

The structure of metahamiltonian groups

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Abstract. A group is called *metahamiltonian* if all its non-abelian subgroups are normal. The aim of this paper is to provide an exhaustive but self-contained reference to the structure of metahamiltonian groups fixing several relevant mistakes appearing in the literature.

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