The seventh Takagi Lectures

November 22, 2009 (Sun) 10:00–11:00
November 23, 2009 (Mon) 11:00–12:00
Graduate School of Mathematical Sciences
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Serre's conjecture and its consequences

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Abstract

I will give a historically motivated account of Serre's conjecture about mod p representations of the absolute Galois group of the rationals. This was proved by J.-P. Wintenberger and myself, together with a certain input of Kisin.

The context in which Serre made his conjecture was the work of Serre and Swinnerton-Dyer which explained the congruences Ramanujan had found for the Ramanujan τ-function. The explanation was via the study of images of Galois representations Deligne attached to the Ramanujan Δ-function (again conjectured by Serre). Here $\Delta(z) = q\Pi(1-q^n)^{24} = \sum_n \tau(n)q^n$ with $q = e^{2\pi iz}$.

I will also explain some of the consequences of Serre's conjecture. For instance it implies Artin's conjecture for 2-dimensional, complex, odd representations of the absolute Galois group of the rationals.

I will also talk about the ideas of the proof and some questions they lead to.