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Infinite-dimensional (dg) Lie algebras and factorization algebras in algebraic geometry*

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Abstract. Infinite-dimensional Lie algebras (such as Kac–Moody, Virasoro etc.) govern, in many ways, various moduli spaces associated to algebraic curves. To pass from curves to higher-dimensional varieties, it is necessary to work in the setup of derived geometry. This is because many feature of the classical theory seem to disappear in higher dimensions but can be recovered in the derived (cohomological) framework. The lectures consist of 3 parts:

(1) Review of derived geometry and of the phenomenon of "recovery of missing features".
(2) The derived analog of the field of Laurent series in n variables ("with poles at a single point"). The corresponding higher current algebras and their relation to derived moduli spaces of G-bundles (based on joint work with G. Faonte and B. Hennion).
(3) Derived Lie algebras of vector fields, their central extensions and cohomology. Role of factorization algebras in studying such cohomology (based on joint with B. Hennion and work in progress with B. Hennion and A. Khoroshkin).

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