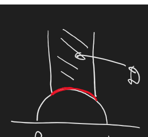
Why $|\tau| \neq 1$?

⇓

well-defined q-parameter

∵) $|\tau| = 1 \Rightarrow \tau, \tau^{-1} \in \mathfrak{D}$

$q = e^{2\pi i \tau}$



$\mathbb{C}_{\text{Im}} \to \mathbb{C}_{\text{Im}}/q^{\mathbb{Z}} \cong E$

$\mathbb{Z} \leftarrow \pi_1(E) \cong \mathbb{Z}^2$

$E \rightsquigarrow SL_2(\mathbb{Z}) \cdot \tau$

$(|\tau| \neq 1 \Rightarrow)$ $\exists!\, \tau \in \mathfrak{D}$ (up to possibly ± 1)

± 1 does not affect q-par.

$\tau^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tau$

Hodge-Arakelov Theory

$\Gamma(E^{\dagger}, \mathcal{L}|_{E^{\dagger}})^{<\ell} \overset{\sim}{\to} \mathcal{L}|_{E[\ell]}$

can be considered over

$\overline{\mathcal{M}} := \overline{\mathcal{M}}_{ell}/\mathbb{Q}$

LHS ⇝ v.b. on $\overline{\mathcal{M}}$ of rk ℓ^2 $\xrightarrow{\sim}$ RHS ⇝ v.b. on $\overline{\mathcal{M}}$ of rk ℓ^2

- $\overset{\sim}{\to}$ in a formal nbhd of ∞ ($\Leftarrow$ classical theory of theta fns. and their derivatives)
- deg (v.b. on LHS) = deg (v.b. on RHS)

$\alpha \circlearrowright V$: v.s. / fld F (free module / comm. rg with 1)

$\det(\alpha) \in F$

V