

RIMS/Symmetries and Correspondences Workshop: Inter-universal Teichmüller Theory Summit 2021

Organizers: Yuichiro Hoshi (RIMS, Kyoto University)
Shinichi Mochizuki (RIMS, Kyoto University)
Ivan Fesenko (The University of Nottingham)
Yuichiro Taguchi (Tokyo Institute of Technology)
Benjamin Collas (RIMS, Kyoto University/The University of Lille)

Dates: September 7 – September 10, 2021

Place: RIMS, online

	September 7 (Tues.)	September 8 (Wed.)	September 9 (Thu.)	September 10 (Fri.)
16:30 – 17:30	Ess. Lgc. Str. (Mochizuki)	Multirad. & Est. (Hoshi)	Galois orbit (Hoshi)	Overview (Porowski)
17:50 – 18:50	Ess. Lgc. Str. (Mochizuki)	Hodge-Arak. (Yamashita)	Galois orbit (Hoshi)	BGT (Tsujimura)
19:10 – 20:10	Expl. estimates (Minamide)	Siegel zeroes (Táfula)	Mod. Nm. Th. (Fesenko)	BGT (Tsujimura)
20:30 – 21:30	Expl. estimates (Minamide)	Princ. of IUT (Collas)	Q & A (Mochizuki)	Q & A (Mochizuki)

Program

September 7 (Tuesday)

16:30 – 17:30 Shinichi Mochizuki (RIMS, Kyoto University)

On the essential logical structure of inter-universal Teichmüller theory IV

17:50 – 18:50 Shinichi Mochizuki (RIMS, Kyoto University)

On the essential logical structure of inter-universal Teichmüller theory V

19:10 – 20:10 Arata Minamide (RIMS, Kyoto University)

Explicit Estimates in Inter-universal Teichmüller Theory I

20:30 – 21:30 Arata Minamide (RIMS, Kyoto University)

Explicit Estimates in Inter-universal Teichmüller Theory II

September 8 (Wednesday)

16:30 – 17:30 Yuichiro Hoshi (RIMS, Kyoto University)

Multiradial representations and log-volume estimates

17:50 – 18:50 Go Yamashita (RIMS, Kyoto University)

A Motivation of Θ -link from Hodge-Arakelov theory

19:10 – 20:10 Christian Táfula Santos (University of Montreal)

From ABC to L: On singular moduli and Siegel zeroes

20:30 – 21:30 Benjamin Collas (RIMS, Kyoto University)

Principles of Inter-universal Teichmüller Geometry

September 9 (Thursday)

16:30 – 17:30 Yuichiro Hoshi (RIMS, Kyoto University)

On the Galois orbit version of inter-universal Teichmüller theory I: a progress report

17:50 – 18:50 Yuichiro Hoshi (RIMS, Kyoto University)

On the Galois orbit version of inter-universal Teichmüller theory II: a progress report

19:10 – 20:10 Ivan Fesenko (The University of Nottingham)

IUT and modern number theory

20:30 – 21:30 Shinichi Mochizuki (RIMS, Kyoto University)

Q & A

September 10 (Friday)

16:30 – 17:30 Wojciech Porowski (The University of Nottingham)

Overview of IUT theory

17:50 – 18:50 Shota Tsujimura (RIMS, Kyoto University)

Combinatorial construction of the absolute Galois group of the field of rational numbers
(I)

19:10 – 20:10 Shota Tsujimura (RIMS, Kyoto University)

Combinatorial construction of the absolute Galois group of the field of rational numbers
(II)

20:30 – 21:30 Shinichi Mochizuki (RIMS, Kyoto University)

Q & A

Abstracts

Speaker: Benjamin Collas

Title: Principles of Inter-universal Teichmüller Geometry

Abstract: Grothendieck's original functorial algebraic geometry of rings and schemes, once exposed to further classical questions as can be seen in the examples of the sheaf model category of motives and the group-theoretic anabelian geometry, crystallizes into new fields which then thrive on new techniques and new questions.

We here review some key techniques and principles of IUT geometry, a categorical monoidal weakened version of scheme theory whose symmetries reflect the anabelian geometric point-reconstruction of spaces in terms of Galois-rational data, which then results in a new kind of anabelian-diophantine geometry.

This talk should be considered as a modest and first attempt to throw a transversal light on IUT theory.

Speaker: Ivan Fesenko

Title: IUT and modern number theory

Abstract: I will discuss the potential of IUT and anabelian geometry to cause or influence developments in parts of modern number theory including the Langlands program, higher class field theory and arithmetic of elliptic curves.

Speaker: Yuichiro Hoshi

Title: Multiradial representations and log-volume estimates

Abstract: In this talk, I first give a brief review of the content of the main theorem of inter-universal Teichmüller theory. After the review, I also explain the relationship between the main theorem and an inequality of log-volumes.

Speaker: Yuichiro Hoshi

Title: On the Galois orbit version of inter-universal Teichmüller theory I, II: a progress report

Abstract: In these two talks, I explain some technical aspects of work in progress concerning the Galois orbit version of inter-universal Teichmüller theory, that may be regarded as a refinement of the theory, developed by Shinichi Mochizuki and the speaker.

Speaker: Arata Minamide

Title: Explicit Estimates in Inter-universal Teichmüller Theory I, II

Abstract: In the final paper of a series of papers concerning inter-universal Teichmüller theory, Mochizuki verified various numerically non-effective versions of the Vojta, ABC, and Szpiro Conjectures over number fields. In this series of two talks, we will give various numerically effective versions of Mochizuki's results. This is joint work with Shinichi Mochizuki, Ivan Fesenko, Yuichiro Hoshi, and Wojciech Porowski.

Speaker: Shinichi Mochizuki

Title: On the essential logical structure of inter-universal Teichmüller theory IV, V

Abstract: Introductory lectures and expositions on inter-universal Teichmüller theory — such as, for instance, [Alien] — have a tendency to concentrate on exposing the technical details surrounding the various mathematical objects that appear in the theory. To a certain extent, of course, this is unavoidable. On the other hand, concentrating on such technical details can lead to a situation where one is overwhelmed with seemingly meaningless technicalities to such an extent that one loses sight of the essential logical structure of the theory. The purpose of this series of talks is to discuss this essential logical structure of the theory, as exposed in [EssLgcIUT].

Speaker: Wojciech Porowski

Title: Overview of IUT theory

Abstract: In this talk we will give a brief overview of the structure of IUT theory and explain a role played by individual papers. We will also discuss various notions such as étale-like and Frobenius-like structures, holomorphic vs. mono-analytic structures and indeterminacies.

Speaker: Christian Táfula Santos

Title: From ABC to L: On singular moduli and Siegel zeroes

Abstract: In 2000, using analytical, algebraic, and arithmetical ideas, Granville and Stark showed that the “uniform” ABC for number fields implies that odd Dirichlet L-functions have no “Siegel zeroes”, which are a severe type of (not yet unconditionally ruled out) counterexample to the Generalized Riemann Hypothesis. In this talk we are going to focus on the structure of their main argument, and discuss recent work that allows us to get more precise relations between the analysis (zero-free regions of L-functions) and the arithmetics (heights of singular moduli).

Speaker: Shota Tsujimura

Title: Combinatorial construction of the absolute Galois group of the field of rational numbers (I), (II)

Abstract: In my talk in the workshop “Combinatorial Anabelian Geometry and Related Topics”, we gave the definition of a purely combinatorial/group-theoretic candidate BGT for the absolute Galois group of the field of rational numbers in the Grothendieck-Teichmüller group and constructed an algebraic closure of the field of rational numbers associated to BGT. In this talk, by imposing further purely combinatorial/group-theoretic conditions (“AA-property”) on BGT, we discuss a purely combinatorial/group-theoretic construction of an algebraic closure of the function field of a tripod associated to such BGT. As a consequence of this construction, we characterize the conjugacy class of the absolute Galois group of the field of rational numbers in the Grothendieck-Teichmüller group as the maximal closed subgroups satisfying AA-property. The construction of the function field of a tripod associated to BGT is closely related to the construction of function fields of hyperbolic curves of strictly Belyi type that has appeared in a S. Mochizuki’s previous work and played a key role in his inter-universal Teichmüller theory. This is joint work with Y. Hoshi and S. Mochizuki.

Speaker: Go Yamashita

Title: A Motivation of Θ -link from Hodge-Arakelov theory

Abstract: I will explain a motivation of Θ -link from the point of view of Hodge-Arakelov theory.