Sharp global well posedness for the non-elliptic derivative Schrödinger equation

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Abstract. We consider the following DNLS:

$$\mathrm{i}u_t - \Delta_{\pm} u = F(u, \bar{u}, \nabla u, \nabla \bar{u}), \quad u(0,x) = u_0(x), \tag{0.1}$$

where $\Delta_{\pm} = \partial_{x_1}^2 \pm ... \pm \partial_{x_n}^2$. $F(z) = O(|z|^{\alpha})$ with $\alpha \geqslant 3$ for $n \geqslant 2$ and $\alpha \geqslant 4$ for $n = 1$. Applying the Gabor frame, we get some time-global dispersive estimates for the Schrödinger semi-group in anisotropic Lebesgue spaces. By resorting to the smooth effect estimate together with the dispersive estimates in anisotropic Lebesgue spaces, we show that DNLS has a unique global solution if the initial data in modulation spaces and weighted Sobolev spaces are sufficiently small.

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