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# SHARP SPECTRAL STABILITY ESTIMATES FOR UNIFORMLY ELLIPTIC DIFFERENTIAL OPERATORS

We consider the eigenvalue problem for the operator

$$Hu = (-1)^m \sum_{|\alpha|=|\beta|=m} D^\alpha \left( A_{\alpha\beta}(x) D^\beta u \right), \quad x \in \Omega,$$

subject to homogeneous Dirichlet or Neumann boundary conditions, where $m \in \mathbb{N}$, $\Omega$ is a bounded open set in $\mathbb{R}^N$ and the coefficients $A_{\alpha\beta}$ are real-valued Lipschitz continuous functions satisfying $A_{\alpha\beta} = A_{\beta\alpha}$ and the uniform ellipticity condition

$$\sum_{|\alpha|=|\beta|=m} A_{\alpha\beta}(x)\xi_\alpha \xi_\beta \geq \theta |\xi|^2$$

for all $x \in \Omega$ and for all $\xi_\alpha \in \mathbb{R}, |\alpha| = m$, where $\theta > 0$ is the ellipticity constant.

We consider open sets $\Omega$ for which the spectrum is discrete and can be represented by means of a non-decreasing sequence of non-negative eigenvalues of finite multiplicity $\lambda_1[\Omega] \leq \lambda_2[\Omega] \leq \dots \leq \lambda_n[\Omega] \leq \dots$ Here each eigenvalue is repeated as many times as its multiplicity and $\lim_{n\to\infty} \lambda_n[\Omega] = \infty$.

The aim is sharp estimates for the variation $|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]|$ of the eigenvalues corresponding to two open sets $\Omega_1$, $\Omega_2$ with continuous boundaries, described by means of the same *fixed* atlas $\mathcal{A}$.

There is vast literature on spectral stability problems for elliptic operators. However, very little attention has been devoted to the problem of spectral stability for higher order operators and in particular to the problem of finding explicit qualified estimates for the variation of the eigenvalues.

Our analysis comprehends *operators of arbitrary even order*, *with homogeneous Dirichlet or Neumann boundary conditions*, and *open sets admitting arbitrarily strong degeneration.*

Three types of estimates will be under discussion: for each $n \in \mathbb{N}$ for some $c_n > 0$ depending only on $n$, $\mathcal{A}, m, \theta$ and the Lipschitz constant $L$ of the coefficients $A_{\alpha\beta}$

$$|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]| \leq c_n d_{\mathcal{A}}(\Omega_1, \Omega_2),$$

where $d_{\mathcal{A}}(\Omega_1, \Omega_2)$ is the so-called *atlas distance* of $\Omega_1$ to $\Omega_2$,

$$|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]| \leq c_n \omega(d_{\mathcal{HP}}(\partial\Omega_1, \partial\Omega_2)),$$

where $d_{\mathcal{HP}}(\partial\Omega_1, \partial\Omega_2)$ is the so-called *lower Hausdoff-Pompeiu deviation* of the boundaries $\partial\Omega_1$ and $\partial\Omega_2$ and $\omega$ is the common modulus of continuity of $\partial\Omega_1$ and $\partial\Omega_2$, and, under certain regularity assumptions on $\partial\Omega_1$ and $\partial\Omega_2$,

$$|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]| \leq c_n \text{meas}\left(\Omega_1 \Delta \Omega_2\right),$$

where $\Omega_1 \Delta \Omega_2$ is the symmetric difference of $\Omega_1$ and $\Omega_2$.

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