Finite-time blowup in the two-dimensional parabolic Keller-Segel system

We consider radially symmetric solutions of the fully parabolic Keller-Segel chemotaxis system

$$(*) \qquad \begin{cases} u_t = \nabla \cdot (\nabla u - u\nabla v) & \text{in } \Omega \times (0,\infty), \\ v_t = \Delta v - v + u & \text{in } \Omega \times (0,\infty) \end{cases}$$

in a disk $\Omega \subset \mathbb{R}^2$ under the Neumann boundary condition. Keller and Segel proposed this system as a model of aggregation of cellular slime molds moving towards higher concentration of a chemical substance produced by themselves. There are quite a number of papers on the finite-time blowup in a simplified system with the second equation replaced by an elliptic equation $0 = \Delta v - v + u$. However it has been an open question for several decades whether the blow-up in finite time is a generic feature in the original system $(*)$. In this talk, we give an affirmative answer to the question. We also show that any global classical solution of $(*)$ is uniformly bounded. This excludes the possibility of growup at time infinity. Based on this, a new blowup criterion for $(*)$ is provided, which is optimal with respect to the natural energy functional.

This is a joint work with M. Winkler of University of Paderborn.