

RIMS/Symmetries and Correspondences Workshop: Invitation to Inter-universal Teichmüller Theory

Organizers: Yuichiro Hoshi (RIMS, Kyoto University)
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Dates: August 31 – September 3, 2021

Place: RIMS, online

	August 31 (Tues.)	September 1 (Wed.)	September 2 (Thu.)	September 3 (Fri.)
16:30 – 17:30	Model Fr'ds. (Tsujimura)	Brauer cyc. rid. (Tsujimura)	[IUTchI] (Yang)	[IUTchIII] (Minamide)
17:50 – 18:50	Ess. Lgc. Str. (Mochizuki)	Mono-th. env. (Porowski)	[IUTchI] (Yang)	[IUTchIII] (Minamide)
19:10 – 20:10	Ess. Lgc. Str. (Mochizuki)	[AbsTopIII] (Higashiyama)	[IUTchII] (Porowski)	[IUTchIII] (Minamide)
20:30 – 21:30	Ess. Lgc. Str. (Mochizuki)	[AbsTopIII] (Higashiyama)	[IUTchII] (Porowski)	Q & A (Mochizuki)

Program

August 31 (Tuesday)

16:30 – 17:30 Shota Tsujimura (RIMS, Kyoto University)
Model Frobenioids

17:50 – 18:50 Shinichi Mochizuki (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory I

19:10 – 20:10 Shinichi Mochizuki (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory II

20:30 – 21:30 Shinichi Mochizuki (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory III

September 1 (Wednesday)

16:30 – 17:30 Shota Tsujimura (RIMS, Kyoto University)
Cyclotomic Rigidity via Brauer Groups

17:50 – 18:50 Wojciech Porowski (The University of Nottingham)
Étale theta function and cyclotomic rigidity

19:10 – 20:10 Kazumi Higashiyama (RIMS, Kyoto University)
Mono-anabelian geometry over sub- p -adic fields via Belyi cuspidalization

20:30 – 21:30 Kazumi Higashiyama (RIMS, Kyoto University)
Log-shell and Log-volume

September 2 (Thursday)

16:30 – 17:30 Yu Yang (RIMS, Kyoto University)
Constructions of Hodge Theaters I

17:50 – 18:50 Yu Yang (RIMS, Kyoto University)
Constructions of Hodge Theaters II

19:10 – 20:10 Wojciech Porowski (The University of Nottingham)
Evaluation of the theta function I

20:30 – 21:30 Wojciech Porowski (The University of Nottingham)
Evaluation of the theta function II

September 3 (Friday)

16:30 – 17:30 Arata Minamide (RIMS, Kyoto University)
Log-Theta Lattice: Symmetries and Indeterminacies I

17:50 – 18:50 Arata Minamide (RIMS, Kyoto University)
Log-Theta Lattice: Symmetries and Indeterminacies II

19:10 – 20:10 Arata Minamide (RIMS, Kyoto University)
Log-Theta Lattice: Symmetries and Indeterminacies III

20:30 – 21:30 Shinichi Mochizuki (RIMS, Kyoto University)
Q & A

Abstracts

Speaker: Kazumi Higashiyama

Title: Mono-anabelian geometry over sub- p -adic fields via Belyi cuspidalization

Abstract: In this talk, we study mono-anabelian geometry. In more concrete terms, we prove the following assertion ([AbsTopIII], Theorem 1.9): Let k_0 be a number field, $k \supseteq k_0$ a sub- p -adic field, $\bar{k}$ an algebraic closure of k , and U_0/k_0 a hyperbolic curve which is isogenous to a hyperbolic curve of genus zero. Write $\bar{k}_0$ for the algebraic closure of k_0 in $\bar{k}$. Then we reconstruct group-theoretically the function field $\text{Funct}(U_0 \times_{k_0} \bar{k}_0)$ from

$$1 \rightarrow \pi_1(U_0 \times_{k_0} \bar{k}) \rightarrow \pi_1(U_0 \times_{k_0} k) \rightarrow \text{Gal}(\bar{k}/k) \rightarrow 1$$

(regarded as an exact sequence of abstract profinite groups) via the technique of Belyi cuspidalization.

$$\pi_1(U_0 \times_{k_0} k) \twoheadrightarrow \text{Gal}(\bar{k}/k) \rightsquigarrow \text{Funct}(U_0 \times_{k_0} \bar{k}_0)$$

Speaker: Kazumi Higashiyama

Title: Log-shell and Log-volume

Abstract: Let k be an MLF and $\bar{k}$ an algebraic closure of k . It is known that we can reconstruct the multiplicative group $\bar{k}^\times$ from the absolute Galois group G_k , despite we cannot reconstruct the field structure of $\bar{k}$.

$$G_k \rightsquigarrow G_k \curvearrowright \bar{k}^\times : \text{the multiplicative group}, \quad G_k \not\curvearrowright \bar{k} : \text{the field}$$

Therefore, we consider a new field $\bar{k}^\sim$ whose additive structure is derived from the multiplicative structure of $\bar{k}$, and we reconstruct the new field $\bar{k}^\sim$ from $\bar{k}$.

$$G_k \curvearrowright \bar{k} : \text{the field} \rightsquigarrow G_k \curvearrowright \bar{k}^\sim : \text{a field}$$

Then we define “log-volume” (notion essentially corresponding to Haar measure) and “log-shell” (a subgroup of $\bar{k}^\sim$ normalizing the log-volume). These are conventionally defined using the additive structure.

$$G_k \curvearrowright \bar{k} : \text{the field} \rightsquigarrow \text{the log-shell} \subseteq \bar{k}^\sim, \text{ the log-volume}$$

Finally, we reconstruct the additive group $\bar{k}^\sim$, the log-shell, and the log-volume from G_k , despite we cannot reconstruct the field structure of $\bar{k}^\sim$.

$$\begin{aligned} G_k \rightsquigarrow G_k \curvearrowright \bar{k}^\sim : \text{the additive group}, \\ \rightsquigarrow \text{the log-shell} \subseteq \bar{k}^\sim, \text{ the log-volume} \end{aligned}$$

Speaker: Arata Minamide

Title: Log-Theta Lattice: Symmetries and Indeterminacies I, II, III

Abstract: In this series of three talks, we will discuss multiradial algorithms — i.e., algorithms that make sense from the point of view of an “alien arithmetic holomorphic structure” — for constructing log-shells, theta values, and number fields.

Speaker: Shinichi Mochizuki

Title: On the essential logical structure of inter-universal Teichmüller theory I, II, III

Abstract: Introductory lectures and expositions on inter-universal Teichmüller theory — such as, for instance, [Alien] — have a tendency to concentrate on exposing the technical details surrounding the various mathematical objects that appear in the theory. To a certain extent, of course, this is unavoidable. On the other hand, concentrating on such technical details can lead to a situation where one is overwhelmed with seemingly meaningless technicalities to such an extent that one loses sight of the essential logical structure of the theory. The purpose of this series of talks is to discuss this essential logical structure of the theory, as exposed in [EssLgcIUT].

Speaker: Wojciech Porowski

Title: Étale theta function and cyclotomic rigidity

Abstract: In this talk we will construct the étale theta function, which is an (étale version of) a certain analytic function on an infinite tempered cover of a Tate curve over a local field. We will also introduce the notion of a mono-theta environment.

Speaker: Wojciech Porowski

Title: Evaluation of the theta function I

Abstract: This talk is based on the theory of IUT2. Our main topic will be Galois theoretic evaluation of theta function at torsion points. We will discuss the notions of multiradiality and conjugate synchronization.

Speaker: Wojciech Porowski

Title: Evaluation of the theta function II

Abstract: We continue the study of IUT2. We will introduce Gaussian monoids and globalize constructions introduced in the previous talk. We will also give Frobenioid theoretic versions of various notions appearing in this talk.

Speaker: Shota Tsujimura

Title: Model Frobenioids

Abstract: In, S. Mochizuki's Inter-universal Teichmüller Theory (IUT), various notions that appear [such as Hodge theater which is a central object in IUT] are introduced in terms of Frobenioids. In this talk, we discuss basic definitions and examples of model Frobenioids that appear in IUT.

Speaker: Shota Tsujimura

Title: Cyclotomic Rigidity via Brauer Groups

Abstract: In anabelian geometry or Inter-universal Teichmüller Theory, various types of cyclotomic rigidity [i.e., synchronization of cyclotomes] has been discussed. In this talk, we discuss one of them, i.e., a cyclotomic rigidity via the use of Brauer groups.

Speaker: Yu Yang

Title: Constructions of Hodge Theaters I, II

Abstract: I will explain Mochizuki's constructions concerning Hodge theaters which play a central role in his inter-universal Teichmüller theory. Roughly speaking, each Hodge theater may be considered as a miniature model of conventional scheme theory that simulates a situation in which the module of l -torsion points of the given elliptic curve over a number field admits a global multiplicative subspace.