# The Navier-Stokes System with Moving Boundaries in Unbounded Domains

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Consider the instationary (Navier-)Stokes system on an unbounded domain with moving boundary $\partial\Omega(t)$ and Dirichlet boundary conditions. Via a coordinate transform the problem reduces to a modified non-autonomous (Navier)-Stokes system

$$\partial_t u(t) + A(t)u(t) = P(t)F - P(t)u \cdot \nabla^{\phi(t)} u, \quad u(0) = u_0,$$

in a fixed reference domain $\Omega_0$. Here $A(t)$ is a $t$-dependent modified Stokes operator on $\Omega_0$, $P(t)$ a modified Helmholtz projection, and $\nabla^{\phi(t)}$ a $\phi(t)$-dependent gradient.

To solve the initial-boundary value problem or find time-periodic solutions in unbounded domains we have to construct the fundamental operator $\{U(t,s)\}$ of the non-autonomous system in case that the operators $A(t)$ are not boundedly invertible. The aim is to get estimates independent of the time parameter $t$ in $A(t)$, e.g. to get Sobolev embeddings for fractional Stokes operators $A(t)^\theta$ with $t$-independent bounds. The adjoint operators $A(t)^*$ will be analyzed for the same reasons. The final main problem is to get global-in-time estimates of the fundamental operator $\{U(t,s)\}$ and to establish sufficiently fast decay rates.

These steps are performed for bounded domains and mainly for the half space $\mathbb{R}^n_+$ with compact perturbations. The results are based on joint papers with K. Tsuda (Kyushu Sangyo University, Fukuoka).