

2nd Kyoto-Hefei Workshop on Arithmetic Geometry

Organizers: Yuichiro Hoshi (KU), Zhi Hu (JGU-M), Arata Minamide (KU), Yu Yang (KU), Runhong Zong (NJU)

Date: August 17, 2020–August 21, 2020 ([Japanese Local Time, GMT+9](#))

Zoom Link: Please contact yuyang@kurims.kyoto-u.ac.jp

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Schedule

August 17 (Mon)

15:15 – 15:30 Opening

15:30 – 16:30 Christian Tafula Santos (Kyoto University)

Title: ABC...L: The uniform abc -conjecture and zeros of Dirichlet L -functions

Abstract: In 2000, using a beautiful blend of analytical, algebraic, and arithmetical ideas, Granville and Stark showed that the uniform abc -conjecture for number fields implies that odd Dirichlet L -functions have no “Siegel zeros”, which are a severe type of (not yet unconditionally ruled out) counterexample to the Generalized Riemann Hypothesis. In this talk, we are going to discuss recent and ongoing work on the framework underlying this *bridge*, with particular emphasis on the direction that takes us from the analysis (zero-free regions of L -functions) to the arithmetics (heights of singular moduli).

16:45 – 17:45 Tomoki Yuji (Kyoto University)

Title: Category-theoretic reconstruction of schemes from categories of reduced schemes

Abstract: Let S be a quasi-separated normal Noetherian schemes and \bullet/S a set of properties of S -schemes. Then we shall write Sch_{\bullet}/S for the full subcategory of the category

of S -schemes Sch_\bullet/S determined by the objects $X \in \text{Sch}_\bullet/S$ that satisfy every property of \bullet/S . In this talk, we shall mainly be concerned with the properties “reduced”, “quasi-compact over S ”, “quasi-separated over S ”, and “separated over S ”. We give a functorial category-theoretic algorithm for reconstructing S from the intrinsic structure of the abstract category Sch_\bullet/S . This result is analogous to a result of Mochizuki and may be regarded as a partial generalization of a result of de Bruyn in the case where S is a quasi-separated normal Noetherian schemes.

18:00 – 19:00 Kang Zuo (Mainz University)

Title: Higgs bundle in p -adic nonabelian Hodge theory

Abstract: I shall overview recent progress on Higgs bundle in nonabelian Hodge theory and give a detailed discussion on deformation of local systems and Higgs bundles, which provides basic tools for proving Lefschetz hyperplane section theorem and the finiteness of the set of crystalline local systems (will be talked by J.B. Yang). This is a joint program with Raju Krishnamoorthy and Jinbang Yang.

August 18 (Tue)

15:30 – 16:30 Benjamin Collas (Kyoto University)

Title: Irreducibility in special loci with arithmetic motivations

Abstract: Characterizing irreducible components of Hurwitz spaces or special loci is a keystone in the stack arithmetic of moduli spaces of curves and in the regular inverse Galois problem. Classical methods rely on group theoretic methods (Fried et al.) from mapping class group (Broughton et al.) or complex algebraic geometry (Catanese et al.) and require a case-by-case study for even ”simple” groups or actions. This talk, based on a joined work with S. Maugeais, presents a new arithmetic-algebraic approach based on Grothendieck’s mixed étale cohomology. It produces, under a certain heuristic, some towers of irreducible components, and also illustrates a certain duality between rationality and irreducibility of the loci.

16:45 – 17:45 Ryoji Shimizu (Kyoto University)

Title: The Neukirch-Uchida theorem with restricted ramification

Abstract: The Neukirch-Uchida theorem is one of the most important results in anabelian geometry. In this talk, I will discuss the following generalization of the Neukirch-Uchida theorem under some assumptions: “For $i = 1, 2$, let K_i be a number field and S_i a set of primes of K_i . We write $K_{i,S_i}/K_i$ for the maximal extension of K_i unramified outside S_i and G_{K_i,S_i} for its Galois group. If G_{K_1,S_1} and G_{K_2,S_2} are isomorphic, then K_1 and K_2 are isomorphic.” The assumptions include: the Dirichlet density of S_i is not zero for $i = 1, 2$; K_i is Galois over \mathbf{Q} for $i = 1, 2$; K_1 is totally imaginary; and so on.

18:00 – 19:00 Yasuhiro Wakabayashi (Tokyo Institute of Technology)

Title: Frobenius-projective structures on higher dimensional varieties

Abstract: A projective structure on a complex manifold is a locally homogenous structure modeled on projective space. Such structures admit (local) projective geometries in the spirit of Klein's Erlangen program and have been investigated in the context of uniformization. In the case of curves, Y. Hoshi introduced a positive characteristic analogue of this notion, called a Frobenius-projective structure. In this talk, I would like to explain mainly my recent study concerning its higher dimensional generalization, while referring to some related topics and classical works of complex projective structures.

August 19 (Wed)

15:30 – 16:30 Jeehoon Park (Pohang University of Science and Technology)

Title: Higher Residue pairing for projective smooth complete intersections

Abstract: K. Saito introduced the concept of higher residue pairings for a universal unfolding space for isolated singularities. The Higher residue pairing is a quantum version of the cup product of the cohomologies for certain families of algebraic varieties, which is compatible with the Gauss-Manin connection. In this talk, we will present an algorithm to construct a higher residue pairing for a universal unfolding space for certain non-isolated singularities, which is closely related to deformations of projective smooth complete intersection varieties. Our algorithm also works for p -adic cohomologies of projective smooth complete intersection varieties over a finite field. Joint work with Jaehyun Yim.

16:45 – 17:45 Naganori Yamaguchi (Kyoto University)

Title: The m -step solvable Grothendieck conjecture for genus 0 curves over finitely generated field

Abstract: Grothendieck's anabelian conjecture is one of the most important problems in anabelian geometry. For curves, many results concerning this conjecture were obtained so far (Nakamura, Tamagawa, Mochizuki, et al). As a variant, we can also consider the m -step solvable Grothendieck conjecture. Briefly, this is the problem of reconstructing the isomorphism class of a curve from the maximal geometrically m -step solvable quotient of the arithmetic fundamental group of the curve. For this variant, previous results were obtained by Nakamura and Mochizuki. In this talk, I will outline the proof of my result on the m -step solvable Grothendieck conjecture for curves of genus 0 over fields finitely generated over the prime field.

18:00 – 19:00 Emmanuel Lepage (Institut de Mathematiques de Jussieu)

Title: Resolution of non-singularities and Absolute Anabelian Conjecture.

Abstract: My talk will be focused on the absolute anabelian conjecture of S. Mochizuki over finite extensions of \mathbf{Q}_p : given two hyperbolic curves over such fields, is every isomorphism between their tame fundamental groups induced by an isomorphism of curves? The conjecture is known to be true for curves of quasi-Belyi type. In this talk, I will extend this result to a wider class of curves, that includes for example Mumford curves.

August 20 (Thu)

15:30 – 16:30 Chunhui Liu (Kyoto University)

Title: Counting rational points, the determinant method and the pseudo-effective threshold

Abstract: The determinant method is widely applied in the problem of counting rational points with bounded height. In this talk, I will introduce this method by the reformulation of Arakelov geometry. The pseudo-effective thresholds of certain line bundles can be applied to describe some key invariants, and I will explain such a connection.

16:45 – 17:45 Haoyu Hu (Nanjing University)

Title: Semi-continuity of conductor divisors of ℓ -adic sheaves

Abstract: Let k be a perfect field of characteristic $p > 0$, let X be a smooth k -scheme, let D be a reduced Cartier divisor of X and let F be a locally constant and constructible étale sheaf of A -modules on $U := X \setminus D$, where A is a finite field with p invertible in A . A. Abbes and T. Saito's ramification theory starting from 2002 offers two slope decompositions for the ramification of F at the generic point of each irreducible component of D . The two versions of highest slopes defines two divisors with rational coefficients on X supported on D , called the conductor divisor of F and the logarithmic conductor divisor of F , respectively. In this talk, I will explain the semi-continuity property for the two divisors when D is a divisor with simple normal crossings. It is an ℓ -adic analogue of Y. André's result for Poincaré-Katz ranks of algebraic D -modules in 2007. If time permits, I will introduce an application on the study of ramification bound for nearby cycles and cohomology of ℓ -adic sheaves, which gives a positive answer to an I. Leal's conjecture (published in 2019) and extends a main result in a manuscript of J.-B. Teyssier and the speaker in 2018, in the geometric setting.

18:00 – 19:00 Fangzhou Jin (University of Duisburg-Essen)

Title: Absolute purity in motivic homotopy theory

Abstract: The absolute purity conjecture, introduced by Grothendieck, plays an important role in the cohomological studies of schemes. While this conjecture has been solved by Gabber for étale cohomology, its counterpart in motivic homotopy theory remains an interesting open question. In this talk we show that the rational motivic sphere spectrum satisfies absolute purity, and discuss some applications. This is a joint work with F. Deglise, J. Fasel and A. Khan.

August 21 (Fri)

15:30 – 16:30 Shun Ishii (Kyoto University)

Title: On a variant of the uniform boundedness conjecture for Drinfeld modules.

Abstract: Drinfeld modules are function field analogues of abelian varieties. Based on this analogy, Poonen conjectured the uniform boundedness conjecture for torsion of Drinfeld modules. In this talk, we prove a result on the uniform boundedness conjecture for 1-dimensional families of Drinfeld modules over finitely generated fields of generic characteristic. This is an analogue of Cadoret-Tamagawa's result on the uniform boundedness conjecture for 1-dimensional families of abelian varieties.

16:45 – 17:45 Wenbin Luo (Kyoto University)

Title: The continuity of χ -volume functions over adelic curves

Abstract: In the setting of Arakelov geometry over adelic curves, we introduce the χ -volume function and show some general properties. This article is dedicated to talk about the continuity of χ -volume function. By discussing its relationship with volume function, we prove its continuity around adelic \mathbf{Q} -ample \mathbf{Q} -Cartier divisors and its continuity in the trivially valued case. The study of the variation of arithmetic Okounkov bodies leads us to its continuous extension on arithmetic surfaces.

18:00 – 19:00 Jinbang Yang (Mainz University)

Title: Finiteness of logarithmic crystalline representations.

Abstract: Let K be an unramified p -adic local field and let W be the ring of integers of K . Let $(X, S)/W$ be a smooth proper scheme together with a normal crossings divisor. We show that there are only finitely many local systems over $X_{\bar{K}} \setminus S_{\bar{K}}$ which are restrictions of \mathbb{Z}_p -local systems over $X_K \setminus S_K$ with geometrically absolutely irreducible representations. The proof uses p -adic nonabelian Hodge theory and a finiteness result due Abe/Lafforgue. This is a joint work with Raju Krishnamoorthy and Kang Zuo.

19:00 – 19:15 Closing