

## The lattice of varieties of monoids

Sergey V. Gusev · Edmond W.H. Lee · Boris M. Vernikov

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*In memory of Professor Lev N. Shevrin (1935–2021)*

**Abstract.** We survey results devoted to the lattice of varieties of monoids. Along with known results, some unpublished results are given with proofs. A number of open questions and problems are also formulated.

*Keywords and phrases:* monoid, variety, lattice of varieties

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S.V. GUSEV

Institute of Natural Sciences and Mathematics, Ural Federal University, Lenina str. 51,  
620000 Ekaterinburg, Russia  
(e-mail: [sergey.gusb@gmail.com](mailto:sergey.gusb@gmail.com))

E.W.H. LEE

Department of Mathematics, Nova Southeastern University, Fort Lauderdale,  
Florida 33314, USA  
(e-mail: [edmond.lee@nova.edu](mailto:edmond.lee@nova.edu))

B.M. VERNIKOV

Institute of Natural Sciences and Mathematics, Ural Federal University, Lenina str. 51,  
620000 Ekaterinburg, Russia  
(e-mail: [bvernikov@gmail.com](mailto:bvernikov@gmail.com))

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