FOUNDATIONS AND PERSPECTIVES OF ANABELIAN GEOMETRY,
RIMS WORKSHOP, JUNE 28–JULY 2 2021, BY ZOOM
SCHEDULE

Organisers: Benjamin Collas (RIMS), Ivan Fesenko (Univ. of Nottingham), Arata Minamide (RIMS), Fucheng Tan (RIMS)

June 28 2021 (Monday)

17:30-18:00 (Kyoto time) Opening

18:00-19:00 Akio Tamagawa *Developments of anabelian geometry of curves over finite fields* 11

19:15-20:15 Shinichi Mochizuki *The technique of tripodal transport in arithmetic geometry* 6

20:30-21:30 Arata Minamide *The Grothendieck–Teichmüller group and the outer automorphism groups of the profinite braid groups* 5

June 29 2021 (Tuesday)

17:30-18:00 Q&A

18:00-19:00 Hiroaki Nakamura *On arithmetic and geometry around the adelic Eisenstein function* 8

19:15-20:15 Shinichi Mochizuki *The technique of tripodal transport in arithmetic geometry* 6

20:30-21:30 Fedor Bogomolov *Birational geometry and group theory* 1

June 30 2021 (Wednesday)

17:30-18:00 Q&A

18:00-19:00 Yuichiro Hoshi *The absolute anabelian geometry of quasi-tripods* 3

19:15-20:15 Emmanuel Lepage *The absolute anabelian conjecture for curves with resolution of non-singularities* 4

20:30-21:30 Yu Yang *Moduli spaces of fundamental groups of curves in positive characteristic* 14

July 1 2021 (Thursday)

17:30-18:00 Q&A

18:00-19:00 Takahiro Murotani *A p-adic analytic approach to the absolute Grothendieck conjecture* 7

19:15-20:15 Wojciech Porowski *Pro-p anabelian geometry of punctured elliptic curves* 9

20:30-21:30 Shota Tsujimura *Combinatorial Belyi cuspidalization and its applications* 12

July 2 2021 (Friday)

17:30-18:00 Q&A

18:00-19:00 Koichiro Sawada *On surjective homomorphisms from a configuration space group to a surface group* 10

19:15-20:15 Shota Tsujimura *Anabelian group-theoretic properties of the absolute Galois groups of higher local fields* 13

20:30-21:30 Kazumi Higashiyama *The mono-anabelian geometry of geometrically pro-p arithmetic fundamental groups of second configuration spaces* 2

TITLES AND ABSTRACTS

1. **Fedor Bogomolov.** *Birational geometry and group theory*

I discuss birational invariants of algebraic varieties which can be derived from the structure of the corresponding Galois groups. I also discuss some related conjectures concerning the structure of the Galois groups and their subgroups.

2. **Kazumi Higashiyama.** *The mono-anabelian geometry of geometrically pro-p arithmetic fundamental groups of second configuration spaces*

The n-th configuration space of a hyperbolic curve is the scheme which parametrizes n-tuples of pairwise distinct points in the hyperbolic curve. Mochizuki proved the Grothendieck conjecture for hyperbolic curves. We discuss a certain pro-p version of the Grothendieck conjecture for hyperbolic curves. In this talk, we reconstruct group-theoretically the function field of a hyperbolic curve of type $(0,3)$ from the pro-p fundamental group of the associated second configuration space equipped with the collection of decomposition groups.

3. **Yuichiro Hoshi.** *The absolute anabelian geometry of quasi-tripods*

A quasi-tripod is defined to be a hyperbolic orbicurve such that there exists a chain consisting of elementary operations (i.e., the operations of passing to a finite etale covering, passing to a finite etale quotient, de-cuspidalization, and de-orbification) whose initial orbicurve is the given orbicurve and whose terminal orbicurve is a tripod. In particular, a hyperbolic orbicurve isogenous to a hyperbolic curve of genus zero gives an example of a quasi-tripod. In this talk, we discuss the absolute anabelian geometry of quasi-tripods. Moreover, we also discuss some applications to the absolute anabelian geometry of higher dimensional varieties.

4. **Emmanuel Lepage.** *The absolute anabelian conjecture for curves with resolution of non-singularities*

The absolute anabelian conjecture asks whether every isomorphism between the étale fundamental groups of two hyperbolic curves over p-adic fields comes from an isomorphism of curves. This was proved by S. Mochizuki for curves of quasi-Belyi type. In this talk, I will discuss an extension of this result to a wider family of curves.

5. **Arata Minamide.** *The Grothendieck–Teichmüller group and the outer automorphism groups of the profinite braid groups*

The Grothendieck–Teichmüller group GT is a profinite group closely related to (and possibly isomorphic to) the absolute Galois group of the field of rational numbers. In this talk, we will determine the outer automorphism groups of the profinite braid groups with four or more strings in terms of GT. This is joint work with Hiroaki Nakamura.

6. **Shinichi Mochizuki.** *The technique of tripodal transport in arithmetic geometry*

The technique of "tripodal transport" allows one to "transport" various deep arithmetic properties of the tripod (i.e., the projective line minus three points) to more general hyperbolic curves over arithmetic fields. This technique is closely related to various results in anabelian geometry. In this talk, we shall survey three important examples of this technique (and its relation to anabelian geometry), namely, p-adic Teichmüller theory, Belyi-type injectivity results for arbitrary hyperbolic curves, and inter-universal Teichmüller theory.

7. **Takahiro Murotani.** *A p-adic analytic approach to the absolute Grothendieck conjecture*

Consider a hyperbolic curve X over a finite extension of the field of p-adic numbers. Let $\pi_1(X)$ be its étale fundamental group. The p-adic absolute Grothendieck conjecture asks: Is it possible to recover X group-theoretically, solely from $\pi_1(X)$? To consider this problem, we introduce a certain p-adic analytic invariant defined by Serre (which we call i-invariant) and discuss the relation between the invariant and the problem.

8. **Hiroaki Nakamura.** *On arithmetic and geometry around the adelic Eisenstein function*

We introduce the adelic Eisenstein function arising from the monodromy representation in the universal family of punctured elliptic curves, and illustrate its basic aspects. If time allows, we discuss a new relation to some knot theoretic invariants.

9. **Wojciech Porowski.** *Pro-p anabelian geometry of punctured elliptic curves*

Let E be an elliptic curve over a local field of characteristic zero and let X be a hyperbolic curve of type $(1,1)$ obtained by removing the origin from E. Consider the geometrically pro-p étale fundamental group of X, where p is the characteristic of the residue field. In this talk we are going to study the problem whether this fundamental group determines the reduction type of the elliptic curve E. We will prove that under the assumption $p > 3$ the answer is positive.

10. **Koichiro Sawada.** *On surjective homomorphisms from a configuration space group to a surface group*

In this talk, we classify all surjective homomorphisms from the étale fundamental group of the configuration space of a hyperbolic curve (over an algebraically closed field of characteristic zero) to the étale fundamental group of a hyperbolic curve. More specifically, we show that such a surjective homomorphism factors through one of the homomorphisms which arise from certain morphisms of schemes. Moreover, we discuss an application to anabelian geometry.

11. **Akio Tamagawa.** *Developments of anabelian geometry of curves over finite fields*

This is a survey talk on anabelian geometry of curves over finite fields.

It will cover various topics, from Uchida's theorem for function fields in 1970s to several recent developments.

12. **Shota Tsujimura.** *Combinatorial Belyi cuspidalization and its applications*

I will introduce the theory of combinatorial Belyi cuspidalization, which is a certain combinatorial version of the theory of Belyi cuspidalization developed by S. Mochizuki in the context of absolute anabelian geometry. This combinatorial version is based on combinatorial anabelian geometry developed by Y. Hoshi and S. Mochizuki. I will also talk about the following applications of the theory of combinatorial Belyi cuspidalization:

(I) The construction of a splitting of the natural inclusion of the absolute Galois group of the field of p-adic numbers to the (largest) p-adic Grothendieck–Teichmüller group.

(II) Absolute version of the Grothendieck conjecture for higher dimensional (> 1) configuration spaces associated to hyperbolic curves of genus 0 over finite extensios of the maximal abelian extension of rationals. [This application is a consequence of recent joint work with Y. Hoshi and S. Mochizuki.]

13. **Shota Tsujimura.** *Anabelian group-theoretic properties of the absolute Galois groups of higher local fields*

Classically, it is well-known that the absolute Galois groups of various fields (e.g., p-adic local fields, number fields) appearing in anabelian geometry satisfy the slimness (i.e., every open subgroup is center-free) and elasticity (i.e., every nontrivial topologically finitely generated closed normal subgroup is open). In this talk, we will explain that the absolute Galois groups of higher local fields of mixed or positive characteristic also satisfy the slimness and elasticity. This is joint work with A. Minamide.

14. **Yu Yang.** *Moduli spaces of fundamental groups of curves in positive characteristic*

In this talk, I will explain some current developments in the theory of anabelian geometry of curves over algebraically closed fields of positive characteristic from the point of view of "Homeomorphism Conjecture" which was formulated by the speaker. This conjecture shows a new anabelian phenomenon which says that the moduli spaces of curves can be reconstructed group-theoretically as topological spaces from fundamental groups. In particular, I will give a sketch of my proof of Homeomorphism Conjecture when the dimension of $\overline{\mathscr{M}}_{g,n}$ is one.