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From: 望月新一 <shinichimochizuki1@gmail.com>

Date: 2022 年 12 月 30 日(金) 22:18

Subject: Invitation to reopen dialogue concerning IUT

To: Jakob Stix <stix@math.uni-frankfurt.de>

Cc: 望月新一 <motizuki@kurims.kyoto-u.ac.jp>, <faltings@mpim-bonn.mpg.de>, <scholze@math.uni-bonn.de>

Dear Professor Stix (CC: Professor Faltings, Professor Scholze),

Roughly six months have passed since my last e-mail (a PDF version of which is available at the following [link](#), for which the password will be sent to you in a subsequent e-mail) to you on June 30, 2022. I continue to hope that my e-mails find you and your colleagues in healthy and productive circumstances.

My last e-mail to you (on June 30, 2022) was addressed to you in your capacity as the de facto "corresponding author" for the 10pp. manuscript that you sent to me in (May/August) 2018, since this arrangement (i.e., with you serving as the "corresponding author" for the manuscript) was, to my understanding, the arrangement that we had between March and August 2018. This time, however, I am also sending (CC'ing) my e-mail to Professor Scholze (i.e., in his capacity as co-author of the manuscript), as well as to Professor Faltings, since the impression that I got from a recently aired (in April 2022) NHK television documentary on IUT was that Professor Scholze was not interested in responding to inquiries from the outside world concerning your manuscript, and that Professor Faltings was acting as a sort of senior mentor/representative for Professor Scholze in case of such inquiries.

I must say that I was most disappointed by the fact that you have not responded to my e-mail of June 30, 2022. I have consulted with numerous colleagues concerning this situation. One colleague reassured me that, considering your character, he was sure that you would respond in due time and advised me simply to wait patiently. I responded to this advice from my colleague by saying that, while I appreciated his advice, it was by no means clear to me that just waiting patiently would lead to meaningful results.

One colleague that I consulted with was Benjamin Collas. I understand that you had a Zoom meeting with him subsequent to receiving my e-mail on June 30, 2022. I am certainly not familiar with all of the details of your Zoom meeting with Benjamin Collas, but I understand that you responded to his inquiries by making the following points, which, indeed, have frequently appeared in various quotes in the mass media attributed to Professors Scholze and/or Faltings:

(R1) Nothing has changed in recent years concerning IUT.

(R2) Mochizuki (as well as other mathematicians involved with IUT) continue(s) to refuse to explain IUT in an understandable fashion.

(R3) Mochizuki (as well as other mathematicians involved with IUT) continue(s) to refuse to address the main points made in your 10pp. manuscript.

I think that it must be stated clearly that each of these assertions (R1), (R2), (R3) is simply **factually incorrect**, as I intend to discuss in detail in the remainder of this e-mail. Moreover, none of these assertions (R1), (R2), (R3) constitutes any sort of justification whatsoever for refusing to engage in **mathematical dialogue** concerning any mathematical assertions that you might have concerning IUT, that is to say, in accordance with article (6.)

"Mathematicians should not make public claims of potential new theorems or the resolution of particular mathematical problems unless they are able to provide full details in a timely manner."

of the subsection entitled "Responsibilities of authors" of the [EMSCOP](#) (i.e., the Code of Practice of the European Mathematical Society), as I emphasized in my e-mail of June 30, 2022.

Before proceeding, I think that it would be appropriate to review what appear to be the two **key points** of your 10pp. manuscript:

(KP1) "In order for a meaningful inequality to be concluded, one must consistently identify all of these." (l. -2 of p. 9 of the August 2018 version);

(KP2) "with the simplifications outlined above, such as identifying identical copies of objects along the identity, the critical [IUTT-3, Theorem 3.11] does not become false, but trivial." (footnote 12 of the August 2018 version).

Here, the identification of various copies of \mathbb{R} (the real numbers) asserted in (KP1) may be regarded as a sort of weakened version of the identification of Hodge theaters asserted in (KP2). As I emphasized in my e-mail of June 30, 2022, I **completely agree** that (KP2), or its weakened version (KP1), leads to a contradiction/completely meaningless situation. The point is that imposing the identifications of either (KP1) or (KP2) yields a situation that is **non-isomorphic to/structurally completely different from** and, moreover, **logically unrelated** to the situation treated in IUT. This is explained in detail in [\[EssLgc\]](#), § 3. That is to say, in a word,

the logical AND/OR structure of the situation that arises from imposing the identifications of either (KP1) or (KP2) is **completely different** from and **logically unrelated** to the situation treated in IUT.

Judging from your assertions during our March 2018 discussions, as well as the contents of your 10pp. manuscript and subsequent blog posts of Professor Scholze, I must say that the only consistent explanation for your persistent assertion of the identifications of either (KP1) or (KP2) is that

your assertions are a reflection of a **fundamental misunderstanding** on your part concerning the logical AND/OR structure of IUT.

Put another way, you seem to be thinking in terms of what I call the "OR approach", or "diagram commutativity approach", to IUT (cf. the discussion of [\[EssLgc\]](#), Examples 3.10.1, 3.10.2), in which one takes the point of view that

the "abstract" prime-strip data in the theta-link maps **either** to the q-pilot data **OR** to the theta-pilot data, and there does not seem to be any reason that one can conclude any relationship between these two possibilities (q-pilot and theta-pilot) for the output "concrete" prime-strip data.

Indeed, this is precisely what you seem to be expressing in the hexagonal diagram in the argument of the final 2pp. of your 10pp. manuscript. Again, as I have emphasized countless times, I **completely agree** that if one takes this "OR approach"/"diagram commutativity approach" to IUT, then one does indeed obtain a completely meaningless situation. The point is that this is simply **not** how the argument proceeds in IUT. Rather, in IUT, one takes the "AND approach", i.e., in which, precisely by working with **distinct labels** (which are **never identified** with one another!) for the domain/codomain of the theta-link, one obtains a situation in which

the "abstract" prime-strip data in the theta-link always maps
tautologically to both the q-pilot data in the codomain
AND, simultaneously, to the theta-pilot data in the domain.

Indeed, this "AND approach" is the content of the **gluing** constituted by the theta-link between the **distinctly labeled** Hodge theaters in the domain/codomain of the theta-link. One then gradually introduces **indeterminacies** which are relatively mild, but sufficient to allow one to **embed** both sides (i.e., the q-pilot data in the codomain of the theta-link and the theta-pilot data in the domain of the theta-link) in a **common container** while keeping the **gluing intact** (i.e., the fundamental AND relation discussed above!); the discrepancy between these two sides may then be computed quantitatively inside the common container and, in particular, bounded from above.

In the final portion of your 10pp. manuscript, you say that you cannot understand this argument involving indeterminacies. On the other hand, as I explained in my e-mail of June 30, 2022, this set-up in IUT is **qualitatively completely similar** to such well-known elementary examples as

- the gluing of two affine lines along a once-punctured affine line to form the projective line,
or,

- over the complex numbers, the metric geometry of the sphere, regarded as the result of gluing together the northern and southern hemispheres along the equator

(cf. [EssLgc], Example 2.4.7). Alternatively, the set-up in IUT is **qualitatively completely similar** to

- the classical theory of **crystals** on a scheme X , i.e., where one obtains a relationship between the two pull-backs of some sheaf on X to a product of two copies of X --- not by arbitrarily identifying the two copies of X (!!), but rather --- by embedding the two pull-backs in the **common container** constituted by a suitable PD-envelope

(cf. the discussion of [\[Alien\]](#), § 3.1, (v); the discussion of (RdVar) in [EssLgc], § 3.1; the discussion of [EssLgc], § 3.5, § 3.10; the discussion of [EssLgc], Example 3.10.2). Here, it is interesting to note that this analogy between multiradiality in IUT and the classical theory of crystals is particularly noteworthy in that it is compatible with the classical proof of the geometric version of the Szpiro inequality via the Kodaira-Spencer morphism (cf. the discussion of [Alien], § 3.1, (v)).

Over the past 10 years since the release of the preprints on IUT, the **essential logical structure** of IUT --- i.e., as exposed in numerous talks and week-long workshops on IUT, as well as in the survey manuscripts [Alien] and [EssLgc] --- has been thoroughly understood by quite a number (roughly, 10 to 20) of mathematicians (cf. (R1)!). This includes European-educated mathematicians such as Mohamed Saidi (educated in France), Emmanuel Lepage (educated in France), and Wojciech Porowski (educated in Poland and the UK). In this context, it is worth mentioning in addition the following important developments (cf. (R1)!):

- the videos of the 2021 anabelian/IUT workshops, which are available at the following URL

[[https://www.***]]

username: video

password: (will be sent to you in a subsequent e-mail)

lecture notes: [WS1](#) [WS2](#) [WS3](#) [WS4](#)

(where I should remind you again that this URL/username/password should **NOT** be made available to people other than the recipients of this e-mail!);

- the publication of the four main IUT papers [IUTchI-IV] in the special issue of PRIMS with a special Editorial Board consisting, in particular, of such highly internationally respected mathematicians as M. Kashiwara, A. Tamagawa, S. Mukai, H. Nakajima, and T. Mochizuki;

- the publication of the sequel [\[ExpEst\]](#) to [IUTchI-IV] on explicit estimates in IUT in the Kodai Math. J.;

- the official establishment of the

International Center for Research in Next-Generation Geometry

(for which the webpage is still in preparation) here at RIMS, Kyoto University, which is devoted to promoting research activities in "next-generation geometry", **centering on IUT**;

- the recent approval of the [AHGT](#) (Arithmetic and Homotopic Galois Theory) program led by B. Collas, P. Dèbes, A. Mézard, and Y. Hoshi for the years 2023 - 2027, a program that is centered at Lille University (France) and RIMS, Kyoto University (Japan), supported by the French CNRS, and devoted to promoting increased interaction between French (and more generally European) and Japanese researchers in various fields of arithmetic geometry, **explicitly including** (though by no means limited to!) IUT.

The **essential logical structure** of IUT referred to above may be explained in terms of closely related classical mathematical ideas and theories such as

- the gluing of the projective line/sphere (as discussed above);

- the classical theory of crystals (as discussed above);

- the classical theory of the functional equation of the theta function on the upper half-plane (cf., e.g., [EssLgc], Example 3.3.2);

- a special case (i.e., in the case of elliptic curves) of the argument of Faltings' proof of the invariance of the height under isogenies (cf., e.g., [EssLgc], Example 3.2.1),

all of which have been well-known/well-understood for many decades throughout the international arithmetic geometric community. In particular,

there is **absolutely no reason** that you should be unable to understand this essential logical structure of IUT

(cf. (R2)!). Moreover, I have spent a tremendous amount of time and effort over the last 4+ (almost 5!) years discussing your 10pp. manuscript in quite substantial technical detail with numerous colleagues! As I have stated many times previously,

there is a **categorically unanimous** consensus among all of these colleagues that there are most certainly

no "unaddressed issues"

in your 10pp. manuscript, and that the argument (involving (KP1), (KP2)) in the final 2pp. of your 10pp. manuscript amounts to nothing more than a **fundamental misunderstanding** on your part of the AND/OR logical structure of IUT

(cf. (R3)!). Here, it should be noted that this unanimous consensus includes (European-educated) mathematicians such as Emmanuel Lepage, who (as discussed in more detail in my e-mail of June 30, 2022) took a rather skeptical/negative position concerning IUT prior to his acknowledgement in the fall of 2021 that he had misunderstood IUT, and that he no longer had any mathematical reason not to acknowledge the mathematical validity of IUT.

In this context, it should also be noted that in general, in mathematics, when asserting some sort of **logical relationship** between a **new version** of a mathematical theory and the original version of the theory,

the **burden of proof** of the existence of such a logical relationship lies solely in the hands of the **proponents** of the new version of the theory, i.e., (most certainly!) **not** in the hands of the author of the original version of the theory

(cf. the discussion of [EssLgc], § 1.10, § 1.12). That is to say, as I have emphasized countless times, I **completely agree** that the identifications of either (KP1) or (KP2) --- i.e., the "OR approach"/"diagram commutativity approach" to IUT --- lead(s) to a meaningless situation. The **burden of proof**, however, of any sort of logical relationship between this "OR approach"/"diagram commutativity approach" to IUT, on the one hand,

and the actual "AND approach"-based content of IUT, on the other hand, lies entirely ****in your hands****. No such proof of a logical relationship can be found either in our March 2018 discussions, in your 10pp. manuscript, or in subsequent blog posts due to Professor Scholze. The only sort of "justification" that I have been able to find --- despite quite a formidable investment of time and effort, both in internet searches and in discussions with numerous colleagues! --- is statements such as

- "I don't see why not" or

- "we are sure" that "restoring all the details" would not have any effect on the argument in the final 2pp. of your 10pp. manuscript.

Needless to say, this sort of statement does not by any means or any stretch of the imagination constitute a ****mathematical proof**** of the sort envisaged in the passage from the EMSCOP quoted above.

As emphasized in my e-mail of June 30, 2022, the importance of providing an explicit, written proof with ****full details****, in accordance with the EMSCOP passage quoted above --- i.e., in short,

the issue of the existence/nonexistence of a genuine mathematical ****proof**** of any asserted ****logical relationship**** between the situation described in your 10pp. manuscript and the actual argument of IUT

--- cuts to the ****very core**** of the discipline of mathematics and has highly serious ****historical implications**** for the future development of the field of mathematics (cf. the discussion of [EssLgc], § 1.5). In my e-mail of June 30, 2022, I mentioned the examples of

- Fermat's unsubstantiated assertion of a proof of "Fermat's last theorem" and

- Deligne's unsubstantiated (and subsequently explicitly withdrawn!) assertion of a proof that the Section Conjecture over number fields implies the Mordell Conjecture.

Another interesting example in this context may be seen in

- certain technical errors that have been found in Faltings' theory of almost étale extensions.

In the case of this last example, I should mention that I was most impressed by the experience that I had when I was a graduate student at Princeton University in the late 1980's - early 1990's: when I pointed out one of these technical errors in the theory of almost étale extensions to Professor Faltings, he immediately acknowledged the error without any further ado and, to my recollection, even went on to say to me that "this teaches me not to trust authority"!

In general, when sorting out such situations, there is absolutely **no justification** for the sort of venomous, defamatory remarks concerning me and my work that have appeared on the internet and in the English mass media during the last decade. Rather it is in the interests of **all parties** --- and indeed in the interests of the mathematical community as a whole --- to maintain a **strictly mathematical** stance, motivated by **pure intellectual curiosity** and a deeply shared sense that we are all fighting on the **same side** (i.e., not fighting against some opposing faction!) for the common goal of elucidating mathematical truth. In particular, with regard to the **fundamental issue** that I continue to emphasize, i.e.,

the fundamental importance of your providing an explicit, written proof with **full details**, in accordance with the EMSCOP passage quoted above, of any asserted logical relationship between

- the "OR approach"/"diagram commutativity approach" to IUT of your 10pp. manuscript, on the one hand, and

- the actual "AND approach"-based content of IUT, on the other hand

--- or (if the truth of the matter is that in fact you simply do **not** have a proof of such a logical relationship!) providing **explicit acknowledgment** to the effect that you do not have a proof of such a logical relationship --- I would like to take the opportunity to **strongly encourage** you to ponder the content of the famous Latin proverb

VERITAS VOS LIBERABIT

("the truth will liberate you").

Before concluding this e-mail, I would like to emphasize once again (as I did in my e-mail of June 30, 2022) that I am always available (modulo scheduling issues arising from numerous other commitments) for discussions/mathematical dialogue concerning the content of this e-mail via e-mail or Zoom. Moreover, you (and your colleagues) are always most welcome (modulo possible scheduling or financial support issues) to visit RIMS to discuss mathematical topics of mutual interest, if you are interested.

Finally, I would like to conclude the present e-mail by observing that

- continued **refusal** on your (i.e., the authors!) part to take a **mathematical attitude** with regard to the content of your 10pp. manuscript by complying with the EMSCOP passage mentioned above,

- in favor of a **political attitude**, in which the logical relationship between IUT and the mathematical content of the 10pp. manuscript is left unaddressed, and the 10pp. manuscript is used (as it has been used for the last 4 to 5 years!) not as a mathematical document to stimulate further **mathematical dialogue** concerning its mathematical content, but rather as a **political tool/weapon** for the purpose of **politically suppressing** mathematical dialogue concerning an established mathematical theory that has been published in a well established journal with an editorial board of highly internationally respected mathematicians and well absorbed/digested/understood by a substantial community of professional mathematicians for over a decade

has had, and will continue to have, over the long term, the effect of slowly but steadily --- much in the fashion of **climate change**! --- tearing apart the **cultural and moral fabric** that underlies the **operational normalcy** of the international mathematical community, and indeed, more generally, international society as a whole. Cooperation with regard to taking such a **mathematical attitude to established mathematical results**, as opposed to a **political attitude directed toward political suppression**, does not require any technical knowledge in any specific field of mathematics whatsoever (i.e., such as anabelian geometry or IUT) and is in the **vital interest** of every professional mathematician, as well as in the general interest of civilized society as a whole.

Sincerely,

Shinichi Mochizuki