COMMENTS ON "ON THE COMBINATORIAL CUSPIDALIZATION OF HYPERBOLIC CURVES"

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(1.) In lines 4–5 of the third paragraph of the discussion entitled *"The Étale Fundamental Group of a Log Scheme"* in §0, the phrase "*unramified* over $R_\circ$" should read "*unramified* over $U_{S_\circ}$".

(2.) In Fig. 1, the subscripts "b", "c" in the notation "F_b", "F_c" should be *reversed*.

(3.) In the statement of Proposition 1.3, (i), the phrase "the set of divisors $\mathcal{D}_n^{\mathrm{ver}}$ which" should read "the set of divisors which"; the notation "$\delta \in \mathcal{D}_n^{\mathrm{hor}}$" should read "$\delta \in \mathcal{D}_n^{\mathrm{ver}}$".

(4.) In the statement of Proposition 1.3, (iv), the notation "$\mathcal{D}_n^{\mathrm{hor}}$" should read "$\mathcal{D}_n^{\mathrm{ver}}$".

(5.) In the first paragraph of the proof of Corollary 1.14, the text "First, let us ... Now let" should read as follows:

> First, let us observe that relative to the *natural isomorphism* $X_n^{\log} \xrightarrow{\sim} (\overline{\mathcal{M}}_{0,n+3}^{\log})_k$ [cf. Definition 1.1, (vi)], the divisors of X_n that belong to $\mathcal{D}_n$ are precisely the *divisors at infinity* of $(\overline{\mathcal{M}}_{0,n+3}^{\log})_k$. [Indeed, this follows immediately from the well-known geometry of $(\overline{\mathcal{M}}_{0,n+3}^{\log})_k$.] In particular, the automorphisms of $(\overline{\mathcal{M}}_{0,n+3}^{\log})_k$ arising from the permutations of the ordering of the cusps *permute* the divisors that belong to $\mathcal{D}_n$. Thus, we conclude that the *outer modular symmetries* $\in \mathrm{Out}(\Pi_n)$ *normalize* $\mathrm{Out}^{\mathrm{QS}}(\Pi_n) = \mathrm{Out}^{\mathrm{FC}}(\Pi_n)^{\mathrm{cusp}}$ [cf. Proposition 1.3, (vi), (vii)]. Now let

(6.) In Definition 5.2, (ii), the phrase "of x in $\mathcal{U}$" should read "of x in $\mathcal{N}$".

(7.) In the argument given in Remark 1.1.5 [i.e., beginning with "Indeed, ..."], it is necessary to apply Proposition 1.2, (iii). [One verifies immediately that there are no vicious circles in the reasoning.]