6TH KYOTO+ WORKSHOP ON ARITHMETIC AND GEOMETRY

Date: October 25-October 29, 2024 (Chinese Local Time, GMT+8)

Place: West Building Room 108 (October 26, October 27) and Room 308 (October 25, October 28, October 29), School of Mathematics, Nanjing University, and Zoom.

Zoom Information:

Link: https://us04web.zoom.us/j/5922030941?pwd=V3NmZHRSZ3k3MlVvTDdOb1JFd0dzZz09 ID: 592 203 0941 PW: 7Zj4xf

Speakers:

Joseph Ayoub (University of Zurich) Paolo Cascini (Imperial College London) Huayi Chen (Westlake University) Jingren Chi (Chinese Academy of Sciences) Ning Guo (Harbin Institute of Technology) Christopher Hacon (University of Utah) Yongquan Hu (Chinese Academy of Sciences) Pengfei Huang (Max Planck Institute for Mathematics in the Sciences) Chun Yin Hui (The University of Hong Kong) Bao Le Hung (Northwestern University) Zhiyuan Li (Fudan University) Ngaiming Mok (The University of Hong Kong) Yukiyoshi Nakajima (Tokyo Denki University) Bertrand Toën (CNRS and Universite de Toulouse) Yupeng Wang (Peking University) Liang Xiao (Peking University) Junyi Xie (Peking University) Bin Xu (Tsinghua University) Daxin Xu (Chinese Academy of Sciences) Huanhuan Yu (Peking University) Shuji Saito (The University of Tokyo) Wei Zhang (Massachusetts Institute of Technology)

Organizers: Yuichiro Hoshi (RIMS, Kyoto), Zhi Hu (NJUST, Nanjing), Pengfei Huang (MPI, Leipzig), Yu Yang (RIMS, Kyoto), Runhong Zong (NJU, Nanjing)

See https://www.kurims.kyoto-u.ac.jp/~yuyang/confer/Kyoto-Nanjing-6th.html.

Titles and Abstracts

Joseph Ayoub (University of Zurich)

Title: Are there positive-degree operations on Weil cohomologies?

Abstract: We discuss a possible construction of degree-one operations on Betti cohomology. This is motivated by the observation that a nonzero positive-degree operation contradicts the existence of a motivic t-structure on Voevodsky 's triangulated category of motives.

Paolo Cascini (Imperial College London)

Title: On the MMP for algebraically integrable foliations *Abstract:* The purpose of this talk is to provide an overview of the recent developments on the Minimal Model Program for algebraically integrable foliations over a complex projective variety.

Huayi Chen (Westlake University)

Title: Harder-Narasimhan Filtration : a game-theoretic point of view

Abstract: In this talk, we explore the concept of Harder-Narasimhan filtration and semistability from a game-theoretic perspective. By introducing the notion of Harder-Narasimhan games, we provide a unified framework that not only proves the existence and uniqueness of the Harder-Narasimhan filtration but also offers a new interpretation of semi-stability conditions. As an application, we establish the existence and uniqueness of a coprimary filtration for finitely generated modules over a commutative Noetherian ring, interpreting it as a Harder-Narasimhan filtration. The presentation will also include a comparison with classical Harder-Narasimhan theory, highlighting the connections between the filtration and game theory. This is a joint work with Marion Jeannin.

Jingren Chi (Chinese Academy of Sciences)

Title: Geometry of affine Springer fibers and generalizations

Abstract: Affine Springer fibers are analogues of Springer fibers for the loop Lie algebras of reductive groups. They were first studied by Kazhdan and Lusztig and they have played important roles in various problems from geometric representation theory and automorphic representation theory. In this talk I will review the basic geometric properties of affine Springer fibers and report on recent work on some of their generalizations, including the group version and the mixed characteristic analogue.

Ning Guo (Harbin Institute of Technology)

Title: Gersten's injectivity for reductive principal bundles

Abstract: Gersten's injectivity conjecture predicts that a motivic type functor F is injective from a regular semilocal domain to its fraction field. In this talk, we discuss when F is a cohomology set $H^n(-, G)$ for a reductive group scheme G. When n = 1, it is the Grothendieck-Serre conjecture, that is, every generically trivial reductive principal bundle

on a regular semilocal ring is trivial. The Grothendieck-Serre conjecture is still widely open in the mixed characteristic case. I will introduce several known cases of this conjecture and Gersten's injectivity for tori when n > 1. The talk will include an analysis of principal bundles over affine lines and their rigidity over projective lines. Finally, we will talk about the purity of Brauer groups on smooth algebras over valuation rings. These are individual joint works with Ivan Panin, Fei Liu, and Yisheng Tian.

Christopher Hacon (University of Utah)

Title: Recent progress in the minimal model program for Kähler varieties *Abstract:* The minimal model program is an ambitious program that aims to understand the geometry of complex projective varieties (e.g. manifolds defined by polynomial equations). In this talk I will discuss some recent results and challenges encountered trying to extend the minimal model program to the context of Kähler varieties.

Yongquan Hu (Chinese Academy of Sciences)

Title: Finite length results for the mod p cohomology of GL_2

Abstract: In the mod p Langlands program for GL_2 , it is important to study the Hecke eigenspaces of mod p cohomology of Shimura curves. Inspired by the work of Breuil-Paskunas (around 2007), it is conjectured that such representations have finite length and a special shape. In this talk, I will explain the proof of the (expected) upper bound of the length under some reasonable hypotheses. This is joint work with Breuil, Herzig, Morra and Schraen.

Pengfei Huang (Max Planck Institute for Mathematics in the Sciences)

Title: Parahoric reduction theory of formal connections

Abstract: The celebrated reduction theory of formal connections is due to Hukuhara, Levelt, Turrittin, and Babbitt-Varadarajan, among others. In this talk, we will demonstrate the parahoric reduction theory of formal parahoric connections, which generalizes the aforementioned results and also extends Boalch's result for the case of regular singularities. As applications, we will establish the equivalence between extrinsic and intrinsic definitions of regular singularities, as well as a parahoric version of Frenkel-Zhu's Borel reduction theorem for formal connections. This is based on a recent joint work with Z. Hu, R. Sun, and R. Zong.

Chun Yin Hui (The University of Hong Kong)

Title: Weak abelian direct summands and irreducibility of Galois representation Abstract: Let ρ_{ℓ} be a semisimple ℓ -adic representation of a number field K that is unramified almost everywhere. We introduce a new notion called weak abelian direct summands of ρ_{ℓ} and completely characterize them, for example, if the algebraic monodromy of ρ_{ℓ} is connected. If ρ_{ℓ} is in addition *E*-rational for some number field *E*, we prove that the weak abelian direct summands are locally algebraic (and thus de Rham). We also show that the weak abelian parts of a connected semisimple Serre compatible system form again

such a system. Using our results on weak abelian direct summands, when K is totally real and ρ_{ℓ} is the three-dimensional ℓ -adic representation attached to a regular algebraic cuspidal automorphic, not necessarily polarizable representation π of $GL_3(\mathbb{A}_K)$ together with an isomorphism $\mathbb{C} \xrightarrow{\sim} \overline{\mathbb{Q}}_{\ell}$, we prove that ρ_{ℓ} is irreducible. We deduce in this case also some ℓ -adic Hodge theoretic properties of ρ_{ℓ} if ℓ belongs to a Dirichlet density one set of primes. This is a joint work with Gebhard Böckle.

Bao Le Hung (Northwestern University)

Title: The generic Fontaine-Laffaille spectral Hecke algebra

Abstract: The mod p categorical Langlands program conjectures an embedding of the (derived) category of mod p smooth representations of $GL_n(\mathbb{Q}_p)$ into a suitable (derived) category of quasi-coherent sheaves on the Emerton-Gee stack X classifying n-dimensional representations of the absolute Galois group of \mathbb{Q}_p . In particular, this predicts an identification of the derived endomorphism of certain standard representations of $GL_n(\mathbb{Q}_p)$ and the derived endomorphism of specific sheaves on X. I will explain the computation of the cohomology algebra of the latter in the most accessible case, where the sheaf is expected to be a line bundle on a well-understood locus (the Fontaine-Laffaille locus). This is joint work in progress with my student Junho Won.

Zhiyuan Li (Fudan University)

Title: Rigidity of pointed families of Hyper-Kähler varieties

Abstract: In this talk, I will talk about the rigidity of pointed families of hyper-Kähler varieties. It was asked if there are only finitely many isomorphic classes of pointed families of hyper-Kähler varieteis.

I will first prove this has an affirmative answer for K3 surface and show this fails in higher dimensional cases. Then I will give the correct version of this problem and confirm this for hyper-Kahler varieties with terminanl singularities. This is a joint work with L. Fu, T. Takamatsu and H. Zou.

Ngaiming Mok (The University of Hong Kong)

Title: Journey to Number Theory through Complex Geometry

Abstract: The speaker has long been interested in applications of complex geometry to number theory, and will trace the trajectory of his involvement concerning (1) abelian schemes over complex function fields, (2) commutants of Hecke correspondences on bounded symmetric domains Ω and (3) functional transcendence concerning Shimura varieties and more generally $X_{\Gamma} := \Omega/\Gamma$ for arbitrary lattices Γ .

For (1) we recall first results of Mok (1991) and Mok-To (1993) concerning the finiteness of Mordell-Weil groups of universal abelian varieties \mathbf{A}_{Γ} without fixed parts over modular function fields $K = \mathbb{C}(\overline{X}_{\Gamma})$ and applications by Mok-Ng of the underlying methodology of our differential-geometric approach to the study of the Betti map, including finiteness results on points of Betti multiplicities ≥ 2 of a section $\sigma \in \mathbf{E}(\mathbb{C}(\overline{X}))$ of infinite order in the case of an elliptic scheme over a quasi-projective curve, a result obtained by Corvaja-Masser-Zannier (2022), which was rendered effective by Ulmer-Urzúa (2021). We

$6 \mathrm{TH} \mathrm{KYOTO^{+}}$

will give a differential-geometric proof of the latter by Mok-Ng (2023). For (2) we recall the reduction of a problem of Clozel-Ullmo (2003) concerning commutants of Hecke correspondences which reduces to a conjecture characterizing measure-preserving germs of holomorphic maps on Ω , and the solution of Mok-Ng (2012) using results in CR geometry due to S. Webster and X. Huang. For (3) we will discuss the Ax-Schanuel theorem of Mok-Pila-Tsimerman (2019) for Shimura varieties and its applications to the study of rational points, notably to the proof of the uniform Mordell Theorem of Dimitrov-Gao-Habegger (2022) for number fields. We will discuss a uniformization theorem of Chan-Mok (2022) for projective varieties uniformized by algebraic subvarieties of bounded symmetric domains which imply a characterization of bi-algebraic subvarieties in compact quotients $X_{\Gamma} = \Omega/\Gamma$ of bounded symmetric domains for not necessarily arithmetic lattices, and furthermore a proof of the analogue of Ax-Lindemann theorem for X_{Γ} .

Yukiyoshi Nakajima (Tokyo Denki University)

Title: p-adic analogue of Rapoport-Zink-Nakayama's l-adic weight spectral sequence *Abstract:* In this talk we construct a p-adic weight spectral sequence for a proper SNCL(=simple normal crossing log) scheme) over a family of log points by using the log convergent topos of the log scheme and the convergent topos of the underlying scheme of it. This spectral sequence is a p-adic analogue of Rapoport-Zink-Nakayama's l-adic weight spectral sequence.

Bertrand Toën (CNRS and Universite de Toulouse)

Title: Derived techniques for foliations in arbitrary characteristics *Abstract:* In this talk I will report on recent, as well as less recent, results on algebraic foliations, in arbitrary characteristics, obtained using techniques from derived algebraic geometry. (Joint with Vezzosi).

Yupeng Wang (Peking University)

Title: A stacky *p*-adic Riemann-Hilbert correspondence on Hitchin-small locus *Abstract:* Let X be a smooth rigid analytic variety over C with a liftable smooth (or semi-stable) formal model over \mathcal{O}_C . We will establish an equivalence between the moduli stack of Hitchin-small de Rham local systems on X_v and the moduli stack of Hitchin-small integrable connections on $X_{\text{ét}}$, based on a new period sheaf with connection on X_v . This is a joint work with Yudong Liu, Chenglong Ma, Xiecheng Nie and Xiaoyu Qu.

Liang Xiao (Peking University)

Title: Reduction of 2-dimensional triangulline representations over the halo region, and Emerton–Gee stack

Abstract: Associated to each finite-slope eigenform, there is a Galois representation, whose local component at p is known to be triangulline. Conversely, a fixed residual Galois representation, the space of all its possible triangulline lifts typically contains a dense subset

coming from the finite-slope eigenforms (of various levels). So understanding the U_p eigenvalues of modular forms is essentially equivalent to understanding the triangulline representations. Previously, we have worked mostly on the "automorphic side" by studying the *p*-adic valuations of U_p -operators, known as *p*-adic slopes of modular forms. In this talk, I will report on a joint work in progress with John Bergdall, Brandon Levin, and Yong-Suk Moon, in which we hope to recover the spectral halo conjecture of Coleman– Mazur using tools from the Galois side by studying the reduction of triangulline (ϕ , Γ)modules, and at the same time relate this to the reduced fiber of Emerton–Gee stack. The technical novelty is that, we hope to find a way to bypass Kedlaya's slope filtration theory to prove the etaleness of this family of (ϕ , Γ)-modules.

Junyi Xie (Peking University)

Title: Algebraic dynamics and recursive inequalities

Abstract: We get three basic results in algebraic dynamics: (1). We give the first algorithm to compute the dynamical degrees to arbitrary precision. (2). We prove that for a family of dominant rational self-maps, the dynamical degrees are lower semi-continuous with respect to the Zariski topology. This implies a conjecture of Call and Silverman. (3). We prove that the set of periodic points of a cohomologically hyperbolic rational self-map is Zariski dense.

In fact, for every dominant rational self-map, we find a family of recursive inequalities of some dynamically meaningful cycles. Our proofs are based on these inequalities.

Bin Xu (Tsinghua University)

Title: Arthur's conjectures for GSp(2n) and application to the zeta functions of Siegel modular varieties

Abstract: The Eichler-Shimura theory describes the zeta functions of modular curves in terms of L-functions of weight 2 cuspidal modular forms. Langlands, Kottwitz initiated a program generalizing this theory to Shimura varieties through the stable trace formula. In their program, Arthur's conjectures on automorphic representations play an important role. In this talk I will report on the current status of Arthur's conjectures for GSp(2n) and discuss the potential application to Siegel modular varieties after the work of Morel.

Daxin Xu (Chinese Academy of Sciences)

Title: On new developments in the *p*-adic Simpson correspondence

Abstract: The *p*-adic Simpson correspondence aims at establishing an equivalence between generalized representations and Higgs bundles over a *p*-adic variety. In this talk, we will present the background, recent developments in the subject, and survey some open problems. The talk is partially based on my joint work with Ben Heuer.

Huanhuan Yu (Peking University)

Title: A refinement of the Coherence Conjecture of Pappas-Rapoport

Abstract: The Coherence Conjecture of Pappas-Rapoport, proven by X. Zhu, establishes a relationship between the geometry of different affine Schubert varieties, notably providing dimension equalities for the sections of line bundles on (unions of) affine Schubert varieties in different affine partial flag varieties. In this talk, I will present a refinement of this conjecture, demonstrating that these spaces of global sections are isomorphic as representations of certain group. I will also discuss applications of this result, particularly to affine Demazure modules, accompanied by concrete examples. This is joint work with Jiuzu Hong.

Shuji Saito (The University of Tokyo)

Title: A pro-cdh topology and motivic cohomology of schemes

Abstract: We address a conjecture of Beilinson predicting the existence of a motivic complex, which is a complex of Zariski sheaves of abelian groups on schemes. The most important property is its relation to algebraic K-theory of schemes via an Atiyah-Hirzebruch spectral sequence. In this talk, we propose a construction of a motivic complex using a new Grothendieck topology, pro-cdh topology, and state a comparison theorem of the procdh motivic complex and another motivic complex also recently constructed by Elmanto and Morrow. This is a joint work with Shane Kelly.

Wei Zhang (Massachusetts Institute of Technology)

Title: Some results on algebraic cycles and L-functions

Abstract: The conjecture of Birch and Swinnerton-Dyer predicts that, for elliptic curves over number fields, the data from congruence equations mod n should actually encode deeper information on the solutions in rational numbers. We will discuss a generalization of the question to certain high dimensional varieties (such as the product of several elliptic curves, or certain Shimura varieties), where, instead of rational points, we look for algebraic cycles modulo suitable equivalence relations (rational equivalence, Abel-Jacobi or its *p*-adic version). I'll then report some (old and new) results on the generalization of B-SD conjectures by Beilinson, Bloch, and Kato for such arithmetic varieties, relating algebraic cycles to *L*-functions (including some *p*-adic variants).