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A. 研究概要

2010 年度は前年度に引き続き、離散群と関数解析についての研究を行った。 $\varepsilon > 0$ に対して、群 Γ から距離付き位相群 U への写像 $\pi: \Gamma \rightarrow U$ が ε -準同型であるとは、任意の $g, h \in \Gamma$ に対して $d(\pi(gh), \pi(g)\pi(h)) < \varepsilon$ が成り立つときをいう。 ε -準同型が本物の準同型の摂動であるか否かを問うのが Ulam の問題 (1960) である。特に、 U として Hilbert 空間 H 上のユニタリ作用素全体 $U(H)$ に作用素ノルムで距離を入れたものを考える。この場合の Ulam の問題は、従順群に対しては肯定的に解けること、しかし一般には反例があることが知られていた (Kazhdan, 1982)。私は M. Burger (ETHZ) 及び A. Thom (Leipzig) との共同研究 [8] において以下の事実を明らかにした: 自由群を部分群として含む任意の群に対して反例が構成できるが、高階数単純 Lie 群の格子 (これらは自由群を部分群として含む) などの場合は有限次元の反例は存在しない。その他にも関連するいくつかの定量的な研究を行った。他にも弱従順性の研究 [9] を行い、先行する Haagerup (1988) の結果と Ozawa–Popa (2010) の結果を統合・拡張する成果を得た。

In the academic year 2010, N. Ozawa studied functional analytic aspects of discrete groups. A map $\pi: \Gamma \rightarrow U$ from a group Γ into a metric group U is called an ε -homomorphism if it satisfies $d(\pi(gh), \pi(g)\pi(h)) < \varepsilon$ for all $g, h \in \Gamma$. Ulam (1960) asked whether an ε -homomorphism is a perturbation of an honest homomorphism. Here we focus on a particularly interesting case where U is the group $U(H)$ of unitary operators on a Hilbert space H , equipped with the norm metric. A counterexample to this Ulam problem was given by Kazhdan (1982), who also provided a positive solution in the case of Γ being amenable. N. Ozawa ([8]) in collaboration with M. Burger (ETHZ) and A. Thom (Leipzig) proved that a counterexample can be constructed for any group containing a noncommutative free group, but certain groups, e.g. lattices in $SL_{n \geq 3}(\mathbb{R})$, do not admit counterexamples of finite dimension. They also did some qualitative study of related problems.

Other researches of N. Ozawa include a study of weak amenability ([9]), which generalized the works of Haagerup (1988) and Ozawa–Popa (2010).

B. 発表論文

1. N. P. Brown and N. Ozawa; “ C^* -algebras and finite-dimensional approximations,” Graduate Studies in Mathematics, 88. American Mathematical Society, Providence, RI, 2008. xvi+509 pp.
2. N. Ozawa; “Weak amenability of hyperbolic groups,” Groups Geom. Dyn., **2** (2008), 271–280.
3. N. Ozawa and S. Popa; “On a class of II_1 factors with at most one Cartan subalgebra,” Ann. of Math. (2), **172** (2010), 713–749.
4. N. Ozawa; “An example of a solid von Neumann algebra,” Hokkaido Math. J., **38** (2009), 557–561.
5. N. Ozawa and S. Popa; “On a class of II_1 factors with at most one Cartan subalgebra II,” Amer. J. Math., **132** (2010), 841–866.
6. N. Monod and N. Ozawa; “The Dixmier problem, lamplighters and Burnside groups,” J. Funct. Anal., **258** (2010), 255–259.
7. N. Ozawa; “Quasi-homomorphism rigidity with noncommutative targets,” J. reine angew. Math., to appear.
8. M. Burger, N. Ozawa and A. Thom; “On Ulam stability,” preprint.
9. N. Ozawa; “Examples of groups which are not weakly amenable,” preprint.

C. 口頭発表

1. *von Neumann algebras and ergodic theory* (Minicourse); (1) von Neumann algebras, Ergodic theory and Geometric Group theory, IMSc (Chennai), February 09. (2) Ergodic Theory of Group Actions, Göttingen, August 09.

2. *Dixmier's Similarity Problem*; (1) 東北大学情報数理談話会, April, 09. (2) Noncommutative L_p spaces, operator spaces and applications, CIRM, June 09. (3) 東大作用素環セミナー, July 09. (4) Geometry and Rigidity of Groups, Münster, August 09. (5) Operators and Operator Algebras, Edinburgh, December 09. (6) 東大数理談話会, December 09.
3. *Hyperlinearity, sofic groups and applications to group theory* (Mini Course); Operator Spaces and Approximation Properties of Discrete Groups, Texas A&M University, August 09.
4. *Quasi-homomorphism rigidity with non-commutative targets*; (1) Rigidity in cohomology, K -theory, geometry and ergodic theory, HIM (Bonn), November 09. (2) Colloquium at University of Hawaii, March 10. (3) Danish-Norwegian Operator Algebra Seminar, Copenhagen, April 10. (4) Geometry and Ergodic Theory Seminar, EPFL, April 10. (5) Recent Developments in Operator Algebras, 東京大学, June 10. (6) Satellite Conference on Operator Algebras to ICM 2010, IMSc (Chennai), August 10. (7) Séminaire d'Algèbres d'Opérateurs, Institut de Mathématiques de Jussieu, September 10. (8) AMS Fall Western Section Meeting, UCLA, October 10.

D. 講義

1. 解析学 IV (理数・3年)・解析学特別演習 I: 測度論と Lebesgue 積分論. 演習つき.
2. 数学 II (理 I・1年)・数学 II 演習: 線形代数, 通年.

E. 修士・博士論文

1. (博士) 見村 万佐人 (MIMURA, Masato): Rigidity theorems for universal and symplectic universal lattices.

F. 对外研究サービス

1. 学術雑誌「Groups, Geometry, and Dynamics」の editor.
2. アメリカ数学会秋季西地区学会 (UCLA, 10月) の「Special Session on Rigidity in von Neumann Algebras and Ergodic Theory」の organizer.

G. 受賞

1. ICM 招待講演 (Operator Algebras and Functional Analysis), 2006 年 8 月.
2. 春季賞 (日本数学会), 2009 年 4 月.
3. 日本学術振興会賞, 2010 年 3 月.