Last update: 14 March, 2025

Inter-universal Teichmüller Theory Summit 2025

Organizers: Benjamin Collas (RIMS, Kyoto University)
Yuichiro Hoshi (RIMS, Kyoto University)
Emmanuel Lepage (Institut de Mathématique de Jussieu)
Shinichi Mochizuki (RIMS, Kyoto University)

Dates: March 17 – March 20, 2025
Place: RIMS, Kyoto University

	March 17 (Mon.)	March 18 (Tues.)	March 19 (Wed.)	March 20 (Thu.)
10:30 – 11:30		GT (Collas)	Class Field (Lagarias)	Mordell-Weil (Taguchi)
13:00 – 14:00	[Gateway] (Mochizuki)	[GSHP-I] (Hoshi)	[ArMCG] (Saïdi)	Galois Section (Porowski)
14:20 – 15:20	[OvEssLgcI] (Porowski)	[GSHP-II] (Hoshi)	[OvEssLgcIV] (Minamide)	Tilting (Kedlaya)
15:40 – 16:40	[OvEssLgcII] (Porowski)	[ArGT] (Tsujimura)	[OvEssLgcV] (Minamide)	TS-IUT (Yamashita)
17:00 – 18:00	[OvEssLgcIII] (Porowski)	[ArGT] (Tsujimura)	[OvEssLgcVI] (Minamide)	History of IUT (Boyd)
18:20 – 19:00	Q & A (Mochizuki)	Q & A (Mochizuki)	Q & A (Mochizuki)	Applications of IUT (Zhou)

Program

March 17 (Monday)

12:45 – 13:00
Opening Address

13:00 – 14:00 Shinichi Mochizuki (RIMS, Kyoto University)
Inter-universal Teichmüller Theory as an Anabelian Gateway to Diophantine Geometry and Analytic Number Theory (IUT Summit 2025 Version)

14:20 – 15:20 Wojciech Porowski (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory I

15:40 – 16:40 Wojciech Porowski (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory II

17:00 – 18:00 Wojciech Porowski (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory III

18:20 – 19:00 Shinichi Mochizuki (RIMS, Kyoto University)
Q & A

March 18 (Tuesday)

10:30 – 11:30 Benjamin Collas (RIMS, Kyoto University)
Grothendieck-Teichmüller theory: from Galois-Teichmüller theory to anabelian geometry

13:00 – 14:00 Yuichiro Hoshi (RIMS, Kyoto University)
On Galois Sections of Hyperbolic Polycurves over Arithmetic Fields I

14:20 – 15:20 Yuichiro Hoshi (RIMS, Kyoto University)
On Galois Sections of Hyperbolic Polycurves over Arithmetic Fields II

15:40 – 16:40 Shota Tsujimura (RIMS, Kyoto University)
On the arithmeticity of the Grothendieck-Teichmüller group I

17:00 – 18:00 Shota Tsujimura (RIMS, Kyoto University)
On the arithmeticity of the Grothendieck-Teichmüller group II

18:20 – 19:00 Shinichi Mochizuki (RIMS, Kyoto University)
Q & A

March 19 (Wednesday)

10:30 – 11:30 Jeffrey Lagarias (The University of Michigan)
Problems on class fields of orders of quadratic fields

13:00 – 14:00 Mohamed Saïdi (Exeter University)
On the arithmeticity of the mapping class group

14:20 – 15:20 Arata Minamide (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory IV

15:40 – 16:40 Arata Minamide (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory V

17:00 – 18:00 Arata Minamide (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory VI

18:20 – 19:00 Shinichi Mochizuki (RIMS, Kyoto University)
Q & A

March 20 (Thursday)

10:30 – 11:30 Yuichiro Taguchi (Institute of Science Tokyo)
Mordell-Weil groups over large algebraic extensions of number fields

13:00 – 14:00 Wojciech Porowski (RIMS, Kyoto University)
Locally conjugate Galois sections

14:20 – 15:20 Kiran Kedlaya (University of California San Diego)
Tilting and Fargues-Fontaine curves

15:40 – 16:40 Go Yamashita (RIMS, Kyoto University)
On tilts and inter-universal Teichmüller theory

17:00 – 18:00 James Douglas Boyd (SciSci)
On the Historical Development of Inter-Universal Teichmüller Theory (IUT): A Science Journalist's Perspective, Based on Recent Interviews

18:20 – 19:00 Zhong-Peng Zhou (Institute for Theoretical Sciences, Westlake University)
Applications of Inter-universal Teichmüller Theory to Diophantine Geometry and Equations over the rational numbers

Abstracts

Speaker: James Douglas Boyd

Title: On the Historical Development of Inter-Universal Teichmüller Theory (IUT): A Science Journalist's Perspective, Based on Recent Interviews

Abstract: This talk presents a series of brief summaries of perspectives on the historical development of IUT gathered from a four-part interview series conducted with Mochizuki-sensei during a RIMS visit in the autumn of 2024. Although some overlap with existing expository material will be found, emphasis will be placed on mathematical or historical observations from our discussions that are not otherwise on the written record or were regarded by Mochizuki-sensei as novel and substantive ways of thinking about IUT. Topics subject to brief summary will include: 1) The genealogical origins of the theta-link found in scheme-theoretic Hodge-Arakelov theory and (the prospect of) arithmetic Kodaira-Spencer theory; 2) the complex-analytic legacy (e.g., the functional equation of the theta function) of the "Membership Equation" ($a \in a$), and the common motivation of prime-strips (simulating a global multiplicative subspace) in IUT I and species (simulating $\in$-loops) in IUT IV; 3) the relationship between Grothendieck's work on connections/crystals and the multiradial representation in IUT, the relationship to arithmetic Kodaira-Spencer theory, and the deeper arithmetic significance (and Teichmüller-theoreticity) of the multiradial representation relative to certain Diophantine inequalities; 4) the origins of the log-link found in p-adic absolute anabelian geometry; 5) arithmetic holomorphic structure and the generalization of Teichmüller-theoreticity from complex-analytic Teichmüller Theory to IUT; 6) the possible relationship between abstract Galois groups in IUT and the inverse Galois problem; and 7) the influence of reconstruction and canonicity in anabelian geometry on the notion of mutually alien ring structure and coricity.

Speaker: Benjamin Collas

Title: Grothendieck-Teichmüller theory: from Galois-Teichmüller theory to anabelian geometry

Abstract: Grothendieck-Teichmüller theory, which originates from an encounter between the absolute Galois groups of rational numbers, Teichmüller theory, and braids-mapping class group theory, and was developed in the 90s as such by Hiroaki Nakamura, Leila Schneps and Pierre Lochak, shapes a triangle between arithmetic, geometry, and combinatorial.

The Minamide-Hoshi-Mochizuki 2017 paper, by establishing the anabelianity of the genus 0 GT group, opens the way to bringing the latest insight and techniques of anabelian geometry in the theory, in terms of canonicity, combinatorial anabelian geometry, and variation of analytic structures via absolute mono-anabelian reconstructions.

The goal of this talk is to present the essential and structuring components of GT theory, and to show how they are related to the new insights naturally brought by new anabelian reconstructions.

Speaker: Yuichiro Hoshi

Title: On Galois Sections of Hyperbolic Polycurves over Arithmetic Fields

Abstract: This talk focuses on the study of Galois sections of hyperbolic polycurves. A Galois section of an algebraic variety over a field is defined to be a continuous section

of the natural continuous surjective outer homomorphism from the etale fundamental group of the given variety to the absolute Galois group of the base field. Recall that Grothendieck's section conjecture states that, for a given hyperbolic curve over a number field, an arbitrary Galois section of the curve is geometric, i.e., the image of an arbitrary Galois section of the curve is contained in a decomposition subgroup associated to a closed point of the curve. This talk reports on recent progress on the study of the geometricity of Galois sections of hyperbolic polycurves over arithmetic fields. This is a joint work with Shinichi Mochizuki.

Speaker: Kiran S. Kedlaya

Title: Tilting and Fargues-Fontaine curves

Abstract: An old construction of Fontaine, now commonly known as "tilting", provides a canonical functor converting "sufficiently ramified" nonarchimedean fields in mixed characteristic into nonarchimedean fields in equal positive characteristic. This functor is not fully faithful; if one fixes a field in positive characteristic, then its inverse images under tilting are naturally parametrized by a geometric object (in a sense to be made precise) called a Fargues-Fontaine curve. While this construction plays a fundamental role in p-adic Hodge theory, we will talk only briefly about this role; instead, we focus on the basic geometry with an eye towards the comparison of the theory of Fargues-Fontaine curves with IUT (although we will make no attempt to discuss the details of such a comparison).

Speaker: Jeffrey Lagarias

Title: Problems on class fields of orders of quadratic fields

Abstract: This talk treats two topics related to ray class fields of imaginary quadratic fields and real quadratic fields.

(1) We review Granville and Stark's 2000 result: Uniform ABC for number fields implies no 'Siegel zeros' for L-functions of characters of negative discriminant. It uses ABC-equations encoding modular relations coming from certain singular moduli specifying CM-elliptic curves. The singular moduli generate the ring class field of the order of the imaginary quadratic field giving the complex multiplication.

(2) We present results related to class fields of real quadratic fields conjecturally arising from SIC-POVM's. SIC-POVM's are a oonstruction in quantum information theory, equivalent to a maximal set of d^2 equiangular complex lines in $\mathbb{C}^d$. Conjecturally they exist for all d. They are known to exist in finitely many d, including all $d \leq 25$. For $d \geq 4$, the constructed ones empirically involve algebraic numbers lying in abelian extensions of the real quadratic field with discriminant $(d+1)(d-3)$. We formulate conjectures specifying the abelian extensions as being certain ray class fields of associated orders of this field, and report on tests of the conjectures. This is joint work with Gene Kopp (LSU).

Speaker: Arata Minamide

Title: On the essential logical structure of inter-universal Teichmüller theory IV, V, VI

Abstract: In this series of three talks, we explain technical and logical subtleties of IUT. For instance, we discuss a certain oversimplified version of IUT, the fundamental

importance of the logical AND "$\wedge$" relation of the Θ-link, and the notion of descent to underlying structures. Finally, we revisit the main theorems of IUT.

Speaker: Shinichi Mochizuki

Title: Inter-universal Teichmüller Theory as an Anabelian Gateway to Diophantine Geometry and Analytic Number Theory (IUT Summit 2025 Version)

Abstract: One question that is frequently asked concerning inter-universal Teichmüller theory (IUT) is the following:

> Why/how does IUT allow one to apply anabelian geometry to prove diophantine results?

In this talk, we address this question from various points of view. First, we discuss the fundamental framework underlying the relationship established by IUT between anabelian geometry, on the one hand, and diophantine geometry/analytic number theory, on the other. This discussion centers around the N-th power map on a subring of a field and the difference between regarding a group as a Galois group, on the one hand, and as an abstract group that is not equipped with an embedding into the automorphism group of a field, on the other. Here, we emphasize that this discussion is entirely elementary and only assumes a knowledge of groups/monoids, rings, fields, and the elementary geometry surrounding the projective line/Riemann sphere. This elementary example centering around the N-th power map is important in that it reflects the various aspects of the essential logical structure of IUT that arise in certain misunderstandings concerning IUT that have received substantial public attention. We also briefly discuss certain (again entirely elementary!) set-theoretic/foundational subtleties surrounding the notion of a "gluing" that are closely related to such misunderstandings. Such subtleties include the importance of working with "types/packages of data" called "species" (as opposed to underlying sets!), as well as the importance of obtaining "closed loops" of such types/packages of data in order to establish set-theoretic conclusions. Classical instances of such subtleties include the conjugacy indeterminacies inherent in the construction of the algebraic closure of a field and the closely related use of norms in Galois theory, as well as the classical notions of analytic continuation/Riemann surfaces — which is reminiscent of the famous classical dispute between Riemann and Weierstrass! — and geodesic completeness/closed geodesics. We then proceed to survey recent developments (work in progress) in IUT, many of which are closely related to the Section Conjecture in anabelian geometry for arbitrary hyperbolic curves over number fields, as well as to combinatorial anabelian aspects of Grothendieck-Teichmüller theory. In the case of the Section Conjecture for hyperbolic curves over number fields, recent progress is closely related to 3 new enhanced versions of IUT that are currently under development. One of these new enhanced versions, namely, the Galois-orbit version of IUT, has new applications not only to the Section Conjecture for hyperbolic curves over number fields, but also to the nonexistence of Siegel zeroes of certain Dirichlet L-functions. The application to the Section Conjecture is interesting in that it exhibits and reconfirms the essentially anabelian content of IUT, i.e., as a theory based on anabelian geometry that is applied to prove new results in anabelian geometry. On the other hand, these recent applications, taken together with the original application of IUT to the ABC/Szpiro/Vojta Conjectures, are also noteworthy in that they may be regarded as a striking example of Poincaré's famous quote to the effect that

> "mathematics is the art of giving the same name to different things".

That is to say, the common name "IUT" that may be regarded as describing, in essence, a single mathematical phenomenon that manifests itself, depending on relatively inessential (!) differences of context, as various (at first glance, unrelated!) diverse phenomena in anabelian geometry, diophantine geometry, and analytic number theory. The relationship with Poincaré's famous quote is also fascinating in that it was apparently motivated by various mathematical observations on the part of Poincaré concerning the similarities between transformation group symmetries of modular functions such as theta functions and symmetry groups of the hyperbolic geometry of the upper half-plane — all of which are topics (cf. the discussion above of Galois groups versus abstract groups!) that bear a profound relationship to IUT.

Speaker: Wojciech Porowski

Title: On the essential logical structure of inter-universal Teichmüller theory

Abstract: In this series of three talks we will discuss the logical structure of the IUT theory, following the manuscript "On the essential logical structure..." by S. Mochizuki. We will concentrate on the general structure of the theory, making the talks accessible to a wider audience.

Speaker: Wojciech Porowski

Title: Locally conjugate Galois sections

Abstract: Consider a hyperbolic curve over a number field and the associated short exact sequence of etale fundamental groups; we are interested in splittings of this sequence, called Galois sections. The Grothendieck section conjecture, an important open problem in anabelian geometry, predicts a concrete description of the set of conjugacy classes of Galois sections. In this talk we prove that sections which are conjugate almost everywhere locally are conjugate globally and we will discuss some possible extensions of this result.

Speaker: Mohamed Saïdi

Title: On the arithmeticity of the mapping class group

Abstract: I will discuss new results regarding the mapping class group, which suggest a strong connection between anabelian geometry and Grothendieck-Teichmuller theory, as well as analogies with IUT. Joint ongoing research project with Shinichi Mochizuki and Shota Tsujimura.

Speaker: Yuichiro Taguchi

Title: Mordell-Weil groups over large algebraic extensions of number fields

Abstract: We present some results on the structure of the Mordell-Weil groups of semiabelian varieties over large algebraic extensions of a number field. We consider two types of algebraic extensions; one is of extensions obtained by adjoining the coordinates of certain points of various semiabelian varieties; the other is of extensions obtained as the fixed subfield in an algebraically closed field by a finite number of automorphisms. Some of such fields turn out to be new examples of Kummer-faithful fields which are not sub-p-adic. This is a joint work with Takuya Asayama.

Speaker: Shota Tsujimura

Title: On the arithmeticity of the Grothendieck-Teichmüller group

Abstract: The absolute Galois group of the field of rational numbers ($G_{\mathbb{Q}}$) may be regarded as one of the central objects in modern number theory. One traditional approach [suggested by A. Grothendieck] to understanding $G_{\mathbb{Q}}$ is by studying the geometric Galois action of $G_{\mathbb{Q}}$ on the étale fundamental groups of suitable geometric objects. In this context, in light of Belyi's theorem, it is well-known that $G_{\mathbb{Q}}$ may be embedded into the Grothendieck-Teichmüller group GT [which was originally introduced by V. Drinfeld] via the natural outer action on the étale fundamental group of the projective line minus three points over the field of algebraic numbers. In light of this embedding, GT has been regarded as a sort of purely combinatorial approximation of $G_{\mathbb{Q}}$. In this talk, I will introduce a part of recent progress surrounding this topic that is based on an approach via combinatorial anabelian geometry.

Speaker: Go Yamashita

Title: On tilts and inter-universal Teichmüller theory

Abstract: We will discuss similarities and differences between the theory of tilts of perfectoid fields and the theory surrounding the unit group portions of $\mathcal{F}^{\Vdash\blacktriangleright\times\mu}$-prime-strips in the Θ-link of inter-universal Teichmüller theory. These theories exhibit some superficial similarities, but also deep structural differences, and in conclusion, it does not appear possible to construct any sort of theory that is in some sense structurally analogous to inter-universal Teichmüller theory by using Galois actions on tilts of p-adic completions of algebraic closures of perfectoid fields. The essential contents of this talk are due to Example 3.5.3 of S. Mochizuki's paper "On the essential logical structure of inter-universal Teichmüller theory in terms of logical AND "$\wedge$"/ logical OR "$\vee$" relations".

Speaker: Zhong-Peng Zhou

Title: Applications of Inter-universal Teichmüller Theory to Diophantine Geometry and Equations over the rational numbers

Abstract: In this talk, we explore applications of Inter-universal Teichmüller (IUT) theory to problems in Diophantine geometry and equations over the rational numbers. Specifically, we present novel effective abc-inequalities and establish bounds on non-trivial primitive solutions to the generalized Fermat equations. By combining insights from IUT theory with computational methods, we demonstrate that certain generalized Fermat equations do not admit non-trivial primitive solutions. This work demonstrates the application of IUT theory to specific challenges in Diophantine geometry and classical number theory problems.