Kobe Workshop on Probabilistic Potential Theory and Related Fields

Dates: 7 May 2019 (Tuesday) 09:00 – 8 May 2019 (Wednesday) 17:00
Venue: Room B428-430, Faculty of Science Building B, Kobe University
Organizer: Naotaka Kajino (Kobe University)

Schedule

May 7 (Tuesday)		May 8 (Wednesday)	
09:00–13:00	Free Discussion	09:00–10:00	Free Discussion
		10:00–10:40	Ryokichi Tanaka
		10:50–11:30	Yuki Tokushige
		11:40–12:30	Mathav Murugan
13:00–13:40	Masayoshi Takeda	12:30–13:30	(Lunch Break)
13:50–14:30	Daisuke Shiraishi	13:30–14:10	Kazumi Okamura
14:50–15:40	Omer Angel	14:20–15:00	David Croydon
15:50–16:30	Yoshihiro Abe	15:20–16:00	Kouhei Matsuura
16:40–17:10	Ryosuke Shimuzu	16:10–16:50	Hiroaki Aikawa

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- JSPS Grant-in-Aid for Scientific Research (B) Grant Number 17H02849
  「空間の構造と解析の関わりーフラクタルを出発点としてー」
  (Principal Investigator: Jun Kigami (Graduate School of Informatics, Kyoto University))
- JSPS Grant-in-Aid for Scientific Research (B) Grant Number 18H01123
  「分野横断的視点によるフラクタル及びその上の確率過程の解析・幾何」
  (Principal Investigator: Naotaka Kajino (Graduate School of Science, Kobe University))

Program

7 May 2019 (Tuesday)

- 9:00–13:00 Free Discussion
- 13:00–13:40 Masayoshi Takeda (Kansai University)
  Criticality theory and maximum principle for generalized Schrödinger forms
- 13:50–14:30 Daisuke Shiraishi (Kyoto University)
  Convergence of three-dimensional loop-erased random walk in the natural parametrization

(Tea break for 20 minutes)

- 14:50–15:40 Omer Angel (University of British Columbia)
  Scaling limits of self interacting random walks
- 15:50–16:30 Yoshihiro Abe (Chiba University)
  Exceptional points of two-dimensional random walks at multiples of the cover time
- 16:40–17:10 Ryosuke Shimizu (Kyoto University)
  Generalized resistance metrics on graphs

8 May 2019 (Wednesday)

- 9:00–10:00 Free Discussion
- 10:00–10:40 Ryokichi Tanaka (Tohoku University)
  Cutoff for product replacement on finite groups
- 10:50–11:30 Yuki Tokushige (Kyoto University)
  Jump processes on the Gromov boundary of a hyperbolic group: Probabilistic interpretation of Bourdon–Pajot's Besov spaces
- 11:40–12:30 Mathav Murugan (University of British Columbia)
  Stability of the elliptic Harnack inequality

(Lunch break for 60 minutes)

- 13:30–14:10 Kazuki Okamura (Shinshu University)
  Some results for range of random walk on graph with spectral dimension two
- 14:20–15:00 David Croydon (Kyoto University)
  Quenched and averaged tails of the heat kernel of the two-dimensional uniform spanning tree

(Tea break for 20 minutes)

- 15:20–16:00 Kouhei Matsuura (Kyoto University)
  Tightness property of reflecting Brownian motions on horn-shaped domains
- 16:10–16:50 Hiroaki Aikawa (Chubu University)
  Potential theoretic notions related to integrability of superharmonic functions and supertemperatures

(End of Workshop)

Abstracts

Criticality theory and maximum principle for generalized Schrödinger forms

Masayoshi Takeda (Kansai University)

We define three function spaces related to a Schrödinger form and its semigroup: two of these are spaces of excessive functions defined through the Schrödinger semigroup, one is the space of weak subsolutions defined through the Schrödinger form. We define the maximum principle for each space and prove the equivalence of three maximum principles. Moreover, we give a necessary and sufficient condition for each maximum principle in terms of the principal eigenvalue of time-changed processes.

Convergence of three-dimensional loop-erased random walk in the natural parametrization

Daisuke Shiraishi (Kyoto University)

In this talk, we consider loop-erased random walk (LERW) and its scaling limit in three dimensions, and prove that 3D LERW parametrized by renormalized length converges to its scaling limit parametrized by some suitable measure with respect to the uniform convergence topology in the lattice size scaling limit. This talk is based on joint work with Xinyi Li (University of Chicago).

Scaling limits of self interacting random walks

Omer Angel (University of British Columbia)

We consider several self interacting random walks in the plane. A central example is the random walk in $\mathbb{Z}^2$ that moves vertically on its first visit to a site, and horizontally on subsequent moves. It is conjectured that this random walk is recurrent. We prove that its scaling limit is a one-dimensional Brownian motion. Joint with Mark Holmes and Alejandro Ramirez.

Exceptional points of two-dimensional random walks at multiples of the cover time

Yoshihiro Abe (Chiba University)

I will talk about the local time of the continuous-time simple random walk on a large finite subset of the two-dimensional lattice with the wired boundary condition. The running time of the walk is parametrized by the local time at the "boundary vertex" and approximated by a multiple of the cover time. The talk will focus on statistics of points where the local times are atypically large, called thick points. This is based on joint work with Marek Biskup (UCLA).

Generalized resistance metrics on graphs

Ryosuke Shimizu (Kyoto University)

Albin, Clemens, Fernando and Poggi-Corradini (2018) defined a new metric associated with p-energy, where the metric is the resistance metric if $p = 2$. In their work, a duality theorem for moduli and a connection between moduli and conductances played important roles. On the other hand, a similar duality problem has been studied by Nakamura and Yamasaki (1976). In this talk, we will apply Nakamura–Yamasaki's result to extend the result by Albin et al. to the case of infinite graphs.

Cutoff for product replacement on finite groups

Ryokichi Tanaka (Tohoku University)

We analyze a Markov chain, known as the product replacement chain, on the set of generating n-tuples of a fixed finite group G. Based on joint work with Yuval Peres and Alex Zhai, we show that as $n \to \infty$, the total-variation mixing time of the chain has a cutoff at time $(3/2)n \log n$ with window of order n. This generalizes a result of Ben-Hamou and Peres (who established the result for $G = \mathbb{Z}/2$) and confirms a conjecture of Diaconis and Saloff-Coste that for an arbitrary but fixed finite group, the mixing time of the product replacement chain is $O(n \log n)$.

Jump processes on the Gromov boundary of a hyperbolic group: Probabilistic interpretation of Bourdon–Pajot's Besov spaces

Yuki Tokushige (Kyoto University)

In this talk, we will discuss how Besov spaces on a compact metric space introduced by Bourdon-Pajot arise in the context of random walks on a hyperbolic group and the theory of Markov processes. Specifically, under an assumption on the Ahlfors-regular conformal dimension, we will prove the regularity of Dirichlet forms defined by those Besov spaces, and give to them a concrete probabilistic description by means of random walks. This talk is based on a joint work with P. Mathieu (Aix-Marseille Université).

Stability of the elliptic Harnack inequality

Mathav Murugan (University of British Columbia)

Harnack inequalities have proved to be a powerful tool in PDE (regularity estimates), geometry (geometric flows) and probability (heat kernel estimates). In the early 1990s Grigor'yan and Saloff-Coste gave a characterization of the parabolic Harnack inequality (PHI). This characterization implies that PHI is stable under perturbations (rough isometries). In this talk, I will provide an introduction to Harnack inequalities and discuss the stability of the elliptic Harnack inequality. The proof uses a quasisymmetric transformation of the space and suggests a variant of the quasisymmetric uniformization problem. This is joint work with Martin Barlow.

Some results for range of random walk on graph with spectral dimension two

Kazuki Okamura (Shinshu University)

We consider the range of the simple random walk on graphs with spectral dimension two. We give a form of strong law of large numbers under a certain uniform condition, which is satisfied by not only the square integer lattice but also a class of fractal graphs. Our results imply the strong law of large numbers on the square integer lattice established by Dvoretzky and Erdos. Our proof does not depend on spacial homogeneity of space and gives a new proof of the strong law of large numbers on the lattice. As an application, we establish a form of law of the iterated logarithms for lamplighter random walks in the case that the spectral dimension of the underlying graph is two. If time is permitted, we will discuss an extension to the framework of stable random walks.

Quenched and averaged tails of the heat kernel of the two-dimensional uniform spanning tree

David Croydon (Kyoto University)

I will describe ongoing joint work (with M. T. Barlow and T. Kumagai) in which a detailed investigation of the heat kernel associated with the two-dimensional uniform spanning tree is conducted. This demonstrates the occurrence of log-logarithmic fluctuations around the leading order polynomial behaviour for the on-diagonal part of the quenched heat kernel. Moreover, two-sided estimates are given for the averaged heat kernel. Notably, these demonstrate a discrepancy between the exponents that appear in the off-diagonal parts of the quenched and averaged versions of the heat kernel, and also yield precise asymptotics for the average distance travelled by the associated random walk. Finally, we derive various scaling limits for the heat kernel, the implications of which include enabling us to sharpen the known asymptotics regarding the on-diagonal part of the averaged heat kernel.

Tightness property of reflecting Brownian motions on horn-shaped domains

Kouhei Matsuura (Kyoto University)

We consider a tightness property for symmetric Hunt processes. One dimensional diffusions possess the tightness property if and only if they have no natural boundaries. Therefore, symmetric Hunt processes with tightness property are similar to one-dimensional diffusions having no natural boundaries. In this talk, we give sufficient conditions for reflecting Brownian motions on unbounded domains to possess the tightness property. We also discuss their spectral and ergodic properties.

Potential theoretic notions related to integrability of superharmonic functions and supertemperatures

Hiroaki Aikawa (Chubu University)

Ever since Armitage proved that every nonnegative superharmonic function in a bounded domain $D \subset \mathbb{R}^n$ of bounded curvature is L^p-integrable on the whole D for $0 < p < n/(n-1)$, the global integrability of nonnegative supersolutions has attracted many mathematicians. Illustrating related potential theoretic notions, we show how this problem has been settled and extended to the parabolic case.