mathrm{Obj}(\mathcal{C})$

$\{A_j\}_{j\in J}$ $\quad J =: \pi_0(\{A_j\}_{j\in J})$

$|J|$-capsule

cat.

$\rightsquigarrow$ Capsule($\mathcal{C}$)

a morph. of capsules $\{A_j\}_{j\in J} \to \{A'_{j'}\}_{j'\in J'}$

$\overset{\text{def}}{\Longleftrightarrow}$ $\begin{cases} \cdot\ \iota: J \hookrightarrow J' \\ \cdot\ A_j \to A'_{\iota(j)} \in \mathrm{Hom}_{\mathcal{C}} \text{ for each } j\in J \end{cases}$

a capsule-full poly-morph

$$\{A_j\}_{j\in J} \longrightarrow \{A'_{j'}\}_{j'\in J'}$$

$\mathrm{Ob}(\widehat{\mathrm{Capsule}}(\mathcal{C}))$

fixed

'$\overset{\mathrm{def}}{\Longleftrightarrow}$' the poly-morph assoc. to some $\iota : J \hookrightarrow J'$

'$\overset{\mathrm{def}}{\Longleftrightarrow}$'

'$\overset{\mathrm{def}}{\Longleftrightarrow}$' capsule-full poly-isom

: capsule-full poly-morph

s.t. $\begin{cases} \iota : \text{bij} \\ A_j \xrightarrow{\sim} A'_{\iota(j)} \end{cases}$

morph isom through

$\mathcal{B}^{temp}(\Pi)$

$\mathcal{C} \longrightarrow \mathcal{C}'$

morph

$\Updownarrow$ def

an isom. clas…

functor…

$\longleftarrow$

$\mathcal{B}^{temp}(\Pi)$

$\mathcal{C} \longrightarrow \mathcal{C}'$

morph

$\Updownarrow$ def

an isom. class of functors

$\longleftarrow$

$\mathcal{C}^0 \longrightarrow (\mathcal{C}')^0$

$\Updownarrow$ def

$(\mathcal{C}^0)^\top \longrightarrow ((\mathcal{C}')^0)^\top$

the cat. of formal countable coproducts

$\perp \rightsquigarrow \top$

anabelioid $\rightsquigarrow$ temperoid

temporification

$(\ )^0 + (\ )^\top$

$\Pi_1 \longrightarrow \Pi_2$ up to inner

$\Updownarrow$

$\mathcal{B}^{tmp}(\Pi_1) \longrightarrow \mathcal{B}^{tmp}(\Pi_2)$

$\Updownarrow$

$\mathcal{B}^{tmp}(\Pi_1)^0 \longrightarrow \mathcal{B}^{tmp}(\Pi_2)^0$

(Sets)

$(\ )\times(\ )\ \amalg$

good

$\underline{X}_{\underline{v}}$

$\downarrow$ deg $= l$

tot. ram at cusp

$\underline{X}_{\underline{v}}$

$\Pi_{\underline{X}_{\underline{v}}} \sim$

$\to$ Q

$\mathrm{Aut}_K(\underline{X}) = \mathbb{Z}/l\mathbb{Z}$

$\leftarrow J'$

fixed

$\iota : J \subset J'$

poly-morph

$(\iota : b_i)$

$A_j \xrightarrow{\sim} A'_{\iota(j)}$

... isom through

... theater — Arithmetic Upper Half Plane

$\perp \rightsquigarrow T$

(anabelioid $\rightsquigarrow$ temperoid)

temporification

$(\ )^{\circ} + (\ )^{T}$

(Sets)

$(\ )^{\times}$ II

good

$\underline{X}_{\underline{v}}$ $\mapsto$ a counterpart of $\underline{X}_{\underline{v}}$ in bad case

$\big\downarrow$ deg = $\ell$, tot. ram at cusps

$\underline{X}_{\underline{v}}$

$\Pi_{\underline{X}_{\underline{v}}} \rightsquigarrow \Pi_{\underline{X}_{\underline{v}}}, \ldots$

$\Pi_C$

$\rightsquigarrow Q$

$\mathrm{Aut}_K(\underline{X}) = \mathbb{Z}/\ell \times \{\pm 1\}$

$\mathrm{Aut}_K(\underline{X}_{\underline{v}}) = M_\ell \times \{\pm 1\}$

$\underline{X}_{\underline{v}}$ tot. ram.

## §9.1 Initial Θ-data

Def 9.2 (initial Θ-data)

$(\overline{F}/F,\ X_F,\ l,\ \underline{C}_K,\ \underline{\mathbb{V}},\ \mathbb{V}_{mod}^{bad},\ \underline{\epsilon})$

initial Θ-data

(def) a). $F$ : NF $\ni \sqrt{-1}$, $\overline{F} > F$ alg closure, $G_F := Gal(\overline{F}/F)$

b). $X_F$ : once punctured ell. curve $/F$ w/ stable red. $\forall_{n \in \mathbb{V}(F)^{non}}$

$E_F$

capsule-full poly-morph

$\mathcal{B}^{temp}(\Pi)$

$e \to e'$

$e_i^\circ \to$

$X_F \twoheadrightarrow C_F := X_F /\!/ \{\pm 1\}$ , $F_{mod} \subset F$ field of moduli of $C_F$

$\mathbb{V}_{mod} := \mathbb{V}(F_{mod})$

$\cup$

$\phi \neq \mathbb{V}_{mod}^{bad} \subset \{ v \mid$ X has bad red. at $v$, res. char of $v > 2 \}$

(not nec. $=$)

$\mathbb{V}_{mod}^{good} := \mathbb{V}_{mod} \setminus \mathbb{V}_{mod}^{bad}$ treated as "good"

$\left( \cup\ \mathbb{V}_{mod}^{arc}, \{ v \mid \text{res. char} = 2 \} \right)$

$\mathbb{V}(F)^{good} := \mathbb{V}_{mod}^{good} \times_{\mathbb{V}_{mod}} \mathbb{V}(F)$

$\mathbb{V}(F)^{bad} := \mathbb{V}_{mod}^{bad} \times_{\mathbb{V}_{mod}} \mathbb{V}(F)$

- $F/F_{mod}$ Galois
- $\forall$ pts of $E_F[2\cdot 3]$ : rational $/F$

$\left( \Rightarrow K/F_{mod} \text{ Galois} \right)$

c) $l \geq 5$ prime $G_F \longrightarrow GL_2(\mathbb{F}_l)$

Image contains $SL_2(\mathbb{F}_l)$

$K := F(E_F[l])$

outer hom ($\leftarrow$ up AS [illegible])

— $l$ is prime

d).

§9. Initial $\Theta$-data

— $\ell$ is prime to $\Theta$ ) $\forall$ elt $\in \mathbb{V}_{\mathrm{mod}}^{\mathrm{bad}}$

| $\mathrm{ord}_{F_v}$ of $q$-parameter at $\mathbb{V}(F)^{\mathrm{bad}}$

$\mathrm{ord}$(unif of $F_v$) $= 1$

$\to K/F_{\mathrm{mod}}$ Galois )

df. $\underline{C}_K$ : hyperb. orbicurve of type $(1, \ell\text{-tors})_{\pm}$ $/K$

w/ $K$-core $C_K := C_F \times_F K$

( (c) $\Rightarrow$ ( $C_F \rightsquigarrow \underline{C}_K$ up to isom$/F$ ) ) ( $(1, \mathbb{Z}/\ell)$-type )

$\underline{X}_K$ up to isom$/K$ of type $(1, \ell\text{-tors})$

$\underline{X}_K$ $\quad$ $\underline{C}_K$

$(1, \ell\text{tors})$ $\quad$ $\boxed{(1, \ell\text{-tors})_{\pm}}$

$\boxed{(1, \mathbb{Z}/\ell)}$ $\quad$ $(1, \ell\text{-tors})_{\pm}$

$\boxed{\ell}$ $\quad$ $(1, \mathbb{Z}/\ell)^{\Theta}$

$\underline{X}_K$

$\underline{X}_K$

$\pm 1$

$\underline{C}_K$

[EtTh, Prop 2.

$\begin{pmatrix} * & * \\ 0 & * \end{pmatrix} / \pm$

$\mathbb{V}_{m\text{-}d}^{\text{bad}}$

...l q-parameters ... at $\mathbb{W}(F)^{\text{bad}}$

...il of $F_v$ ) = 1

... hyperb. orbicurve of type $(1, \ell\text{-tors})_\pm$ /K

... $C_K := C_F \times_F K$ $(1, \mathbb{Z}/\ell)$-type

$\ldots_F \rightsquigarrow \underline{C}_K$ up to isom/K

$\underline{X}_K$   $\underline{C}_K$

$(1, \ell\text{-tors})$   $\boxed{(1, \ell\text{-tors})_\pm}$

$\boxed{(1, \mathbb{Z}/\ell)}$   $(1, \ell\text{-tors})_\pm$

$\boxed{\boxed{\ell}}$   $(1, \mathbb{Z}/\ell)^\Theta$

$\underline{\underline{X}}_\sim$

$\underline{X}_K$ up to isom/K of type $(1, \ell\text{-tors})$

$\mathbb{Q}$

$$\begin{array}{ccc} \underline{X}_K & \xrightarrow{\deg_{\text{geo}} = \ell} & X_F \\ \downarrow \pm 1 & & \downarrow \pm 1 \\ \underline{C}_K & \xrightarrow[\text{non-Gal}]{\deg_{\text{geo}} = \ell} & C_F \end{array}$$

def's

$\Delta_{\underline{C}} \supset \Delta_{\underline{\underline{C}}} \supset \Delta_{\underline{X}}$

[EtTh, Prop 2.2] $\underline{\underline{X}}_F$ $\left(\underline{\underline{C}}_F\right)$ of type $(1, \ell\text{-tors}^\Theta)$

$\Pi$   $\underline{\underline{X}}$   Assump (3) e). $\underline{\mathbb{V}} \subset \mathbb{V}(K)$ subset

s.t. 
$$\begin{tikzcd} \mathbb{V}(K) \arrow[d] & \supset \underline{\mathbb{V}} \arrow[dl, "\wr"] & \ni \underline{x} \arrow[d, mapsto] \\ \mathbb{V}_{\mathrm{mod}} & & \ni x \end{tikzcd}$$

$\underline{\mathbb{V}}^{\mathrm{nm}} := \underline{\mathbb{V}} \cap \mathbb{V}(K)^{\mathrm{nm}}$

anc
'good',
'bad'

$\bar{x} \in \mathbb{V}(\bar{F}) \;/\; \underline{x}$

$\underline{X}_{\bar{x}} \xrightarrow{\ \ } \underline{X}$

$\left( \underline{S}_{\bar{x}} \right) \longrightarrow \underline{S}$

$\bar{\underline{v}} \in \mathbb{V}(\bar{F}) \ / \ \underline{v}$

$$\begin{array}{ccccc}
\underline{\underline{X}}_{\bar{\underline{v}}} & \xrightarrow[\Delta_\Theta \overset{1:2}{\cong} \mathbb{Z}/l(1)]{\text{Gal}} & \underline{X}_{\underline{v}} & \xrightarrow{\circledast \cong \mathbb{Z}/l} & X_{\underline{v}} \\
\downarrow \pm 1 & & \downarrow \pm 1 & & \downarrow \pm 1 \\
\left(\underline{\underline{C}}_{\bar{\underline{v}}}\right) & \longrightarrow & \underline{C}_{\underline{v}} & \longrightarrow & C_{\underline{v}}
\end{array}$$

$\underline{v} \in \mathbb{V}^{bad}$

$\Downarrow$

$\underline{C}_{\underline{v}}$ : of $(1, \mathbb{Z}/l)_{\pm}$ type

$\left( (b) \to \overset{\text{pts of}}{\mathbb{E}[2]} : \text{rat} \to K_{\underline{v}} = \dot{K}_{\underline{v}} \right)$

$\underline{v} \in \mathbb{V}^{bad}$

[EtTh, §2]

$\underline{X}_{\underline{v}}$ ha

T

$\underline{n} \in \mathbb{V}^{\mathrm{bad}}$

$\mathbin{\rotatebox[origin=c]{90}{$\in$}}$

$\underline{C}_{\underline{n}}$ : of $(1, \mathbb{Z}/\ell)_{t}$ type

$\left( b) \Rightarrow \overset{\text{pts}}{E[2]} : \text{rat} \Rightarrow K_{\underline{n}} = \dot{K}_{\underline{n}} \right)$

$\underline{n} \in \mathbb{V}^{\mathrm{bad}}$

[E+Th, §2]

$\underline{X}_{\underline{n}}$ has a model $\underline{X}_{\underline{n}} / K_{\underline{n}}$ of type $(1, (\mathbb{Z}/\ell)^{\Theta})$

$\Pi_{\underline{n}} := \Pi^{\mathrm{temp}}_{\underline{X}_{\underline{n}}}$

[IUTchI, Def 3.1]

each $\log|\mu| = 0$ ?

$\{\mathcal{O}^\times_{``\mathbb{C}"}/\mu_N\}_{N\geq 1}$

$\mathcal{O}^\times_{``\mathbb{C}"} \twoheadrightarrow \mathcal{O}^\times_{``\mathbb{C}"}/\mu_N$

log-mult = N

mono-an := mono-analytic

mono-anab := mono-anabelian

f). $\underline{\epsilon}$ : cusp of $\underline{C}_K$

which comes from

a non-zero cusp (of $Q$)

$\underline{v} \in \underline{\mathbb{V}}^{bad} \Rightarrow \underline{\epsilon}_{\underline{v}}$ corresp. to

$\Pi_X \twoheadrightarrow \hat{\mathbb{Z}}$

$\cup$

$\check{\Pi}^{tp}_X \to \mathbb{Z} \to \pm 1$

can gen.

$\left( X_K := X \times_F K,\ C_K\ \underline{\epsilon} \right) \rightsquigarrow \underline{X}_K,\ \underline{C}_K$

$\underline{v} \in \underline{\mathbb{V}}^{good} \Rightarrow \Pi_{\underline{v}} := \Pi_{\underline{X}, \underline{v}}$

§9.2 Prime-Strips

some model

$\underline{\mathbb{V}}^{bad}$

$\underline{\mathbb{V}}^{good} \cap \underline{\mathbb{V}}$

$\mathcal{O}^\triangleright/\mathcal{O}^\times$

$\underline{\mathbb{V}}^{unc}$

e). $\underline{\mathbb{V}} \subset \mathbb{V}(K)$ subset

s.t. $\mathbb{V}(K) \supset \underline{\mathbb{V}} \ni \underline{v}$

$\mathbb{V}_{mod} \ni v$

$\underline{v} \in \mathbb{V}(F)\ /\ \underline{v}$

$\underline{X}_{\underline{v}} \xrightarrow[\Delta_\Theta]{\text{geom}} \underline{X}_{\underline{v}}$

$\pm 1$

$\underline{C}_{\underline{v}}$

:= mono-analytic
:= mono-anabelian

§9.2 Prime-strips

same model obj's

hol. $\mathfrak{D}_{\underline{v}}$ base

mono-an $\mathfrak{D}^{\vdash}_{\underline{v}}$ base

hol. F $\mathcal{F}_{\underline{v}}$

$\underline{\mathcal{F}}_{\underline{v}}$

$\mathcal{F}^{\vdash}_{\underline{v}}$

$(\sim \mathbb{Q})$

$\underline{\mathbb{V}}^{\mathrm{bad}}$

$\mathcal{B}^{\mathrm{temp}}(\underline{X}_{\underline{v}})^0$

$(\Pi_{\underline{v}})$

$\mathcal{B}(K_{\underline{v}})^0$

$(G_{\underline{v}})$

$\underline{\mathcal{F}}^{\mathrm{bs\text{-}fld}}$

$(\Pi_{\underline{v}} \curvearrowright \mathcal{O}^{\triangleright}_{\overline{F}_{\underline{v}}})$

$\underline{\mathcal{F}}$

$(\Pi_{\underline{v}} \curvearrowright (\mathrm{fds}))$

$\Psi_{\mathcal{F}^{\vdash}_{\underline{v}}} \curvearrowright \mathrm{IND}_{\mathrm{og}}$

$\rightsquigarrow$ Frd

$\mathbb{V}^{\text{good non}} \cap \underline{\mathbb{V}}$

$\mathcal{B}(\underline{X}_{\to \underline{v}})^0$

$(\Pi_{\underline{v}})$

$\mathcal{B}(K_{\underline{v}})^0$

$(G_{\underline{v}})$

on $\mathfrak{D}^{\vdash}_{\underline{v}}$: $\Psi_{\mathcal{F}_{\underline{v}}}$, ind$(\mathfrak{D}^{\vdash}_{\underline{v}})^{rf}$

Spec ⟼ Frd

mll-hol. to $\mathfrak{D}_{\underline{v}}$

$\underline{\mathcal{F}}_{\underline{v}} := \mathcal{F}_{\underline{v}}$

$\Psi_{\mathcal{F}^{\vdash}_{\underline{v}}}$: Spec ⟼ ind

$\rightsquigarrow$ Frd $/\mathfrak{D}^{\vdash}_{\underline{v}}$

ind(α)

$\mathcal{O}^{\triangleright}/\mathcal{O}^{\times}$

$\mathcal{O}^{\times}$ $\mathcal{O}^{\mathrm{bad}}$ splitting

$(\mathcal{O}^{\triangleright}_{\overline{A}_{\underline{X}}}, \tau^{\vdash}_{\underline{v}})$

"$\mathcal{O}^{\triangleright}_{\overline{F}}$"

Frd

base = mono morph. cat.

$[0, \lambda]$

$K_{\underline{v}}$

$\underline{\mathcal{F}}_{\underline{v}} := (\mathcal{F}_{\underline{v}}, \mathfrak{D}_{\underline{v}}, K_{\underline{v}})$

Kummer str.

$K_{\underline{v}}: \mathcal{O}^{\triangleright}(\mathcal{F}_{\underline{v}}) \hookrightarrow \mathcal{A}_{\mathfrak{D}_{\underline{v}}}$

"external $\mathcal{O}_{\overline{F}}$" "internal $\mathbb{C}$"

$\mathcal{F}^{\vdash}_{\underline{v}} := \mathcal{F}_{\underline{v}}$

$\mathcal{O}^{\triangleright} = $

$\underline{\mathbb{V}}^{\mathrm{arc}}$

$\underline{X}_{\to}$ Aut-hol

can gen.

$\Pi_{\underline{v}} := \Pi_{\underline{X}_{\underline{v}}}$

hol. base $\mathcal{D}_{\underline{v}}$

$\mathcal{B}^{\mathrm{temp}}(\underline{X}_{\underline{v}})^0$

$(\Pi_{\underline{v}})$

$\mathcal{B}(\underline{X}_{\to\underline{v}})^0$

$(\Pi_{\underline{v}})$

$\underline{X} \to$ Aut-hol

mono-an base $\mathcal{D}^{\vdash}_{\underline{v}}$

$\mathcal{B}(K_{\underline{v}})^0$

$(G_{\underline{v}})$

$\mathcal{B}(K_{\underline{v}})^0$

$(G_{\underline{v}})$

$\mathcal{O}^\times$ $\mathcal{O}^{\mathrm{rad}}$ splitting

$(\mathcal{O}^{\triangleright}_{A_{\underline{X}}}, \tau^{\vdash}_{\underline{v}})$

"$\mathcal{O}^{\triangleright}_{\mathbb{C}}$"

hol. F $\mathcal{C}_{\underline{v}}$

$\underline{\mathcal{F}}^{\mathrm{bs\text{-}Fld}}$

$(\Pi_{\underline{v}} \curvearrowright \mathcal{O}^{\triangleright}_{\overline{F}_{\underline{v}}})$

on $\mathcal{D}^{\vdash}_{\underline{v}}$ $\Phi_{\mathcal{C}_{\underline{v}}}$

Spec $\mapsto$ ind$(\mathcal{O}^\times)^{\mathrm{pf}}$

~ Frd

pull-back to $\mathcal{D}_{\underline{v}}$

Frd

base = one morph. cat.

$(0, 1)$

$K_{\underline{v}}$

$\underline{\mathcal{F}}_{\underline{v}}$

$\underline{\mathcal{F}}$

$(\Pi_{\underline{v}} \curvearrowright (\text{fcts}))$

$\underline{\mathcal{F}}_{\underline{v}} := \mathcal{C}_{\underline{v}}$

$\underline{\mathcal{F}}_{\underline{v}} := (\mathcal{C}_{\underline{v}}, \mathcal{D}_{\underline{v}}, \kappa_{\underline{v}})$

Kummer str. ét

$\kappa_{\underline{v}} : \mathcal{O}^{\triangleright}(\mathcal{C}_{\underline{v}}) \hookrightarrow \mathbb{A}_{\mathcal{D}_{\underline{v}}}$

Frd. "external $\mathcal{O}^{\triangleright}_{\mathbb{C}}$" internal "$\mathbb{C}$"

$\mathcal{C}^{\vdash}_{\underline{v}}$

$\Phi_{\mathcal{C}^{\vdash}_{\underline{v}}} \sim \mathrm{INDiag}(\ldots)$ $\mathcal{D}^{\vdash}_{\underline{v}}$

$\sim$ Frd

$\Phi_{\mathcal{C}^{\vdash}_{\underline{v}}}$ : Spec

$\mapsto$ ind$(\mathbb{Z}^{\triangleright}_{p_{\underline{v}}})$

$\sim$ Frd/$\mathcal{D}^{\vdash}_{\underline{v}}$

$\mathcal{C}^{\vdash}_{\underline{v}} := \mathcal{C}_{\underline{v}}$

$\mathcal{O}^{\triangleright} = \mathcal{O}^\times \times \mathbb{N}$

mono-an F $\mathcal{F}^{\vdash}_{\underline{v}}$

$\mathcal{F}^{\vdash}_{\underline{v}} := (\mathcal{C}^{\vdash}_{\underline{v}}, \tau^{\vdash}_{\underline{v}}) \leftarrow (G_{\underline{v}} \curvearrowright \mathcal{O}^{\times}_{\overline{F}_{\underline{v}}} \times q^{\mathbb{N}}_{\underline{v}})$

splitting

$\mathcal{F}^{\vdash}_{\underline{v}} := (\mathcal{C}^{\vdash}_{\underline{v}}, \tau^{\vdash}_{\underline{v}})$

splitting

$\mathcal{O}^{\triangleright}_{F_{\underline{v}}} = \mathcal{O}^{\times}_{\underline{v}} \times p^{\mathbb{N}}_{\underline{v}}$

$\mathcal{F}^{\vdash}_{\underline{v}} := (\mathcal{C}^{\vdash}_{\underline{v}}, \mathcal{D}^{\vdash}_{\underline{v}}, \tau^{\vdash}_{\underline{v}})$

splitting

mono-an $\mathcal{D}^{\vdash}_{\underline{v}}$ base

$\mathcal{B}(K_{\underline{v}})^0$

$(G_{\underline{v}})$

$\mathcal{B}(K_{\underline{v}})^0$

$(G_{\underline{v}})$

$\mathcal{O}^{\times}$ $\mathcal{O}^{\triangleright}$ rad splitting

$(\mathcal{O}^{\triangleright}_{A_{\infty}}, \tau^{\vdash}_{\underline{v}})$

"$\mathcal{O}^{\triangleright}_{F}$"

hol. F $\mathcal{C}_{\underline{v}}$

$\underline{\mathcal{F}}^{\text{bs-fld}}$

$(\Pi_{\underline{v}} \curvearrowright \mathcal{O}^{\triangleright}_{\overline{F}_{\underline{v}}})$

on $\mathcal{D}_{\underline{v}}$ $\Phi_{\mathcal{C}_{\underline{v}}}$

Spec $L \mapsto \text{ind}(\mathcal{O}^{\triangleright}_{L})^{rf}$

$\rightsquigarrow$ Fr'd

well-def. to $\mathcal{D}_{\underline{v}}$

Fr'd

base = mono morph. cat.

$[0, \lambda]$

$K_{\underline{v}}$

$\underline{\mathcal{F}}_{\underline{v}}$

$\underline{\mathcal{F}}$

$(\Pi_{\underline{v}} \curvearrowright (\text{flds}))$

$\underline{\mathcal{F}}_{\underline{v}} := \mathcal{C}_{\underline{v}}$

$\underline{\mathcal{F}}_{\underline{v}} := (\mathcal{C}_{\underline{v}}, \mathcal{D}_{\underline{v}}, \kappa_{\underline{v}})$

Kummer str. $\mathcal{C}^{\dagger}$

$\kappa_{\underline{v}} : \mathcal{O}^{\triangleright}(\mathcal{C}_{\underline{v}}) \hookrightarrow \mathcal{A}_{\mathcal{D}_{\underline{v}}}$

Frob.- "external $\mathcal{O}^{\triangleright}_{F}$" internal "C"

$\mathcal{C}^{\vdash}_{\underline{v}}$

$\Phi_{\mathcal{C}^{\vdash}_{\underline{v}}} \sim \mathbb{N}\log(q_{\underline{v}})$

$\rightsquigarrow$ Fr'd

$\mathcal{D}^{\vdash}_{\underline{v}}$

$\Phi_{\mathcal{C}^{\vdash}_{\underline{v}}}$ : Spec $L$

$\mapsto \text{ind}(\mathcal{O}^{\triangleright}_{p_{\underline{v}}})$

$\rightsquigarrow$ Fr'd $/\mathcal{D}^{\vdash}_{\underline{v}}$

$\mathcal{C}^{\vdash}_{\underline{v}} := \mathcal{C}_{\underline{v}}$

$\mathcal{O}^{\triangleright} = \mathcal{O}^{\times} \times \mathbb{N}$

$\mathcal{F}^{\vdash}_{\underline{v}}$ mono-an F

$\mathcal{F}^{\vdash}_{\underline{v}} := (\mathcal{C}^{\vdash}_{\underline{v}}, \tau^{\vdash}_{\underline{v}}) \longleftarrow (G_{\underline{v}} \curvearrowright \mathcal{O}^{\times}_{\overline{F}_{\underline{v}}} \times q^{\mathbb{N}}_{\underline{v}})$

splitting up to mult

$\mathcal{F}^{\vdash}_{\underline{v}} := (\mathcal{C}^{\vdash}_{\underline{v}}, \tau^{\vdash}_{\underline{v}})$

splitting

$\mathcal{O}^{\triangleright}_{F_{\underline{v}}} = \mathcal{O}^{\times}_{F_{\underline{v}}} \times p^{\mathbb{N}}_{\underline{v}}$

$\mathcal{F}^{\vdash}_{\underline{v}} := (\mathcal{C}^{\vdash}_{\underline{v}}, \mathcal{D}^{\vdash}_{\underline{v}}, \tau^{\vdash}_{\underline{v}})$

splitting

split Fr'd

‖

Fr'd + splitting

(L1)

later $\leftarrow (\mathcal{D}_{\underline{v}})^{\ddagger}_{\underline{\underline{\Upsilon}}_{\underline{v}}} \supset \mathcal{D}$

$$\mathcal{F}^{\vdash}_{\underline{v}} \xrightarrow{\sim} \mathcal{F}^{\Theta}_{\underline{v}} \qquad \times \ddot{\underline{\Upsilon}}$$

bad

$\mathcal{O}^{\times} \times \underline{\mathbb{I}}^{\mathbb{N}}$ $\quad \mathcal{F}^{\vdash}_{\underline{v}} = (\mathcal{C}^{\vdash}_{\underline{v}}, \tau^{\vdash}_{\underline{v}}) \quad \mathcal{F}^{\Theta}_{\underline{v}} = (\mathcal{C}^{\Theta}_{\underline{v}}, \tau^{\Theta}_{\underline{v}})$ $\quad \times \ddot{\underline{\Upsilon}}$ $\quad \sim$ $\quad \mathcal{O}^{\times} \times \underline{\Theta}^{\mathbb{N}}$

good non

$$\mathcal{F}^{\vdash}_{\underline{v}} \xrightarrow{\sim} \mathcal{F}^{\Theta}_{\underline{v}}$$

$$\mathcal{F}^{\vdash}_{\underline{v}} = (\mathcal{C}^{\vdash}_{\underline{v}}, \tau^{\vdash}_{\underline{v}}) \qquad \mathcal{F}^{\Theta}_{\underline{v}} = (\mathcal{C}^{\Theta}_{\underline{v}}, \tau^{\Theta}_{\underline{v}})$$

arc

$$\mathcal{F}^{\vdash}_{\underline{v}} \xrightarrow{\sim} \mathcal{F}^{\Theta}_{\underline{v}} = (\mathcal{C}^{\Theta}_{\underline{v}}, \mathcal{D}^{\Theta}_{\underline{v}}, \tau^{\Theta}_{\underline{v}})$$

$$\mathcal{F}^{\vdash}_{\underline{v}} = (\mathcal{C}^{\vdash}_{\underline{v}}, \mathcal{D}^{\vdash}_{\underline{v}}, \tau^{\vdash}_{\underline{v}})$$

full rel

$\ddot{\underline{\underline{\Upsilon}}}_{\underline{v}} \; (\curvearrowright) \; \mathcal{D}^{\vdash}_{\underline{v}} \xrightarrow{\sim}$

$A \longrightarrow A$

[IUTch I, Def 3.1]

$\Sigma$ : ...

full rel

$(\mathcal{D}_{\underline{v}})_{\ddot{\underline{\underline{Y}}}_{\underline{v}}} \supset \mathcal{D}^{\Theta}_{\underline{v}}$ ← later

$\times \ddot{\underline{Y}}$

$\uparrow$

$(\text{obj. of } \mathcal{D}^{\vdash}_{\underline{v}}) \times \ddot{\underline{\underline{Y}}}_{\underline{v}}$

triv. line bdl

$\uparrow$ $\in \underline{\Xi}_{\underline{v}}$

$\mathbb{T}_{A^{\Theta}}$ : Frob. triv. obj. assoc. to $A^{\Theta}$

monoid

$\mathcal{O}^{\triangle}_{\mathcal{C}^{\Theta}_{\underline{v}}(-)} / \mathcal{D}^{\Theta}_{\underline{v}}$

Frd

$\mathcal{O}^{\times} \times \underline{\underline{\Theta}}^{\mathbb{N}}$

$\ddot{\underline{\underline{Y}}}_{\underline{v}} (\,,)\ \mathcal{D}^{\vdash}_{\underline{v}} \xrightarrow{\sim} \mathcal{D}^{\Theta}_{\underline{v}}$

$A \longmapsto A \times \ddot{\underline{\underline{Y}}}_{\underline{v}} =: A^{\Theta} \longmapsto \mathcal{O}^{\times}(\mathbb{T}_{A^{\Theta}}) \underline{\underline{\ddot{\Theta}}}_{\underline{v}}^{\mathbb{N}} \Big|_{\mathbb{T}_{A^{\Theta}}} \subset \mathcal{O}^{\times}(\mathbb{T}^{\text{birat}}_{A^{\Theta}})$

$(\,2 \geq 0 \cdots 2\,)$

$\underline{\underline{\Theta}} := \left( (\ddot{\Theta})^{1/l} \right)^{-1}$

compat w/

$\mathcal{C}_{\underline{v}}$

$(\mathcal{C}^{\Theta}_{\underline{v}}, \mathcal{D}^{\Theta}_{\underline{v}}, \mathcal{I}^{\Theta}_{\underline{v}})$

cf. [IUTchI Ex. 3.2, 3.3, 3.4]

bad — good num — arc.

monoid

$\mathcal{O}^{\triangleright}_{\mathcal{C}^{\Theta}_{\underline{v}}}(-) \,/\, \mathcal{D}^{\Theta}_{\underline{v}}$

Frd $\mathcal{C}^{\Theta}_{\underline{v}}$

as split Frds

$\mathcal{F}^{\vdash}_{\underline{v}} \xrightarrow{\sim} \mathcal{F}^{\Theta}_{\underline{v}}$

good omit

... ldl

$\in \Pi_{\underline{v}}$

trivially$_{\Theta}$

... to $A^{\Theta}$

$(\underline{\underline{\Theta}})^{\mathbb{N}}_{\underline{v}}|_{\Pi_{A^{\Theta}}} \subset \mathcal{O}^{\times}(\Pi^{\text{birat}}_{A^{\Theta}})$

$(\mathbb{Z}_{\geq 0} \curvearrowright \mathbb{Z})$

$\mathcal{C}^{\vdash}_{\underline{v}} \xrightarrow{\sim} \mathcal{C}^{\Theta}_{\underline{v}}$

$\underline{\underline{q}}_{\underline{v}}|_{\Pi_{A}} \mapsto \underline{\underline{\Theta}}_{\underline{v}}|_{\Pi_{A^{\Theta}}}$

compat w/ splittings

$\ldots|^{-1}$

recon cat. th'c

$$\begin{tikzcd}
\mathcal{D}_{\underline{v}} \arrow[r, mapsto] & \mathcal{D}^{\vdash}_{\underline{v}} \\
\mathcal{C}_{\underline{v}} \arrow[u, hook, "\text{(non-arch)}"'] \arrow[dr, "\text{recon } \Theta \rightsquigarrow \tau^{\vdash}_{\underline{v}} \text{ 3-lt-splitting}"] & \mathcal{C}^{\vdash}_{\underline{v}} \arrow[u, hook] \\
\mathcal{F}_{\underline{v}} \arrow[u, hook] & \mathcal{F}^{\vdash}_{\underline{v}} \arrow[u, hook]
\end{tikzcd}$$

Kummer + cycl. rig.

$\Theta(\sqrt{-q_{\underline{v}}}) = q_{\underline{v}}$

gl real'd Fr'd [IUTch I, Ex. ...

$\mathcal{F}_{\mathrm{mod}} \rightsquigarrow \mathcal{C}^{\Vdash}_{\mathrm{mod}}$ ← base cat. = one ...

$\Phi_{\mathcal{C}^{\Vdash}_{\mathrm{mod}}} = \bigoplus_{v \in \mathbb{V}_{\mathrm{mod}}} \mathrm{ord}(q^{\triangleright}_{F_{\mathrm{mod}}} ...$

$\mathcal{C}_{\rho_{\underline{v}}} : \mathcal{C}^{\Vdash}_{\mathrm{mod}}$ —

"gl. no. loc."

$\rho_{\underline{v}} : \Phi_{\mathcal{C}^{\Vdash}_{\mathrm{mod}}}$ ...

n part of gl $F_{\mathrm{mod}}$

$\log^{\Vdash}_{\mathrm{mod}}$ ...

gl real'd Fr'd [IUTchI, Ex. 3.5]

$F_{mod} \rightsquigarrow \mathcal{C}^{\Vdash}_{mod}$ ← base cat. = one morph. cat. "$\mathbb{R}_{\geq 0}$"

$\Phi_{\mathcal{C}^{\Vdash}_{mod}} = \bigoplus_{v \in \mathbb{V}_{mod}} \mathrm{ord}((\mathcal{O}^{\triangleright}_{F_{mod}})_v) \otimes \mathbb{R}_{\geq 0}$ ⤳ log-vol.

$\mathcal{C}_{\rho_{\underline{v}}} : \mathcal{C}^{\Vdash}_{mod} \longrightarrow (\mathcal{C}^{\vdash}_{\underline{v}})^{rlf}$

"gl. non-loc."

$\rho_{\underline{v}} : \Phi_{\mathcal{C}^{\Vdash}_{mod}, \underline{v}} \longrightarrow \Phi^{rlf}_{\mathcal{C}^{\vdash}_{\underline{v}}}$

$\log^{\Vdash}_{mod}(f_v) \longmapsto \frac{1}{[K_{\underline{v}} : (F_{mod})_v]} \log_{\Phi}(f_{\underline{v}})$

v part of gl $F_{mod}$

loc.

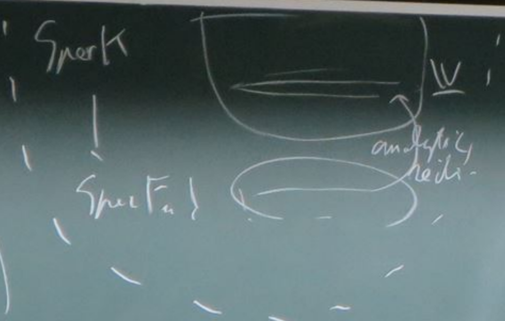
$K_{\underline{v}}$

$\underline{\mathbb{V}} \subset \mathbb{V}(K)$

$\rho^{\log}_{\underline{v}}$

Spec K

Spec $F_{mod}$




log-vol.)

$\underline{\mathbb{V}} \subset \underline{\mathbb{V}}(K)$

$\rho^{\log}_{\underline v}$

$(\ )_{\underline v \in \underline{\mathbb{V}}}$

$(\underline{\underline q}_{\underline v})$

formal symbol

$$\Phi_{\mathcal{C}^{\Vdash}_{\mathrm{tht}}} := \Phi_{\mathcal{C}^{\Vdash}_{\mathrm{mod}}} \log(\Theta)$$

$$\mathcal{C}_{\mathcal{F}^{\Theta}_{\underline v}} : \mathcal{C}^{\Vdash}_{\mathrm{tht}} \xrightarrow{\sim} \left(\mathcal{F}^{\Theta}_{\underline v}\right)^{\mathrm{rlf}}$$

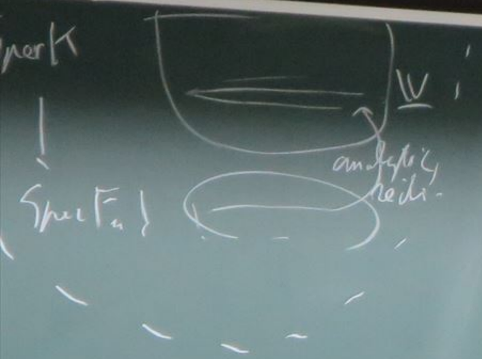
$$\rho^{\Theta}_{\underline v} : \Phi_{\mathcal{C}^{\Vdash}_{\mathrm{tht},\underline v}} \xrightarrow{\sim} \Phi^{\mathrm{rlf}}_{\mathcal{F}^{\Theta}_{\underline v}}$$

formal ones

$$\text{def} \quad \log_{\Phi_{\mathrm{mod}}}(\underline{\underline q}_{\underline v}) \log(\Theta) \longmapsto \frac{\log_{\Phi}(\underline{\underline q}_{\underline v})}{[K_{\underline v} : (F_{\mathrm{mod}})_v]} \left(\frac{\log(\Theta)}{\log_{\Phi}(\underline{\underline q}_{\underline v})}\right)$$

$$\text{good} \quad : \quad \longmapsto \frac{1}{[K_{\underline v} : (F_{\mathrm{mod}})_v]} \log_{\Phi}(\underline{\underline q}_{\underline v}) \underline{\log(\Theta)}$$

formal symbol




formal symbol

$$\underline{\underline{\Phi}}_{\mathcal{C}^{\Vdash}_{\mathrm{tht}}} := \underline{\underline{\Phi}}_{\mathcal{C}^{\Vdash}_{\mathrm{mod}}} \log(\Theta)$$

$$\mathcal{C}_{\mathcal{C}^{\Theta}_{\underline{v}}} : \mathcal{C}^{\Vdash}_{\mathrm{tht}} \xrightarrow{\sim} \left(\mathcal{C}^{\Theta}_{\underline{v}}\right)^{\mathrm{rlf}}$$

$$\mathcal{C}^{\Theta}_{\underline{v}} : \underline{\underline{\Phi}}_{\mathcal{C}^{\Vdash}_{\mathrm{tht},\underline{v}}} \xrightarrow{\wedge} \underline{\underline{\Phi}}^{\mathrm{rlf}}_{\mathcal{C}^{\Theta}_{\underline{v}}}$$

formal ones

$$\text{deg} \quad \log^{\vdash}_{\mathrm{mod}}|p_{\underline{v}}| \log(\Theta) \longmapsto \frac{\log_{\Phi}|p_{\underline{v}}|}{[K_{\underline{v}} : (\mathbb{F}_{\mathrm{mod}})_v]} \left( \frac{\log(\Theta)}{\log_{\Phi}|q_{\underline{v}}|} \right)$$

good

$$: \longmapsto \frac{1}{[K_{\underline{v}} : (\mathbb{F}_{\mathrm{mod}})_v]} \log_{\Phi}|p_{\underline{v}}| \underline{\log(\Theta)}$$

formal symbol

arch $p_{\infty} :=$ "e" $= 2.718281828459045 \ldots$

$$\mathfrak{F}^{\Vdash}_{\mathrm{mod}} := \Big( \underset{\text{gl.}}{\mathcal{C}^{\Vdash}_{\mathrm{mod}}},\ \mathrm{Prime}(\mathcal{C}^{\Vdash}_{\mathrm{mod}}) \xrightarrow{\sim} \underline{\mathbb{V}},\ \underset{\text{loc}}{\{\mathcal{F}^{\vdash}_{\underline{v}}\}_{\underline{v}\in\underline{\mathbb{V}}}},\ \overset{\text{gl. to loc.}}{\{\rho_{\underline{v}}\}_{\underline{v}\in\underline{\mathbb{V}}}} \Big)$$

$$\mathfrak{F}^{\Vdash}_{\mathrm{tht}} := \Big( \mathcal{C}^{\Vdash}_{\mathrm{tht}},\ \mathrm{Prime}(\mathcal{C}^{\Vdash}_{\mathrm{tht}}) \xrightarrow{\sim} \underline{\mathbb{V}},\ \{\mathcal{F}^{\Theta}_{\underline{v}}\}_{\underline{v}\in\underline{\mathbb{V}}},\ \{\rho^{\Theta}_{\underline{v}}\}_{\underline{v}\in\underline{\mathbb{V}}} \Big)$$

$$\left( \begin{array}{lll} \text{gl.} & \log^{\vdash}_{\mathrm{mod}}(p_v) \mapsto & \log^{\Theta}_{\mathrm{mod}}(p_v)\ \log(\Theta) \\ \text{loc.} & \dfrac{1}{(\ \ )}\log_{\Phi}(p_{\underline{v}}) \mapsto & \dfrac{\log_{\Phi}(p_{\underline{v}})}{(\ \ )} \qquad \dfrac{\log \underline{\underline{\Theta}}_{\underline{v}}}{\log_{\Phi}(\underline{\underline{q}}_{\underline{v}})} \\ & \log(\underline{\underline{q}}_{\underline{v}}) \mapsto \log(\underline{\underline{\Theta}}_{\underline{v}}) & \end{array} \right)$$

"$\Theta$-version of $\mathcal{F}$" (~ "for log-shells")

$\mathfrak{D}^{\Vdash}_{\mathrm{mod}}$ : a copy of $\mathcal{C}^{\Vdash}_{\mathrm{mod}}$

$\overline{\Phi}_{\mathfrak{D}^{\Vdash}_{\mathrm{mod}}}$ : $\mathrm{Prime}(\mathfrak{D}^{\Vdash}_{\mathrm{mod}}) \xrightarrow{\sim} \mathbb{V}_{\mathrm{mod}}$

$\cup$

$\underline{\Phi}_{\mathfrak{D}^{\Vdash}_{\mathrm{mod}}, v} \ni \log^{\mathfrak{D}}_{\mathrm{mod}}(p_v)$

$\mathcal{C}^{\Vdash}_{\mathrm{mod}} \xrightarrow{\sim} \mathfrak{D}^{\Vdash}_{\mathrm{mod}}$

$\underline{v} \in \underline{\mathbb{V}}^{\mathrm{non}} \rightsquigarrow (\mathbb{R}^{\Vdash}_{\geq 0})_{\underline{v}} \leftarrow \mathbb{R}_{\mathrm{non}}(G_v)$

ram. index

Frob. deg.

$\cup$

$\log^{\mathfrak{D}}_{\underline{\Phi}}(p_{\underline{v}}) = e_{\underline{v}} \cdot \mathbb{F}(G_{\underline{v}})$

$\underline{v}$ : arc.

non $\mathcal{O}^{\times}$ non-rig

$\mathbb{Z}$ : rig

arc $\mathcal{O}^{\times}$ : rig

$\mathbb{R}_{\geq 0}$ : non-rig

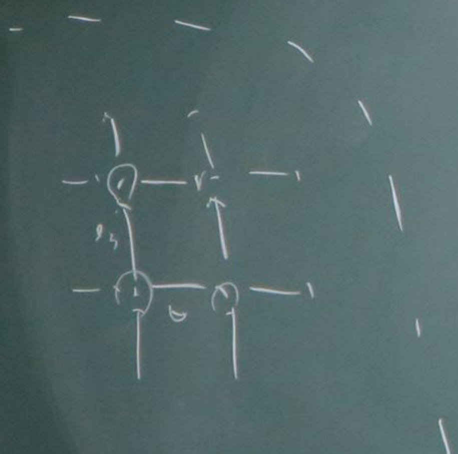
"currency exchange"

initial

$\underline{v} \sim \Phi$

$$\rho^{\mathcal{D}}_{\underline{v}} : \Phi_{\mathcal{D}^{\vdash}_{\mathrm{mod},\underline{v}}} \xrightarrow{\sim} (\mathbb{R}^{\vdash}_{\geq 0})_{\underline{v}}$$

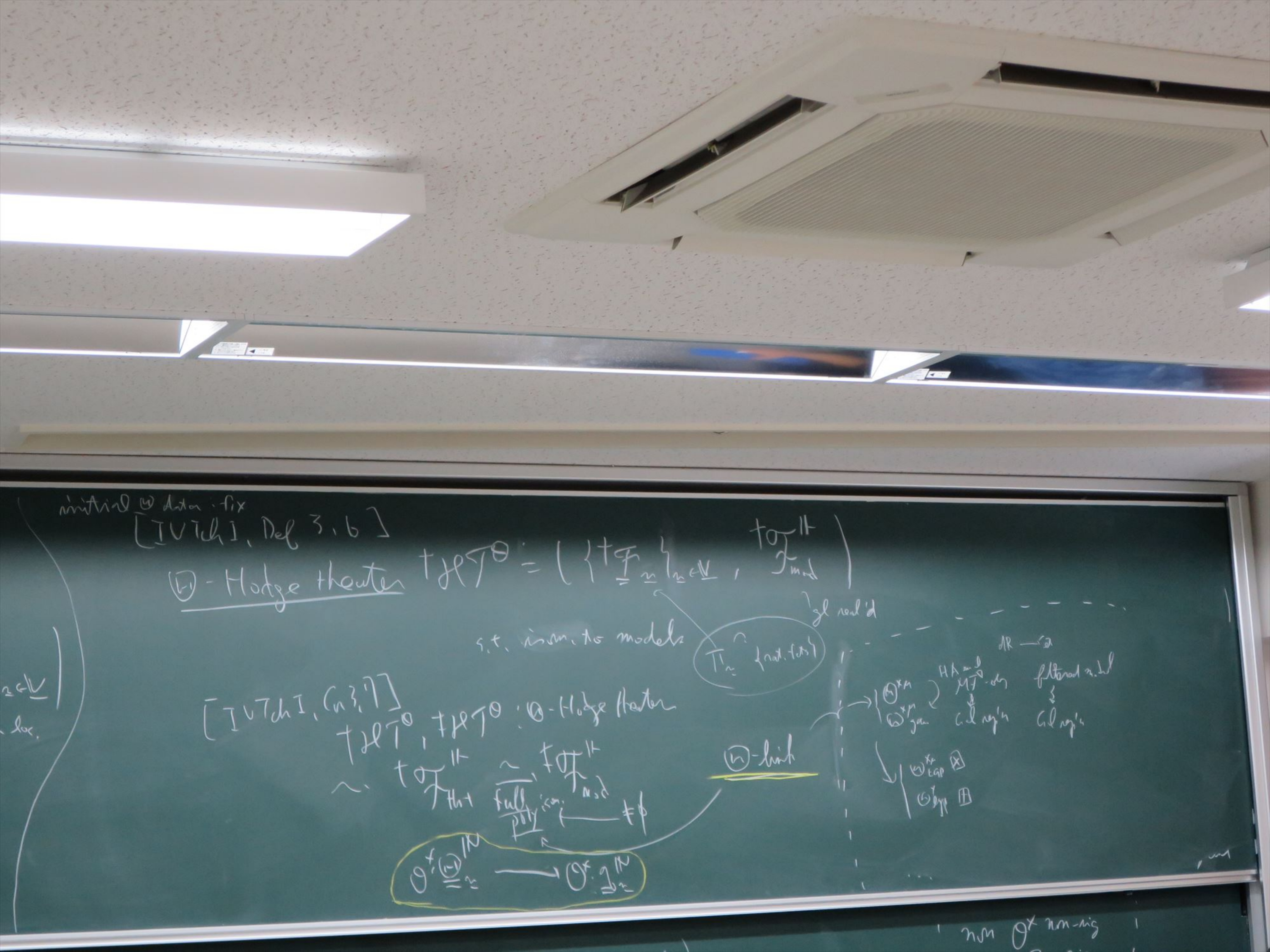
$$\log^{\mathcal{D}}_{\mathrm{mod}}(p_{\underline{v}}) \longmapsto \frac{1}{[K_{\underline{v}}:(F_{\mathrm{mod}})_{v}]} \log^{\mathcal{D}}_{\Phi}(p_{\underline{v}})$$



$$\mathcal{F}^{\Vdash}_{\mathcal{D}} := \Big(\underset{\text{gl.}}{\mathcal{D}^{\vdash}_{\mathrm{mod}}},\ \mathrm{Prime}(\mathcal{D}^{\vdash}_{\mathrm{mod}}) \xrightarrow{\sim} \underline{\mathbb{V}},\ \underset{\text{loc.}}{\{\mathcal{D}^{\vdash}_{\underline{v}}\}_{\underline{v}\in\underline{\mathbb{V}}}},\ \underset{\text{gl. to loc.}}{\{\rho^{\mathcal{D}}_{\underline{v}}\}_{\underline{v}\in\underline{\mathbb{V}}}}\Big)$$

gl. to loc.

$\ldots \xrightarrow{\sim} \underline{\mathbb{V}},\ \{\mathcal{F}^{\vdash}_{\underline{v}}\}_{\underline{v}\in\underline{\mathbb{V}}},\ \{\rho_{\underline{v}}\}_{\underline{v}\in\underline{\mathbb{V}}}$


initial Θ data: fix
[IUTchI, Def 3.6]
Θ-Hodge theater †HT^Θ = ({†F_v}_{v∈V}, †F^⊩_mod)
gl real'd
s.t. isom. to models
Π_v ⊂ {nat. fcts}
[IUTchI, Cor 3.7]
†HT^Θ, ‡HT^Θ : Θ-Hodge theater
~ †F^⊩_tht ≅ ‡F^⊩_mod
full poly isom. ≠ ∅
Θ-link
O^× · Θ^N → O^× · q^N
filtered
Gal reg'n
nm O^× nm-rig

$\left(\begin{array}{c}\text{preservation} \\ \text{of } \mathcal{O}^{\vdash}\end{array}\right)$ $\quad {}^{\dagger}\mathcal{D}^{\vdash}_{\underline{v}} \xrightarrow[\text{as before}]{\sim} {}^{\dagger}\mathcal{D}^{\circleddash}_{\underline{v}} \xrightarrow[\text{induced by } \Theta\text{-link}]{\sim} {}^{\ddagger}\mathcal{D}^{\vdash}_{\underline{v}}$

$\left( \text{ind. } \mathcal{O}^{\times} \right)$ $\quad \mathcal{O}^{\times}_{{}^{\dagger}\mathcal{F}^{\vdash}_{\underline{v}}} \xrightarrow{\sim} \mathcal{O}^{\times}_{{}^{\dagger}\mathcal{F}^{\Theta}_{\underline{v}}} \xrightarrow{\sim} \mathcal{O}^{\times}_{{}^{\ddagger}\mathcal{F}^{\vdash}_{\underline{v}}}$

Frob. picture $\rightarrow$ ${}^{n}\mathcal{HT}^{\mathcal{D}\text{-}\Theta} \xrightarrow[\curvearrowleft \cancel{\ } \curvearrowright]{\Theta} {}^{n+1}\mathcal{HT} \xrightarrow{\Theta} {}^{n+2}\mathcal{HT} \longrightarrow \cdots$

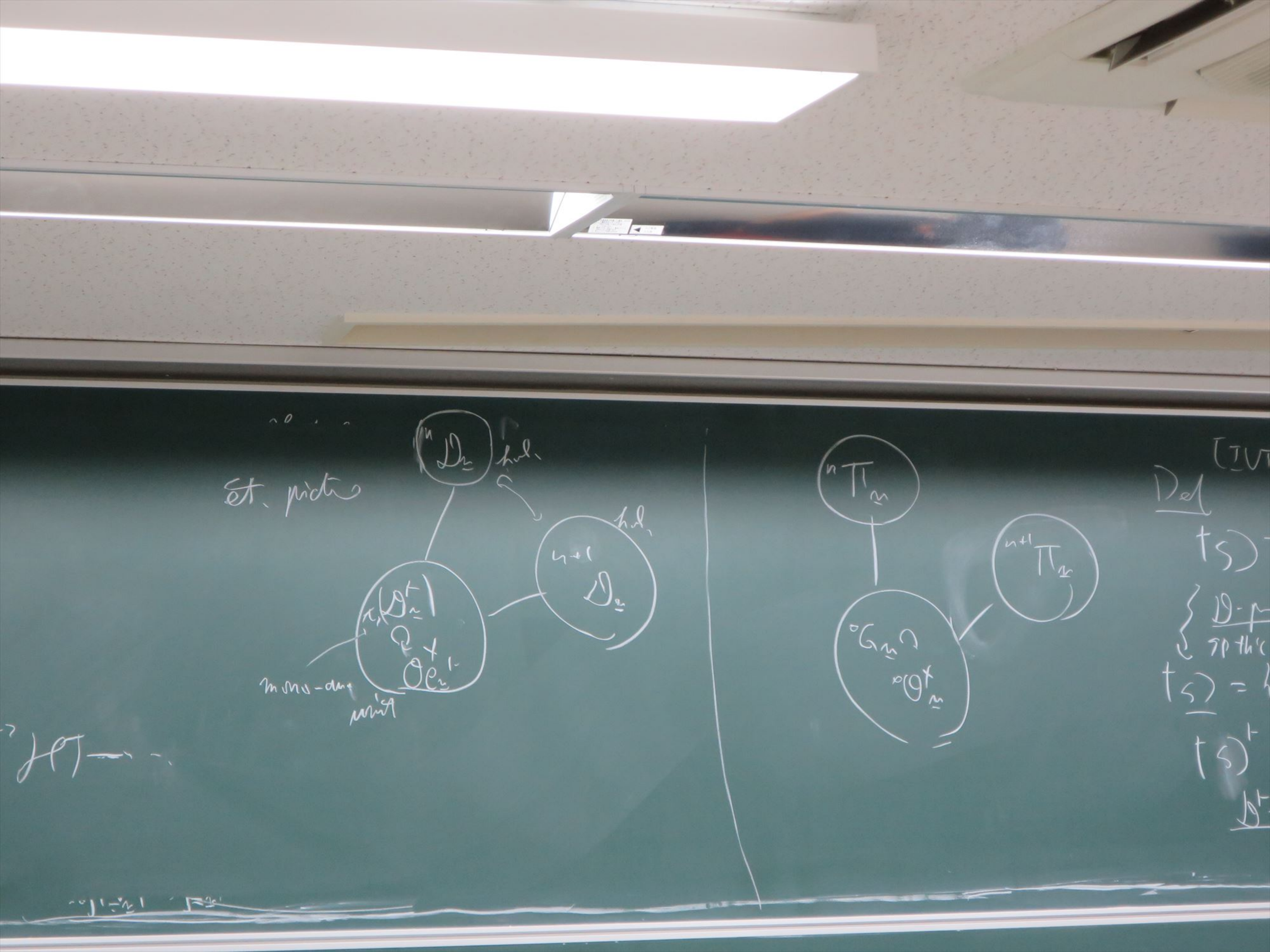
ét. picture

${}^{\dagger}\mathcal{D}^{\vdash}_{\underline{v}}$

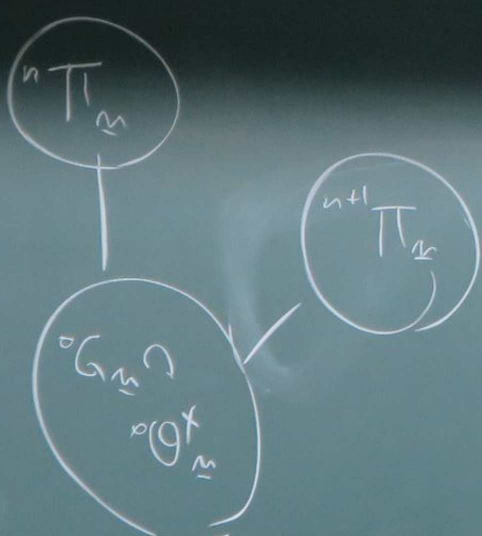
$\curvearrowright \times$

$\mathcal{O}^{\times}_{\mathcal{F}}$

mono-anal. core



hol.
hol.
Def




[IUTch I, Def 4.1]

<u>Def</u>

$\dagger\mathfrak{D} = \{\dagger\mathcal{D}_{\underline{v}}\}_{\underline{v} \in \underline{\mathbb{V}}}$

$\mathcal{D}$-prime-strip s.t. isom. to models

sp th'c

$\dagger\mathfrak{D} = \{\dagger\mathcal{D}_{\underline{v}}\} \leftarrow$ "$\underline{C}_{\underline{v}}$"

$\dagger\mathfrak{D}^{\vdash} = \{\dagger\mathcal{D}^{\vdash}_{\underline{v}}\}_{\underline{v} \in \underline{\mathbb{V}}}$

$\mathcal{D}^{\vdash}$-prime-strip s.t. isom. to models

$\dagger\mathfrak{D} \xrightarrow{\text{sp th'c}} \dagger\mathfrak{D}^{\vdash}$ mono-an'ation

$\left\{ {}^{\dagger}\mathcal{D}_{\underline{v}} \rightarrow {}^{\dagger}\mathcal{D}^{\circledcirc} \right\}_{\underline{v} \in \underline{\mathbb{V}}} \quad \ldots \, \mathfrak{F}) \rightarrow {}^{\dagger}\mathcal{D}^{\ldots}$

(arc, mult)

$\{ {}^{e}\mathfrak{F} \}_{e \in E}$ : a capsule of $\mathcal{D}$-prime-strip

$\{ {}^{e}\mathfrak{F} \}_{e\in E} \rightsquigarrow \{ {}^{e}\mathfrak{F} \}^{|\mathrm{p.b}|}_{e \in E}, {}^{\dagger}\mathcal{D}^{\circledcirc}$

$\{ {}^{e}\mathfrak{F}^{|\mathrm{p.b}|}, {}^{\dagger}\mathcal{D}^{\circledcirc} \}_{e\in E} \rightsquigarrow \{ {}^{e}\mathfrak{F} \}^{|\mathrm{p.b}|}_{e\in E}, {}^{\dagger}\mathcal{D}^{\circledcirc}$

$\{ {}^{e}\mathfrak{F}^{|\mathrm{p.b}|}, {}^{\dagger}\mathfrak{F} \}_{e\in E} \rightsquigarrow \{ {}^{e}\mathfrak{F} \}^{|\mathrm{p.b}|}_{e\in E}, {}^{\dagger}\mathfrak{F}$

[IUTch I, Def 4.1]

Def

${}^{\dagger}\mathfrak{F} = \{ {}^{\dagger}\mathcal{D}_{\underline{v}} \}_{\underline{v} \in \underline{\mathbb{V}}}$

$\mathcal{D}$-prime-strip $\xleftarrow{\xi,\ \dagger,\ \text{isom. to models}}$ (hol. base-prime-strip)

sp th'c

(mono-an. base prime-strip)

${}^{\dagger}\mathfrak{F} = \{ {}^{\dagger}\mathcal{D}_{\underline{v}} \} \leftarrow$ "$\underline{C}_{\underline{v}}$"

(base prime-strip $\underline{\mathbb{V}}$ preserved = collection of isom's)

${}^{\dagger}\mathfrak{F}^{\vdash} = \{ {}^{\dagger}\mathcal{D}^{\vdash}_{\underline{v}} \}_{\underline{v} \in \underline{\mathbb{V}}}$

$\mathcal{D}^{\vdash}$-prime-strip $\{\xi, \dagger,$ isom. to models$\}$

${}^{\dagger}\mathfrak{F} \xrightarrow[\text{sp th'c}]{} {}^{\dagger}\mathfrak{F}^{\vdash}$ mono-an'ation

${}^{n+1}\Pi_{\underline{v}}$

${}^{\infty}\theta^{\times}_{\underline{v}}$

${}^{\dagger}\mathcal{D}_{\underline{n}}$ $\quad \underline{n} \in \underline{\underline{\mathbb{V}}}^{\mathrm{non}}$ $\quad$ the set of cusps

$\widetilde{\ }$ $\quad$ of ${}^{\dagger}\mathcal{D}_{\underline{n}}$

$\mathfrak{p}$ tric

(arc. const)

$\underline{X}_{\underline{n}}$ $\longrightarrow$ $\underline{X}$

$\mathrm{Aut}_{k}(\underline{X}_{\underline{n}})$

$\parallel$

$\mathbb{F}_{\ell}^{\times}/\{\pm 1\}$

<u>label class of cusps</u> of ${}^{\dagger}\mathcal{D}_{\underline{n}}$

$:=$ cusps of ${}^{\dagger}\mathcal{D}_{\underline{n}}$ lying / a non-zero cusp of ${}^{\dagger}\underline{\mathcal{D}}_{\underline{n}}$

$\underline{X}$ $\quad \subseteq$

LabCusp

$\mathbb{F}_{\ell}^{*}$

set of cusps

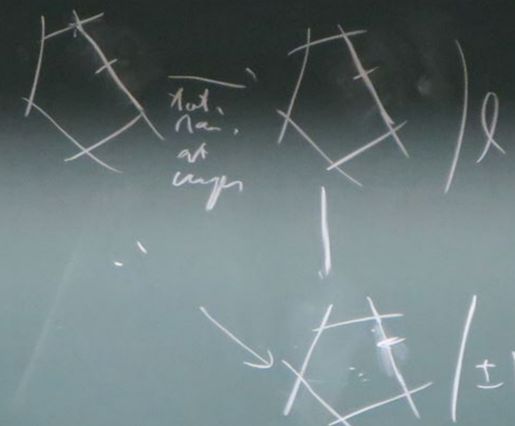
of $^{\dagger}\mathcal{D}_{\succeq}$

$X_{\succeq}$

$\mathrm{Aut}_k(X_{\succeq})$ $\simeq$ $\mu_{\ell}^{\times}\{\pm 1\}$

$X$

$\subseteq$



of $^{\dagger}\mathcal{D}_{\succeq}$

of $^{\dagger}\mathcal{D}_{\succeq}$ lying / a non-zero cusp of $^{\dagger}\mathcal{D}_{\succeq}$

$X$ $\subseteq$

$\mathrm{LabCusp}(^{\dagger}\mathcal{D}_{\succeq}) := \{\text{cusp label classes}\}$

$\curvearrowleft \mathbb{F}_{\ell}^{*}$-torsor $+$ $^{\dagger}\eta_{\succeq}^{\text{corresp.}}$

classes}

$\mathfrak{D}^{\circledcirc} := \mathcal{B}(\underline{C}_K)^0 \underset{\text{sp th'c}}{\xrightarrow{\sim}} \overline{F}, \mathbb{V}(F)$

$\dagger\mathfrak{D}^{\circledcirc}$ } isom.

$\dagger\zeta) = \{\dagger\mathfrak{D}_{\underline{v}}\}$ D-ne-ntg

$\{\dagger\mathfrak{D}_{\underline{v}} \xrightarrow{\text{(poly)}} \dagger\mathfrak{D}^{\circledcirc}\}_{\underline{v}\in\underline{\mathbb{V}}} \rightsquigarrow \dagger\zeta) \xrightarrow{\text{(poly)}} \dagger\mathfrak{D}^{\circledcirc}$

(arc, omit)

$\{{}^e\zeta)\}_{e\in E}$ ; a capsule of D-ne-ntg

$\{{}^e\zeta) \xrightarrow{\text{(poly)}} \dagger\mathfrak{D}^{\circledcirc}\}_{e\in E} \rightsquigarrow \{{}^e\zeta)\}_{e\in E} \xrightarrow{\text{(poly)}} \dagger\mathfrak{D}^{\circledcirc}$

$\{{}^e\zeta) \xrightarrow{\text{(poly)}} \dagger\zeta)\}_{e\in E} \rightsquigarrow \{{}^e\zeta)\}_{e\in E} \xrightarrow{\text{(poly)}} \dagger\zeta)$

...

[IUTch I, Prop 4.2]

$\underline{v}, \underline{w} \in \underline{\mathbb{V}}$

$$\mathrm{LabCusp}({}^\dagger\mathcal{D}_{\underline{v}}) \xrightarrow[\sim]{\exists!} \mathrm{LabCusp}({}^\dagger\mathcal{D}_{\underline{w}})$$

as $\mathbb{F}_l^{*}$-torsors

$\cong$ identify $\left\{ \quad {}^\dagger\eta_{\underline{v}} \longmapsto {}^\dagger\eta_{\underline{w}} \right.$

onto $\mathrm{LabCusp}({}^\dagger\mathcal{S})) \xrightarrow{\sim} \mathbb{F}_l^{*}$

$\curvearrowright \mathbb{F}_l^{*}$-torsor $({}^\dagger\eta) \longrightarrow$

can. elt

[IUTch I,

Rem

$\underline{v} \in \underline{\mathbb{V}}$

If

$\mathrm{LabCusp}(^{\dagger}\mathcal{D}_{\underline{v}})$

-torsors

$\mathbb{F}_l^{*}$

[IUTch I, Rem 4.2.1]

Rem

$\underline{v} \in \underline{\mathbb{V}}^{\mathrm{good}}$

If we use $\underline{C}_K$ instead of $\underline{X}_{\underline{v}}$

$\leadsto \nexists$ bij

$\mathrm{Gal}(K/F) \hookrightarrow GL_2(\mathbb{F}_l)$

$\mathrm{Gal}, \exists \underline{v}$ decp $\to \left\{ \begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix} \right\}$

$\hookrightarrow \mathrm{Aut}(^{\dagger}\mathcal{D}_{\underline{v}}) \curvearrowright_{\text{transitive}} \mathrm{LabCusp}(^{\dagger}\mathcal{D}_{\underline{v}})$

$\mathbb{F}_l^{\rtimes\pm}$

$\mathbb{F}_{\mathrm{mod}}$

[IUTch I,

model

Aut

[IUTchI, Ex 4.3]

model    have $\mathcal{D}$-NF-bridge

inner acts as $\{\pm 1\}$     cusp fix autom's

$$\mathrm{Aut}(\mathcal{D}^{\circledcirc}) \cong \mathrm{Out}(\Pi_{C_K}) \cong \mathrm{Aut}(C_K) \supset \mathrm{Aut}_{\epsilon}(C_K)$$

$$\mathrm{Out}(\Pi_{C_K}) \longrightarrow \mathrm{Aut}(\Delta_X^{ab} \otimes \mathbb{F}_l)/\pm 1 \cong GL_2(\mathbb{F}_l)/\pm 1$$

$$\| \qquad\qquad\qquad\qquad\qquad\qquad \cup \quad \text{Q: isotropic}$$

$$\mathrm{Aut}(C_K) \longrightarrow I_m = \left\{ \begin{pmatrix} * & * \\ 0 & * \end{pmatrix} \right\}$$

$$\mathbb{F}_l^* \left( \;\cup \qquad\qquad\qquad\qquad\qquad \cup \right.$$

$$\mathrm{Aut}_{\epsilon}(C_K) \longrightarrow J_h = \left\{ \begin{pmatrix} * & * \\ 0 & \pm 1 \end{pmatrix} \right\} \quad \epsilon\text{: fixed up to } \pm 1$$

$\mathbb{F}_{\mathrm{mod}}$, $\mathbb{F}_{\mathrm{mod}}$, NF, $\mathbb{F}_l^*$

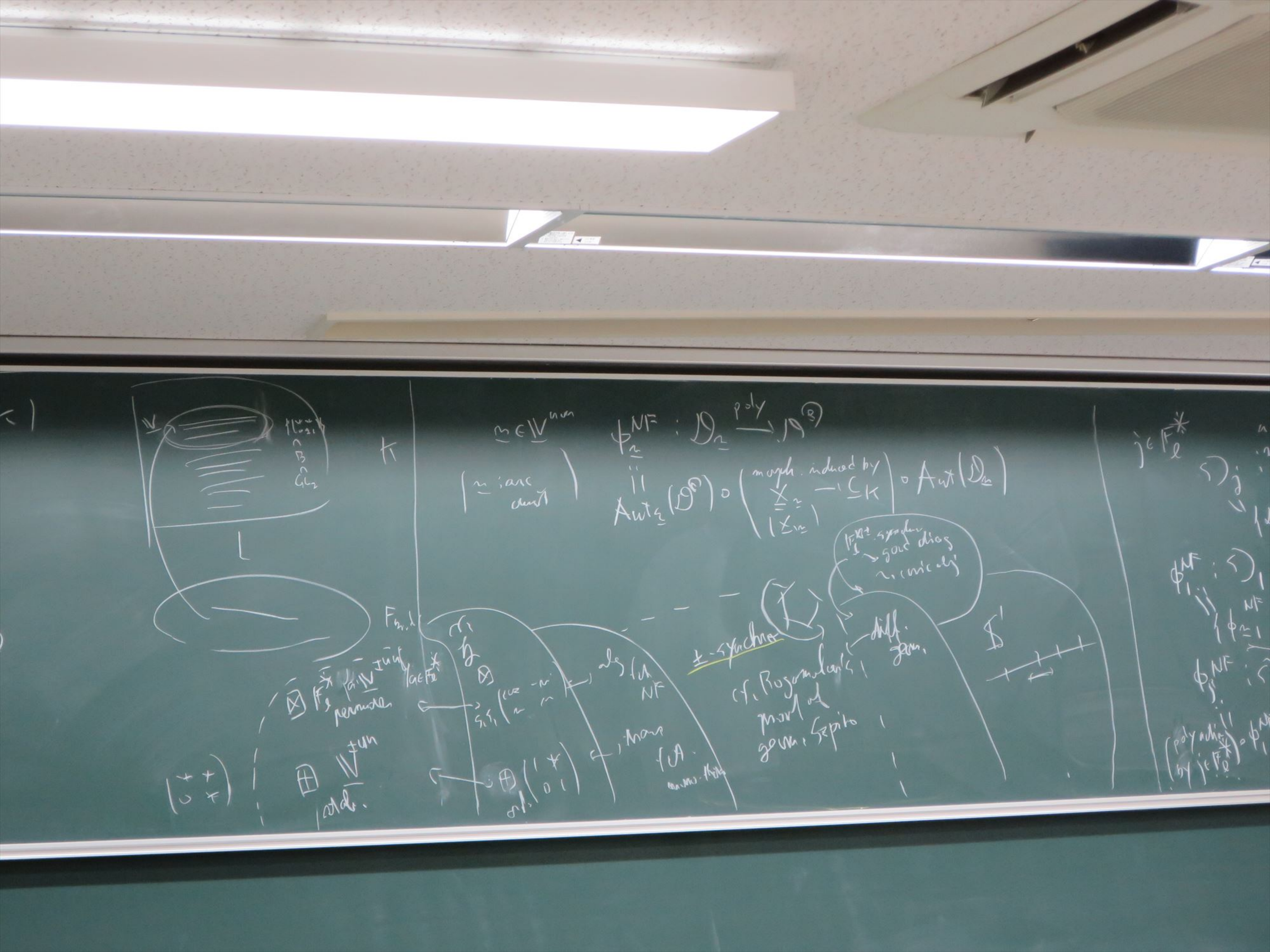
$$\mathcal{D}^{\circledcirc} := \mathcal{B}(C_K)^0 \quad \underbrace{\phantom{xx}}_{\text{sp this}} \quad \overline{F},\ V(F)$$

$$\underline{\mathbb{V}}^{\pm un} := \mathrm{Aut}_{\varepsilon}(\mathcal{C}_K)\,|\,\underline{\mathbb{V}} \subset \mathbb{V}(K)$$

$$\underline{\mathbb{V}}^{Bor} := \mathrm{Aut}(\mathcal{C}_K)\,|\,\underline{\mathbb{V}} \subset$$

$\mathbb{F}_l^{*}$-orbit of $\underline{\mathbb{V}}^{\pm un}$

$$\mathrm{Aut}(\mathcal{D}^{\circledcirc}) \supset \underbrace{\mathrm{Aut}_{\varepsilon}(\mathcal{D}^{\circledcirc})}_{\text{sp th'l}} \;"\; \underbrace{\mathbb{F}_l^{*} \curvearrowright \mathcal{D}^{\circledcirc}}_{\text{poly-act}} \;"$$



poly
morph. induced by
Aut
±-synchro
diff.
alg
NF

$\to D^{\circledcirc}$

morph. induced by $\underline{X}_m \to \underline{C}_K$ $\circ \operatorname{Aut}(D_m)$

$(\underline{X}_m)$

$\mathbb{F}_l^{\times\pm}$-symm. good diag. $\rightsquigarrow$ canonically

diff. gen.

Rugamandan's model of gen. Szpiro

$D'$

$j \in \mathbb{F}_l^{*}$

a copy of model $D$-positive

$S)_j$

$\wr$

$\{D_{m,j}\}_{m \in \mathbb{N}}$

$\phi_1^{NF} : S)_1 \xrightarrow{\text{poly}} D^{\circledcirc}$

$\{\phi_{m,1}^{NF}\}$ copy of $\phi_m^{NF}$

$\phi_j^{NF} : S)_j \xrightarrow{\text{poly}} D^{\circledcirc}$

$\left(\begin{array}{c}\text{poly action} \\ \text{by } j \in \mathbb{F}_l^{*}\end{array}\right) \circ \phi_1^{NF}$

$\Phi^{NF}_{*} : \zeta)_{*} := \{\zeta)_j\}_{j \in \mathbb{F}_l^{*}} \xrightarrow{\text{poly}} \mathcal{D}^{\odot}$

model $\mathcal{D}$-NF-bridge

$\{\phi_j^{NF}\}_{j \in \mathbb{F}_l^{*}}$

$\mathbb{F}_l^{*}$ $\mathbb{F}_l^{*}$ Aut$_\varepsilon$-ulon

$\phi^{NF}_{*}$

$\mathbb{F}_l^{*}$

$\zeta_K$

[IUTch I Ex 4.4]

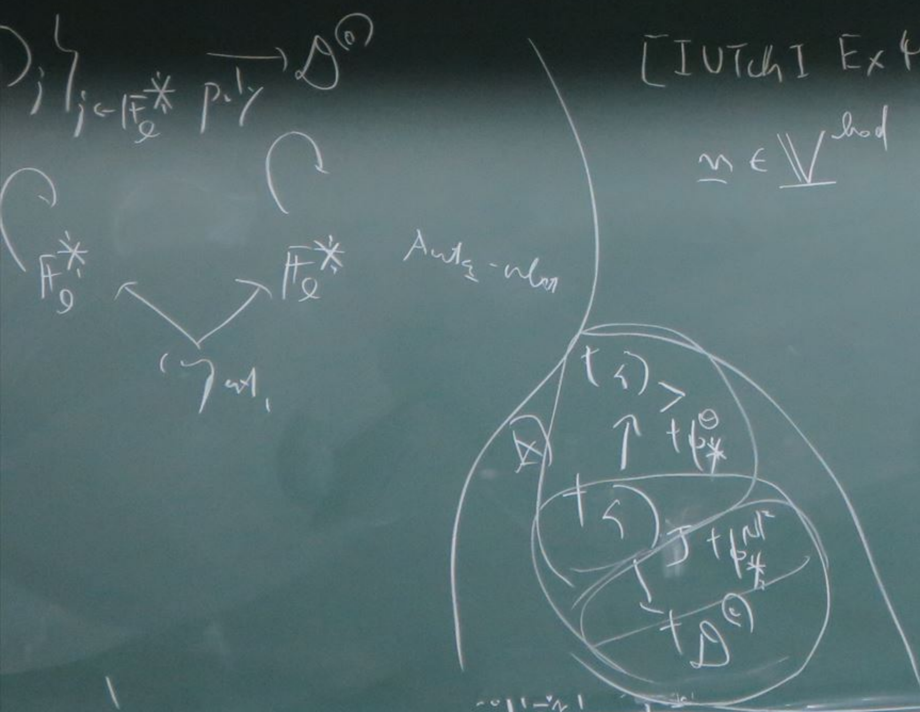
$\underline{v} \in \mathbb{V}^{bad}$

$|\mathbb{F}_l|$

$^{\dagger}\zeta \supset {}^{\dagger}\phi^{\Theta}_{*}$

$^{\dagger}\zeta \supset {}^{\dagger}\phi^{NF}_{*}$

$^{\dagger}\mathcal{D}^{\Theta}$



[IUTchI Ex 4.4] (model $\mathcal{D}$-$\Theta$-bridge)

$\underline{v} \in \mathbb{V}^{\text{bad}}$

Aut$_\varepsilon$-orbit

$|\mathbb{F}_l| := \mathbb{F}_l/\{\pm 1\} = \{0\} \cup \mathbb{F}_l^{*}$

$* \longmapsto |*|$

$\mu_- \in \underline{X}_{\underline{v}}(K_{\underline{v}})$

for pt of order $= 2$

lying in $0 \in |\mathbb{F}_l|$-labelled comp.

$|\mathbb{F}_l| \ni j$-th comp

$K_{\underline{v}} / q_{\underline{v}}^{2}$

"$-1$"

$\pm 1, \pm q^{\frac{1}{2}}$

$\mu_{2l}$-orbit

$\underline{\underline{\Theta}}_{\underline{v}} \; (\sqrt{-1}\, q^{j/2}) = q_{\underline{v}}^{j^2}$

$j \in \{1, \dots, l^{*}\}$

$U \leftarrow \ddot{U}$

a pt of $\underline{X}_{\underline{v}}$

evaluation

(...ridge)

$= \{0\} \cup \mathbb{F}_q^*$

$K_{\sim}/q^2_{\sim}$

"$-1$" $\pm 1, \pm q^{\frac{1}{2}}$

$U \leftarrow U''$

...der $= 2$

...-labelled comp.

$\mu_{2\ell}$-mult

$\left(\sqrt{-1}\, q^{j/2}\right) = q^{\frac{j^2}{2}}$

$\underline{\underline{\mathbb{Z}}}_{\sim}$ $\quad \frac{j}{2} \in \mathbb{I}_{\ell} + \ell\mathbb{Z}$

a pt of $\underline{X}_{\sim}$

by $-j$ mean

a pt $\quad$ (cusp) $+\ \mu_{-} \in \underline{X}_{\sim}$

translate by $t$-tors of ell. curve

underlying

evaluation pt

$G_{\sim} \longrightarrow \Pi_{\sim}$

evaluation pt

eval section

Borgne (Thu)
19:30-

$\mathcal{D}_{>} = \{\mathcal{D}_{>,\underline{v}}\}_{\underline{v}\in\underline{\mathbb{V}}}$

a copy of the model $\mathcal{D}$-prime-strip

$j \in \mathbb{F}_l^{*}$ $\qquad \phi^{\Theta}_{\underline{v},j} : \mathcal{D}_{\underline{v},j} \xrightarrow{\text{poly}} \mathcal{D}_{>,\underline{v}}$

$\Pi_{\underline{v}} \twoheadrightarrow G_{\underline{v}} \xhookrightarrow{\text{out. rat.}} \Pi_{\underline{v}}$

$\mathrm{Aut}(\mathcal{D}_{>,\underline{v}}) \circ \left(\begin{array}{c}\text{factor through}\\ \text{eval. sect. at}\\ j\text{-labelled comp}\end{array}\right) \circ \mathrm{Aut}(\mathcal{D}_{\underline{v},j})$

compat. w/ actions of $\pi_1^{geo}(\mathcal{D}_{\underline{v},j})$, $\pi_1^{geo}(\mathcal{D}_{>,\underline{v}})$

$\underline{v} \in \underline{\mathbb{V}}^{good}$ admit $\rightsquigarrow$ fully poly isom

$j \in \mathbb{F}_\ell^{*}$ $\quad \phi^{\Theta}_{j} \; ; \varsigma \rangle_j \xrightarrow[\text{poly}]{} \varsigma \rangle >$

$\{\phi^{\Theta}_{\underline{\approx} j}\}$

$\phi^{\Theta}_{*} \; ; \varsigma \rangle_{*} \xrightarrow{\text{poly}} \varsigma \rangle >$

$\{\phi^{\Theta}_{j}\}_{j \in \mathbb{F}_\ell^{*}}$

model

$\Theta$-bridge

$\mathbb{F}_\ell^{*}$-transl.

e-itey

$\mathcal{D}_{>}, \underline{\approx}$

$\Pi_{\underline{\approx}} \to G_{\underline{\approx}} \xleftrightarrow{\text{ord. root}} \Pi_{\underline{\approx}}$

*Mon... react. at labelled comp*

$) \circ \mathrm{Aut}(\mathcal{D}_{\underline{\approx} j}$

$(\mathcal{D}_{\underline{\approx} j}), \pi_1^{geo}(\mathcal{D}_{>}, \underline{\approx}$

$\underline{\approx} < \mathbb{IV}^{\text{good}}$ $\quad$ $\mathcal{M}_{\mathrm{mix}}$ $\rightsquigarrow$ fully poly isom

[IUTch I Ex...

$j \in$

$\underline{\approx} \in$

[IUTch I Ex 4.5]

Arrangement of labels

$j \in \mathbb{F}_l^{*}$

$\underline{v} \in \underline{\mathbb{V}}^{non}$

$\underline{X}_{\underline{v}}$

$\underline{C}_{\underline{v}}$

$\phi^{NF}_{\underline{v}, j} : \mathcal{D}_{\underline{v}, j} \longrightarrow \mathcal{D}^{\circledcirc}$

$\underline{X}_{\underline{v}}$
$\underline{X}_{\underline{v}} \to \underline{X}$
tot. ram. at cusps

cuspidal $\longleftarrow$ sp th's

subgp of index $= l$ of mention contained in the image

$\mathrm{LabCusp}(\mathcal{D}^{\circledcirc}) \xrightarrow{\sim} \mathrm{LabCusp}(\mathcal{D}_{\underline{v}, j})$
as $\mathbb{F}_l^{*}$-torsors

(v : arc ... ) $\longrightarrow \mathrm{LabCusp}(\mathcal{D}^{\circledcirc}) \xrightarrow{\sim} \mathrm{LabCusp}(\mathcal{S})_j$

$\phi^{\Theta}_{j} : \mathcal{S}_j \xrightarrow{\mathrm{poly}} \mathcal{S}_{\dots} \longrightarrow \mathrm{LabCusp}(\mathcal{S})_j \xrightarrow{\sim} \mathrm{LabCusp}(\mathcal{S})_{\dots}$
as $\mathbb{F}_l^{*}$-torsors

$j \in \mathbb{F}_\ell^{*}$, $\phi_j^{MF}$, $\phi_j^{\Theta}$

$\rightsquigarrow \phi_j^{LC} : \mathrm{LabCusp}(D^{\odot}) \xrightarrow{\sim} \mathrm{LabCusp}(S)_{>}$

is $\mathbb{F}_\ell^{*}$-tors. $\quad \mathrm{Aut}_{\varepsilon}$

s.t. $\phi_j^{LC} = \phi_1^{LC} \circ (j \in \mathbb{F}_\ell^{*}$ poly act$)$

$\phi_j^{LC}([\varepsilon]) = j$ under $\mathrm{LabCusp}(S)_{>} \xrightarrow{\sim} \mathbb{F}_\ell^{*}$

$\qquad ||$

$\phi_1^{LC}(j \cdot [\varepsilon])$

)

— $\mathrm{Aut}_{\cong}$

$\mathrm{LabCusp}(\varsigma)_J \,] \xrightarrow{\sim} \mathbb{F}_l^{\circledast}$

---

[IUTch I Def 4.6]

$\mathcal{D}$-NF-bridge

(base-)

$$\dagger\varsigma)_J \xrightarrow{\text{poly}} \dagger\mathcal{D}^{\circledcirc}$$

$$\dagger\phi^{NF}_{\circledast}$$

s.t. isom. to models

(iso)morph $\dagger\varsigma)_J \to \ddagger\varsigma)_{J'}$,

capsule-full poly. isom.

...

---

$\mathcal{D}$-$\Theta$-bridge

(base-)

$$\dagger\varsigma)_J \xrightarrow{\text{poly}} \dagger\phi^{\Theta}_{\circledast}$$

similarly,

(iso)morph.

$\mathcal{D}$-$\Theta$NF-Hodge...

(base-)

$\dagger\mathcal{HT}^{\mathcal{D}\text{-}\Theta NF} = ...$

s.t. ...

(iso)morph...

Def 4.6 ]

NF bridge

$\dagger\mathfrak{S}_{\succ, J} \xrightarrow[\dagger\phi^{NF}_*]{\text{poly}} \dagger\mathcal{D}^{\circledcirc}$

s.t. isom. to models

(iso)morph. $\dagger\mathfrak{S}_{\succ, J} \to \dagger\mathfrak{S}_{\succ, J'}$

capsule-full poly. isom.

...

$\mathcal{D}$-base-$\Theta$-bridge

$\dagger\mathfrak{S}_{\succ, J} \xrightarrow[\dagger\phi^{\Theta}_*]{\text{poly}} \dagger\mathfrak{S}_{\succ}$

similarly,

(iso)morph.

$\mathcal{D}$-base-$\Theta$NF-Hodge theater

$\dagger\mathcal{HT}^{\mathcal{D}\text{-}\Theta NF} = \left( \dagger\mathcal{D}^{\circledcirc} \xleftarrow{\dagger\phi^{NF}_*} \dagger\mathfrak{S}_{\succ, J} \xrightarrow{\dagger\phi^{\Theta}_*} \dagger\mathfrak{S}_{\succ} \right)$

s.t. ....

(iso)morph. bij's of index set coincide

capsules

$$\begin{array}{ccc} {}^{\dagger}\mathfrak{S}_{\succ} & \cdots & {}^{\ddagger}\mathfrak{S}_{>} \\ & \text{partial} & \\ \uparrow & & \uparrow \\ {}^{\dagger}\mathfrak{S}_{T} & \cdots & {}^{\ddagger}\mathfrak{S}_{T} \\ \downarrow & & \downarrow \\ {}^{\dagger}\mathcal{D}^{\circledcirc\pm} & & {}^{\ddagger}\mathcal{D}^{\circledcirc} \end{array}$$

$${}^{\dagger}\mathcal{HT}^{\ominus}$$

$$\begin{array}{ccc} {}^{\dagger}\mathfrak{F}_{\succ} & \cdots & {}^{\ddagger}\mathfrak{F}_{>} \\ \uparrow & & \uparrow \\ {}^{\dagger}\mathfrak{F}_{T} & \cdots & {}^{\ddagger}\mathfrak{F}_{T} \\ \downarrow & & \downarrow \\ {}^{\dagger}\mathcal{D}^{\circledcirc\pm} & & {}^{\ddagger}\mathcal{F}^{\circledcirc} \\ & & \vdots \\ & & {}^{\ddagger}\mathcal{F}^{\circledast} \end{array}$$

[IUTchII, 4.6, 4.7]

[IUTchI, Prop 4.8]

(i) $\mathrm{Isom}({}^{\dagger}\phi^{NF}_{\circledast}, {}^{\ddagger}\phi^{NF}_{\circledast}$

(ii) $\# \mathrm{Isom}({}^{\dagger}\phi^{\ominus}_{\circledast}$

(iii) ${}^{\dagger}\phi^{NF}_{\circledast}$ $\underbrace{{}^{\ddagger}\phi^{\ominus}_{\circledast}}$ gr

${}^{\dagger}\mathfrak{S}_{T} \longrightarrow$ compat

po

$C_{{}^{\dagger}F}^{\circledast} - {}^{\dagger}$

(iv) ${}^{\dagger}\phi^{NF}_{\circledast}$

[IUTch I, 4.6, 4.7]

[IUTch I, Prop 4.8] $^\dagger\phi^{NF}_{*} \mid F_l^{\circledast}$-torsor

(i) $\mathrm{Isom}({}^\dagger\phi^{NF}_{*}, {}^\ddagger\phi^{NF}_{*})$

(ii) $\# \mathrm{Isom}({}^\dagger\phi^{\Theta}_{*}, {}^\ddagger\phi^{\Theta}_{*}) = 1$

(iii) ${}^\dagger\phi^{NF}_{*}, {}^\ddagger\phi^{\Theta}_{*}$ gen. $\{{}^\dagger\mathfrak{S}\} \xrightarrow[\substack{\text{capsule-full}\\\text{poly-isom}}]{} \{{}^\ddagger\mathfrak{S}\}$ s.t. forms a $\mathcal{D}$-$\Theta$NF-Hodge theater

$F_l^{\circledast}$-torsor

(iv) ${}^\dagger\phi^{NF}_{*} \longrightarrow {}^\dagger\mathcal{H}\mathcal{T}^{\mathcal{D}\text{-}\Theta NF}$ up to $F_l^{\circledast}$-indet.

for conih.

proc

capsule-full

et pict

$H_\ell^{*}$ - torsor

$\left| \phi^{\Theta}_{*} \right| = 1$

$J'$ s.t. forms a $\mathcal{D}$-ONF-Hodge datum

$\dagger\mathcal{H}^{\mathcal{D}\text{-ONF}}_{T}$ up to $F_\ell^{*}$-indet.

comm.

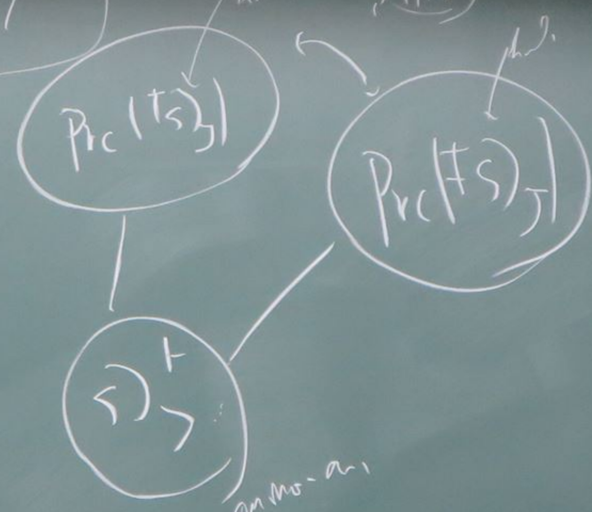
proc

$\{ \ldots \} \subseteq \{ \ldots \} \subseteq \{ \ldots \}$ capsule-full poly

[IUTchI, 4.9, 4.10, 4.11]

(4.12)

et picture




$M^{\circledast}(\dagger\mathfrak{D}^{\circledast})$ $\quad \dagger\mathcal{F}^{\circledast}_{T} \cong \mathcal{F}^{\circledast}$

Frd. gina

et

$M^{\circledast}_{\ldots}$ Frd.

$\dagger S)_J$ : $|J|$-capsule of $\mathcal{D}$-

$\mathbb{V}_j := \{v_j\}_{\cong \mathbb{V}}$

copy of $\mathbb{V}$

[IUTchI, 4.9, 4.10, 4.11]

... ll poly

ctur

hol. (4.12)

$\mathrm{Prc}({}^{\dagger}\mathfrak{S}_J)$

hol.

$\mathrm{Prc}({}^{\ddagger}\mathfrak{S}_J)$

$\mathfrak{S}^{\vdash}_J$

mono-an.

$\mathcal{M}^{\circledast}({}^{\dagger}\mathfrak{D}^{\circledast})$ $\quad {}^{\dagger}\mathcal{F}^{\circledast} \simeq \mathcal{F}^{\circledast}({}^{\dagger}\mathfrak{D}^{\circledast})$

ét $\quad$ Frd. $\quad$ gina $\quad$ ét

$\mathcal{M}^{\circledast}_{\dots}$ $\quad$ Frd.

${}^{\dagger}\mathfrak{S}_J$ : $|J|$-capsule of $\mathcal{D}$-prime-strips

$\underline{\mathbb{V}}_j := \{\underline{v}_j\}_{\underline{v} \in \underline{\mathbb{V}}} \xrightarrow{\sim} \underline{\mathbb{V}}$

copy of $\underline{\mathbb{V}}$

$\underline{v}_j \longmapsto \underline{v}$

$\langle\ \rangle$

$\circ, 1$ diag.

$\rightarrow \Delta$

<u>diagonal</u> $\underline{\mathbb{V}}_{\langle J \rangle} \subset \underline{\mathbb{V}}_J := \prod_{j \in J} \underline{\mathbb{V}}_j$

mod. hij's $\underline{\mathbb{V}}_{\langle J \rangle} \xrightarrow{\sim} \underline{\mathbb{V}}_j \xrightarrow{\sim} \mathrm{Pic}({}^{\dagger}\mathcal{F}^{\circledR}_{\mathrm{mod}}) \xrightarrow{\sim} \mathbb{V}_{\mathrm{mod}}$

$${}^{\dagger}\mathcal{F}^{\circledR}_{\underset{j}{\langle J \rangle}} := \{ {}^{\dagger}\mathcal{F}^{\circledR}_{\mathrm{mod}}, \underline{\mathbb{V}}_{\underset{j}{\langle J \rangle}} \xrightarrow{\sim} \mathrm{Pic}({}^{\dagger}\mathcal{F}^{\circledR}_{\mathrm{mod}}) \}$$

$\Theta^{\mathrm{lin}} \simeq g^{\sim}$

${}^{\dagger}\mathcal{F}^{\circledR}_{\langle}$

$\{\mathrm{IVL}$

$\Theta^{\mathbb{N}} \simeq \underline{q}^{\mathbb{N}}$

$\hookrightarrow \mathbb{V}_{\mathrm{mod}}$

$^{\dagger}\mathcal{F}^{\circledast \mathbb{R}}_{\langle J \rangle} \hookrightarrow {}^{\dagger}\mathcal{F}^{\circledast \mathbb{R}}_{J} := \prod_{j \in J} {}^{\dagger}\mathcal{F}^{\circledast \mathbb{R}}_{j}$

$\mathcal{HT}^{\Theta} \ \ \underline{\underline{\mathcal{F}}}_{\underline{v}} \ \ \Pi_{\underline{v}}$ (non-arch)

$\underline{\underline{\mathcal{F}}}_{\underline{v}} \vdash \Pi_{\underline{v}} \curvearrowright \mathcal{O}^{\triangleright}_{\underline{v}}$

$\mathrm{Prime}({}^{\dagger}\mathcal{F}^{\circledast}_{\mathrm{mod}}) \}$

[IUTch I, Def 5.2]

hol. Frob.-pic-strip $\underline{\mathcal{F}}$

$^{\dagger}\mathfrak{F} = \{{}^{\dagger}\mathcal{F}_{\underline{v}}\}_{\underline{v} \in \underline{\mathbb{V}}}$

s.t. isom. to the models

non $\mathcal{C}_{\underline{v}}$

arc $\underline{\underline{\mathcal{F}}}_{\underline{v}}$

mono-an. Frob.-pic-...

$( \underline{\mathcal{F}}^{\Vdash} - )$

gl. real'd mono-a...

$^{\dagger}\mathfrak{F}^{\Vdash} = ( \ldots$

s.t. ...

$\mathcal{H}\mathcal{T}^{\Theta} \leftarrow \underline{\mathcal{F}}_{\underline{n}} \quad \mathcal{T}^{\text{hol}}_{\underline{n}} \curvearrowright (\text{mod}, \text{-th})$

$\underline{\mathcal{F}}^{\text{hs-Thd}}_{n} \vdash \mathcal{T}^{\varrho}_{\underline{n}}\ \mathcal{G}_{\underline{n}}$

$= \{{}^{\dagger}\mathcal{F}_{\underline{n}}\}_{\underline{n} \in \mathbb{V}}$

inv. to the models

hom $\quad \varrho_{\underline{n}}$

an. $\quad \underline{\underline{\mathcal{F}}}_{\underline{n}}$

mono-an. Frob.-pre-strip $\quad {}^{\dagger}\mathfrak{F}^{\vdash} = \{{}^{\dagger}\underline{\mathcal{F}}^{\vdash}_{\underline{n}}\}_{\underline{n} \in \underline{\mathbb{V}}}$

$(\underline{\mathcal{F}}^{\vdash}\text{-})$

s.t. inv. to the models $\quad \mathcal{F}^{\vdash}_{\underline{n}}$

gl. real'd mono-an. Frob.-pre-strip

$(\underline{\mathcal{F}}^{\Vdash}\text{-})$

$${}^{\dagger}\mathfrak{F}^{\Vdash} = ({}^{\dagger}\mathcal{C}^{\Vdash}, \mathrm{Prime}({}^{\dagger}\mathcal{C}^{\Vdash}) \xrightarrow{\sim} \underline{\mathbb{V}}, {}^{\dagger}\mathfrak{F}^{\vdash}, \{{}^{\dagger}\rho_{\underline{n}}\}_{\underline{n} \in \underline{\mathbb{V}}})$$

gl. $\quad$ lr. $\quad$ gl. to lr.

$\mathcal{F}^{\vdash}$-pre-strip

s.t. ...

( ✓ recon $\mathbb{I}_m$ splitting )

$$^{\dagger}\mathfrak{D} \overset{\text{gp th'c}}{\rightsquigarrow} {}^{\dagger}\mathfrak{D}^{\vdash}$$

$$\overset{\text{gp th'c}}{\rightsquigarrow} {}^{\dagger}\mathfrak{D}^{\Vdash}$$

[IUTch I Cor 5.3]

(i) Frd Isom$({}^{\dagger}\mathcal{F}^{\circledast}$, ‡

(ii) mult Isom

(iii) Isom

(iv) $\Xi$-IV hid

( recon $\mathbb{I}_n$ splitting $\partial$ )

[IUTch I Cor 5.3]

(i)$^{\mathrm{Frd}}$ $\mathrm{Isom}({}^{\dagger}\mathcal{F}^{\circledast\circleddash}, {}^{\ddagger}\mathcal{F}^{\circledast\circleddash}) \xrightarrow{\sim} \mathrm{Isom}(\mathrm{Base}({}^{\dagger}\mathcal{F}^{\circledast\circleddash}), \mathrm{Base}({}^{\ddagger}\mathcal{F}^{\circledast\circleddash}))$

$\mathrm{Isom}({}^{\dagger}\mathfrak{F}, {}^{\ddagger}\mathfrak{F}) \xrightarrow{\sim} \mathrm{Isom}({}^{\dagger}\mathfrak{S}, {}^{\ddagger}\mathfrak{S})$

(ii) mildly $\mathrm{Isom}({}^{\dagger}\mathfrak{F}^{\vdash}, {}^{\ddagger}\mathfrak{F}^{\vdash}) \twoheadrightarrow \mathrm{Isom}({}^{\dagger}\mathfrak{S}^{\vdash}, {}^{\ddagger}\mathfrak{S}^{\vdash})$

(iii) $\mathrm{Aut}(\Xi_{\simeq}) \xrightarrow{\sim} \mathrm{Aut}(\mathfrak{D}_{\simeq})$ ← need some arg's

(iv) $\simeq \in \underline{\mathbb{V}}^{\mathrm{bad}}$

$) \xrightarrow{\sim} \mathrm{Isom}(\mathrm{Base}({}^{\dagger}\mathcal{F}^{\circledast}), \mathrm{Base}({}^{\ddagger}\mathcal{F}^{\circledast}))$

$, {}^{\ddagger}\mathfrak{F}) \xrightarrow{\sim} \mathrm{Isom}({}^{\dagger}\mathcal{G}), {}^{\ddagger}\mathcal{G}))$

$^{\vdash}, {}^{\ddagger}\mathfrak{F}^{\vdash}) \twoheadrightarrow \mathrm{Isom}({}^{\dagger}\mathcal{G})^{\vdash}, {}^{\ddagger}\mathcal{G})^{\vdash})$

$(\underline{\Xi} ) \xrightarrow{\sim} \mathrm{Aut}(\mathcal{D}_{\underline{\ }})$

need some arg's

$\mathcal{O}^{\circledast\times} \hookrightarrow \prod_{H} H^1(\ldots)$

$G \mapsto \mathcal{O}^{\times}_{\bar{k}}(G) \subseteq \bar{k}^{\times}(G)$

$\curvearrowleft \hat{\mathbb{Z}}^{\times}$-action

${}^{\dagger}\mathfrak{F}$

$\phi^{\times}({}^{\ddagger}\mathfrak{F})$ — ‡

$\Gamma$

${}^{\dagger}\mathcal{G}) \xrightarrow[\phi_1 \text{ given}]{} {}^{\ddagger}$

${}^{\dagger}\mathcal{G})^{\vdash} \cong {}^{\ddagger}$

$\mathcal{O}^{\circledast\times} \hookrightarrow \varinjlim_{H} H^1(\ldots)$

$G \mapsto \mathcal{O}^\times(G) \subseteq \bar{k}^\times(G)$

$\curvearrowleft \hat{\mathbb{Z}}^\times$-action

ang's

${}^\dagger\mathfrak{X}$ $\quad \phi^\times({}^\ddagger\mathfrak{X}) \longrightarrow {}^\ddagger\mathfrak{X}$

$\Gamma$

${}^\dagger G \xrightarrow{\ \phi\ } {}^\ddagger G$ not nec. isom.

given

${}^\dagger G^\perp \cong {}^\ddagger G^\perp$

$\phi : {}^\dagger\mathfrak{X} \to {}^\ddagger\mathfrak{X}$

(Ind) ${}^\dagger\mathfrak{X} \cong \phi^\times({}^\ddagger\mathfrak{X})$

Θ-link

~~Hodge~~ morph