

So what is IU $\mathcal{T}$ eich about?  
...an explanation via  
“Sokkuri animation”



The starting point of “IU $\mathcal{T}$ eich” (i.e., Inter-universal Teichmüller Theory) lies in the image of a

**sequence of nested universes.**

This sort of image apparently goes back to ancient times and appears not only in the “Sokkuri Hausu” (i.e., “Identical House”) animation discussed here, but in various stories and myths all over the world. In the case of IU $\mathcal{T}$ eich, the various universes correspond to

**“classical arithmetic geometry theaters  
in which conventional ring theory/scheme theory is valid”.**

In the “Sokkuri animation”, these universes are represented by “houses”. Within each classical arithmetic geometry theater, one has a **theta function**; it is this theta function that plays the role of “Frobenioid-theoretic” (i.e., non-scheme-theoretic! — cf. [Frobenioids I, II](#); [Étale Theta](#)) “bridge” to the “next universe”. In the “Sokkuri animation”, this link furnished by the

**theta function corresponds to  
the gaze of the little girl into the “small house”.**

Indeed, the large eyes of the little girl look somewhat like thetas  $\Theta$ ! In IU $\mathcal{T}$ eich, **Galois groups** and **arithmetic fundamental groups** behave as though they are made of a “mysterious substance” that allows them to pass freely, in an isomorphic fashion, between the various universes without being subject to expansion or contraction. In the “Sokkuri animation”, this “mysterious substance” corresponds to the **“mysterious stars”** that form the link between the “small” and “large” houses. The

**rotation of these “mysterious stars”**

may be thought of, relative to IU $\mathcal{T}$ eich, as representing the nature of the “étale-like structures” that occur in the theory of Frobenioids (cf. [Frobenioids I, II](#)) or, alternatively, as corresponding to the **rotation of addition and multiplication** that occurs in the mono-anabelian theory of [Topics III](#). The deepest and most pivotal portion of IU $\mathcal{T}$ eich consists of a certain **canonical splitting** of the sequence of nested universes, a splitting which is constructed by means of the

**mono-anabelian theory of Topics III** and the  
**various rigidity properties of the étale theta function**

(cf. [Étale Theta](#) for more on the latter). This canonical splitting corresponds to the **canonical liftings** of  $p\mathcal{T}$ eich (i.e.,  $p$ -adic Teichmüller theory) or, at a more elementary level, the **Teichmüller representatives of a Witt ring**. (Incidentally, in the  $p$ -adic theory, the various “universes” of IU $\mathcal{T}$ eich correspond to the various positive characteristic algebraic geometries contained in the various subquotients “ $p^n/p^{n+1}$ ”.)