Kyoto-CAU Joint Meeting on Nonlinear PDEs

Date: March 29 (Wed) – March 30 (Thu), 2023

Venue: Hotel Kanazawa Kenrokusou (Kanazawa, Japan)

– Program –

March 29 (morning)	March 30 (morning)
9:00 – 10:00 <**Opening**> **Kyungkeun Kang (Yonsei University)** Local regularity of weak solution for the Stokes and Navier-Stokes equations near boundary **Hiroyuki Tsurumi (Kyoto University)** Existence of the stationary Navier-Stokes flow in $\mathbb{R}^2$ around a radial flow	9:00 – 10:00 **Shota Sakamoto (Tokyo Institute of Technology)** Global solution to the Boltzmann equation without cutoff on the whole space in $(L^1 \cap L^p)_k$ **In-Jee Jeong (Seoul National University)** On vorticity supported on logarithmic spirals
10:10 – 11:10 **Soonsik Kwon (KAIST)** Blow up solutions of radial self-dual Chern-Simons-Schrödinger equations **Masaya Maeda (Chiba University)** Small energy stabilization for 1D Nonlinear Klein Gordon Equations	10:10 – 11:10 **Kyudong Choi (UNIST)** On vortex stretching for anti-parallel axisymmetric flows **Stephen Gustafson (University of British Columbia)** Growth rates for some axisymmetric Euler flows
11:20 – 11:50 **Hyunseok Kim (Sogang University)** Gagliardo-Nirenberg inequalities in Sobolev-Lorentz spaces and their applications to PDEs	11:20 – 12:20 **Tatsu-Hiko Miura (Hirosaki University)** Nonlinear stability of the two-jet Kolmogorov type flow on the unit sphere under a perturbation with nondissipative part **Jihoon Lee (Chung-Ang University)** Quasi-geostrophic approximation for the rotating Boussinesq equations with strong stratification
11:50 – 13:30 <**Lunch**>	12:20 – 14:00 <**Lunch**>

March 29 (afternoon)	March 30 (afternoon)
13:30 – 14:30 **Yong-Kum Cho (Chung-Ang University)** Sturm's oscillation theory and spirals **Sonae Hadama (Kyoto University)** Stability of steady states for the Hartree equation for random fields	14:00 – 18:00 **<Open Discussion>** **<Closing>**
14:40 – 15:40 **Younghun Hong (Chung-Ang University)** The NLS approximation for the nonlinear Klein-Gordon equation **Jumpei Kawakami (Kyoto University)** Averaging of strong magnetic nonlinear Schrödinger equations in energy space	
15:50 – 16:50 **Yoshihiro Ueda (Kobe University)** Analysis of the delay effect for the viscous Burgers equation **Moon-Jin Kang (KAIST)** Long-time behavior of H^1-perturbations from Riemann data for Navier-Stokes-Fourier system	
17:00 – 18:00 **Keisuke Takasao (Kyoto University)** On volume preserving mean curvature flow in higher dimensions **Jaeyoung Byeon (KAIST)** Solvability of the Born-Infeld model for general sources	
19:00 – 21:00 **<Banquet>**	

– Abstract –

Solvability of the Born-Infeld model for general sources

Jaeyoung Byeon (KAIST)

The solvability of the Born-Infeld equation for general sources is quite nontrivial due to the presence of a singular term in the equation. I would like to introduce some recent developments for the solvability and regularity for the Born-Infeld equation.

Sturm's oscillation theory and spirals

Yong-Kum Cho (Chung-Ang University)

In connection with the differential equation of type

$$u'' + \phi(t)u = 0, \quad t > 0,$$

Sturm's oscillation theory describes oscillatory nature of solutions. In this talk we present how Sturm's theory leads to an abstract formulation of spirals. As illustrations, we take Bessel's equation and investigate the corresponding spirals in detail. In particular, we construct families of spirals which fill the interior or exterior of arbitrary ellipses without interfacing.

On vortex stretching for anti-parallel axisymmetric flows

Kyudong Choi (UNIST)

We consider axisymmetric incompressible inviscid flows without swirl in R^3 under the assumption that the axial vorticity is non-positive in the upper half space and odd in the last coordinate, which corresponds to the flow setup for head-on collision of anti-parallel vortex rings. For any such data, we establish monotonicity and infinite growth of the vorticity impulse on the upper half-space. As an application, we achieve infinite growth of Sobolev norms for certain classical/smooth and compactly supported vorticity solutions in R^3. This is joint work with In-Jee Jeong(SNU).

Growth rates for some axisymmetric Euler flows

Stephen Gustafson (University of British Columbia)

We discuss certain solutions of the Euler equations of fluid mechanics in three (and higher) dimensions which describe, roughly speaking, colliding antiparallel vortex tubes. Inspired by recent work of K. Choi and I. Jeong, we present some rigorous upper and lower bounds on the growth of the vorticity. This is joint work with Evan Miller and Tai-Peng Tsai.

Stability of steady states for the Hartree equation for random fields

Sonae Hadama (Kyoto University)

I will talk about the Hartree equation describing the time evolution of the wave functions of infinitely many fermions interacting with each other. The Hartree equation can be formulated in terms of random field. This formulation was introduced by de Suzzoni(2015). It has infinitely many steady states, and its asymptotic stability has been studied. However, the previous results required somewhat strong conditions for steady states. Indeed, the stability of Fermi gas at zero temperature, which is important from a physics point of view, was left open. This talk discusses the asymptotic stability of steady states in a wide class that allows Fermi gas at zero temperature. Moreover, in the previous studies, the function spaces to which solutions belong are strictly wider than those to which initial data belong. This talk discusses that we can match them if we assume a strong condition for potential function.

The NLS approximation for the nonlinear Klein-Gordon equation

Younghun Hong (Chung-Ang University)

The nonlinear Schrödinger equation (NLS) is a universal model equation describing the envelope dynamics of slowly modulating small wave packets. By multi-scaling analysis, it arises from various Hamiltonian systems even including the water wave equation. In this talk, we consider the NLS approximation for the one-dimensional cubic nonlinear Klein-Gordon equation (NLKG), which is the simplest expository textbook example in Schneider-Uecker. For this problem, we present a new approach based on Fourier analysis methods for dispersive PDEs, and show that the NLS can be derived for a larger class of solutions. This talk is based on joint work with Seokchang Hong.

On vorticity supported on logarithmic spirals

In-Jee Jeong (Seoul National University)

We study logarithmic spiraling solutions to the 2d incompressible Euler equations which solve a nonlinear transport system on the unit circle. We show that this system is locally well-posed in L^p with any $1 \leq p \leq \infty$ as well as for atomic measures, that is logarithmic spiral vortex sheets. We prove global well-posedness for bounded logarithmic spirals as well as data that admit at most logarithmic singularities. Within symmetry we show that the vortex sheet limit holds locally in time. We give a complete characterization of the long time behavior of logarithmic spirals. This is due to the observation that the local circulation of the vorticity around the origin is a strictly monotone quantity of time. We are then able to show a dichotomy in the long time behavior, solutions either blow up (in finite or infinite time) or completely homogenize. In particular, bounded logarithmic spirals converge to constant steady states. For vortex logarithmic spiral sheets, the dichotomy is shown to be even more drastic where only finite time blow up or complete homogenization of the fluid can and does occur. This is joint work with A. Said (Duke University).

Local regularity of weak solution for the Stokes and Navier-Stokes equations near boundary

Kyungkeun Kang (Yonsei University)

We are concerned with local regularity of weak solutions for Stokes system near boundary. There exist weak solutions of the Stokes system whose normal derivatives are unbounded near boundary in a half space. Similar construction can be made for the Navier-Stokes equations as well.

Long-time behavior of H^1-perturbations from Riemann data for Navier-Stokes-Fourier system

Moon-Jin Kang (KAIST)

I will present the latest result on long-time behavior of solutions to the 1D compressible Navier-Stokes-Fourier system with initial H^1-perturbation of Riemann data. Especially, we consider the Riemann data generating Riemann solution composed of shock, rarefaction and contact discontinuity. We use the so-called "a-contraction with shifts" method, which is basically energy based, and can seamlessly handle the superposition of waves of different kinds.

Averaging of strong magnetic nonlinear Schrödinger equations in energy space

Jumpei Kawakami (Kyoto University)

We consider two nonlinear Schrödinger-type models which R L. Frank, F. Méhats, C. Sparber (2017) derived to study nonlinear Schrödinger equations with strong magnetic fields in 3 dimensions. One model is derived by spatial scaling and the other model is obtained by averaging the spatial scaled model in time. We study these models in energy space to obtain global solutions and improve the convergence result to arbitrarily long time. In the case of nonic nonlinear power of the time averaged model, we prove a scattering result under a scaling-invariant small-energy condition, which indicates energy-criticality of the nonic case.

Gagliardo-Nirenberg inequalities in Sobolev-Lorentz spaces and their applications to PDEs

Hyunseok Kim (Sogang University)

It has been recently shown that the classical Gagliardo-Nirenberg interpolation inequalities can be refined by using weak L^p-norms. The goal of the talk is to present further refinements via general Lorentz spaces. We provide interpolation inequalities in Sobolev-Lorentz spaces of arbitrary orders, as special cases of more general results on Triebel-Lizorkin-Lorentz spaces. Then as an application to PDEs, we study global weak solutions to a nonlinear elliptic-parabolic system with fractional diffusions.

Blow up solutions of radial self-dual Chern-Simons-Schrödinger equations

Soonsik Kwon (KAIST)

We consider the self-dual Chern-Simons-Schrödinger equations under radial symmetry (or equivariant index $m = 0$). The radial soliton Q with the least mass has slow spatial decay. Due to this, the blow-up solution obtained by the pseudoconformal transform of Q have infinite energy. One can naturally ask what happens to blow-up solutions in energy class. In this talk, I will present the construction of blow-up solutions from smooth and localized data. It turns out the story goes differently from high equivariant cases ($m \geq 1$). Also, I will add more discussion on possible blow-up rates. This talk is based on joint works with Kihyun Kim(IHES) and Sung-Jin Oh(UC Berkeley).

Quasi-geostrophic approximation for the rotating Boussinesq equations with strong stratification

Jihoon Lee (Chung-Ang University)

We consider quasi-geostrophic approximation for 3D rotating stratified Boussinesq equations in $\mathbb{R}^3$. The quasi-geostrophic approximation for rotating stratified fluids has been conjectured to be invalid in the sense that the convergence rate becomes singular near certain rotation-stratification ratio while the original Boussinesq dynamics changes continuously in that ratio. Such a paradox, called Devil's staircase convergence results, was originally cast by Babin-Mahalov-Nicolaenko-Zhou in Theor. Comput. Fluid Dyn. 9, 223-251 (1997). We discuss the mathematical proof of the Devil's staircase paradox in $\mathbb{R}^3$ by showing that the 3D rotating stratified inviscid Boussinesq flows do not converge to the quasi-geostrophic flows either when the rotation-stratification ratio is fixed to be one or when the ratio tends to one in the regime of slowly increasing intensities of rotation and stratification. As a consequence, we quantitatively prove the blow-up of the obtained convergence rate towards the quasi-geostrophic dynamics. This is the joint work with Min-jun Jo(Univ. Britisch Columbia) and Junha Kim(KIAS).

Small energy stabilization for 1D Nonlinear Klein Gordon Equations

Masaya Maeda (Chiba University)

In this talk, we give a partial extension to dimension 1 of the result proved by Bambusi and Cuccagna on the absence of small energy real valued periodic solutions for the NLKG in dimension 3. We combine the framework in Kowalczyk and Martel with the notion of "refined profile".

Nonlinear stability of the two-jet Kolmogorov type flow on the unit sphere under a perturbation with nondissipative part

Tatsu-Hiko Miura (Hirosaki University)

We consider the nonlinear stability of the two-jet Kolmogorov type flow which is a stationary solution to the vorticity equation on the 2D unit sphere given by the zonal spherical harmonic function of degree two. Our interest is in the long time behavior of a perturbation containing a nondissipative part which is a linear combination of the spherical harmonics of degree one. We show that the nondissipative part of a perturbation is preserved in time and that the dissipative part converges exponentially towards an equilibrium which is expressed explicitly in terms of the nondissipative part of the initial data.

Global solution to the Boltzmann equation without cutoff on the whole space in $(L^1 \cap L^p)_k$

Shota Sakamoto (Tokyo Institute of Technology)

We consider a Cauchy problem of the Boltzmann equation without angular cutoff near the global Maxwellian. When a spatial domain is the torus, it is proved that the problem has a unique solution for small data in the Wiener space, the collection of functions whose Fourier coefficients are absolutely summable. Due to its Banach-algebra property, we do not need the Sobolev embedding on this space.

Following this result, we consider the problem on the whole space. In this case, the control of the L^1 norm on the Fourier side is not sufficient due to low-frequency terms. Therefore, inspired by Kawashima-Nishibata-Nishikawa's work on the viscous conservation laws, we employ the L^p norm estimates with respect to the frequency to control such parts. This $L^1 \cap L^p$ strategy will close a priori estimates when combined with a time-weighted energy method.

This work is based on a joint work with Renjun Duan (CUHK) and Yoshihiro Ueda (Kobe University).

On volume preserving mean curvature flow in higher dimensions

Keisuke Takasao (Kyoto University)

In this talk, we show a global existence of the weak solution (family of integral varifolds) to the volume preserving mean curvature flow in the d-dimensional torus, where $d \geq 2$. This flow is also a distributional BV-solution for a short time, when the perimeter of the initial data is sufficiently close to that of ball with the same volume. To construct the flow, we use the Allen-Cahn equation with non-local term motivated by studies of Mugnai, Seis, and Spadaro (2016), and Kim and Kwon (2020).

Existence of the stationary Navier-Stokes flow in $\mathbb{R}^2$ around a radial flow

Hiroyuki Tsurumi (Kyoto University)

In this talk, we treat the stationary Navier-Stokes equations on the 2D whole plane, and show the existence of a classical solution for a given small and smooth external force around a radial flow. The advantage is that external forces do not necessarily have to be symmetrical. For construction of a solution, we analyze the vorticity-streamfunction system using the polar coordinates and Fourier series. This talk is based on a joint work with Yasunori Maekawa (Kyoto University).

Analysis of the delay effect for the viscous Burgers equation

Yoshihiro Ueda (Kobe University)

The viscous Burgers equation is well known as a simple equation describing fluid phenomena, and it is also known as a mathematical model of traffic flow. As a model of traffic flow, we consider the one-dimensional Cauchy problem of the generalized viscous Burgers equations with a time delay and analyze a delay effect. Indeed, because the viscous Burgers equation is a parabolic partial differential equation, its solution has an infinite speed of propagation. In terms of traffic flow, this means that drivers and their vehicles are assumed to react instantly changing the density and its gradient. In order to improve this troubling feature, we modify the term without a time delay to the one with a time delay.

In this talk, we show the existence of the global in time solution when the product of the size of the delay parameter and the one of the initial history is suitably small. Moreover, we also prove some theorem concerning with the regularity of the global in time solution.

This result is a joint research with Takayuki Kubo of Ochanomizu University.