Triangle-free subgraphs

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August 26, 2013

Abstract

There are many problems dealing with triangle-free subgraphs both in extremal combinatorics and combinatorial optimization. One of the first results is due to Mantel who gave an upper bound on the maximum number of edges in a triangle-free graph on n nodes and also characterized the extremal cases. This talk gives an overview of results on triangle-free graphs, mainly concentrating on Turán numbers and Tuza's conjecture on minimum triangle covers.

Let M_k^r denote the set of r-element multisets over the set $\{1, \ldots, k\}$. We show that M_k^k has the so-called splitting property introduced by Ahlswede et al. Our approach gives a new interpretation of an earlier construction of Sidorenko.

For $r \le t \le n$, a Turán (n, t, r)-system is an r-uniform hypergraph on n nodes such that every t-element subset of the nodes spans at least one edge of the hypergraph. The Turán number $T(n, t, r)$ asks for the minimum size of such a family. We introduce a weighted version of Turán numbers and show that our construction for proving the splitting property for multisets is applicable to give an upper bound on weighted Turán numbers, matching previous bounds.

Finally, we also show how these results are connected to Tuza's conjecture and propose an extension of the conjecture to hypergraphs. As a hypergraphic counterpart of Krivelevich's result, we prove a fractional weakening of the conjecture.

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