Japan. J. Math. 16, 1–48 (2021)
DOI: 10.1007/s11537-020-1920-5

JAPANESE JOURNAL OF MATHEMATICS
SINCE 1924

Information geometry*

Shun-ichi Amari

Received: 10 September 2019 / Revised: 30 June 2020, 6 October 2020,
13 October 2020 / Accepted: 20 October 2020
Published online: 2 January 2021

Communicated by: Toshiyuki Kobayashi

Abstract. Information geometry has emerged from the study of the invariant structure in families of probability distributions. This invariance uniquely determines a second-order symmetric tensor g and third-order symmetric tensor T in a manifold of probability distributions. A pair of these tensors (g, T) defines a Riemannian metric and a pair of affine connections which together preserve the metric. Information geometry involves studying a Riemannian manifold having a pair of dual affine connections. Such a structure also arises from an asymmetric divergence function and affine differential geometry. A dually flat Riemannian manifold is particularly useful for various applications, because a generalized Pythagorean theorem and projection theorem hold. The Wasserstein distance gives another important geometry on probability distributions, which is non-invariant but responsible for the metric properties of a sample space. I attempt to construct information geometry of the entropy-regularized Wasserstein distance.

Keywords and phrases: canonical divergence, dual affine connection, information geometry, Pythagorean theorem, semiparametric statistics, Wasserstein geometry

Mathematics Subject Classification (2020): 53B12

* This article is based on the 23rd Takagi Lectures that the author delivered at Research Institute for Mathematical Sciences, Kyoto University on June 8, 2019.

S. Amari
RIKEN, Hirosawa, Wako-shi, Saitama, 351-0198 Japan
(e-mail: amari@brain.riken.jp)